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ELECTRICAL CIRCUITS AND MACHINERY

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VOLUME I
CONTINUOUS CURRENTS

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PREFACE

The series of three texts on electrical circuits and machinery, of which this volume is the first, is designed primarily for the use of students in engineering schools. The material of this first book, on continuous currents, has been tried out in engineering classes for several years, and seems to be about what the average engineering student wants, whatever may be the branch of engineering in which he expects to specialize. Not only the material, but also the method of presentation, is such that the ordinary college Physics course is an adequate preparation; in fact, for most of the book, an ordinary high-school preparation should be sufficient.

Even though a knowledge of the electron theory is not necessary for the solution of electrical problems, the student familiar with this theory, even in an elementary degree, will generally find his work far more interesting. Many of the phenomena ordinarily given as empirical facts seem reasonable, and even predictable, from the electron viewpoint. This viewpoint is very necessary to the man who anticipates going into the modern development of radio and vacuum tubes.

The fundamental principles of changing magnetic fields and the resulting induced voltages, which underlie the operation of practically all electrical apparatus, seem sometimes to be so loosely presented that even eminent engineers doubt their universal applicability. The authors have, therefore, given more than a normal amount of space to the consideration of changing magnetic fields and currents and have analyzed these effects from different viewpoints, wherever possible.

Circuits, both electric and magnetic, have been treated in sufficient detail for the solution of all ordinary problems; for more complicated networks, books dealing especially with such problems must be consulted. The authors have used oscillograms to an extent which is more than usual for a book of this character, feeling that the average student grasps the significance of an oscillogram more quickly and easily than he does that of a formula. Armature reaction and commutation have been dealt with as completely as is necessary for any but a designer of electrical machinery, and the oscillograms show easily and exactly the need for commutating poles and compensating windings in certain types of motors.

G. Mr. Laurence H. Cleveland

The films which show the current in armature coils during commutation are to be especially mentioned as illustrations of the value of the oscillograph in making a difficult subject easy to understand.

The questions of choice of machine types for various services, voltage regulation, effect of temperature and ventilation upon capacity, batteries, automobile equipment, and care and operation of electrical machinery have been dealt with to an extent which seems suitable and reasonable for all engineering students, except those who are preparing themselves particularly for electrical engineering. Even to these the book may well serve as an interesting introduction to more comprehensive texts.

J. H. M.
F. W. H.

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CONTINUOUS CURRENT CIRCUITS AND MACHINERY

CHAPTER I

ELECTROSTATICS

1. Nature of Electricity.—Everyone is more or less familiar with elementary experiments having to do with electrically charged bodies. Fur, if rubbed on a dry day, crackles and gives off minute sparks; a glass rod rubbed with a cloth becomes electrified and will attract small bits of paper, cotton, etc.; owing to wind friction and other causes, clouds become intensely electrified and are able to break down the insulating strength of the air and produce sparks thousands of feet long.

In what way does an electrified body, or an electrically charged body, differ from one in the uncharged, or neutral, state? A reasonable answer to this question is found in the modern conception of the constitution of matter.

2. Electrons.—It has been firmly established, as the result of a great deal of experimental work, that every atom of matter is charged with minute particles, or "somethings," of negative electricity, called electrons. An electron, when detached from the atom of matter with which it was associated, shows none of the properties of ordinary matter. It does not react chemically with other electrons to produce some new substance; moreover, all electrons are similar, and act in precisely the same way whether extracted from atoms of hydrogen, mercury, iron, or any other substance. It seems that the *electron is nothing but electricity.* It is, however, definite in amount, always being exactly the same, and is generally believed to be the smallest possible quantity of electricity, i.e., the electron cannot be subdivided.

The commonly accepted constants of the electron are: radius $=2\times10^{-13}$ cm.; mass $=8.8\times10^{-28}$ grams; charge $=1.59\times10^{-19}$ coulomb. The mass of the electron depends upon the velocity with which it is moving; the value given holds good only if the electron is traveling at velocities considerably less than the velocity of light, say with a velocity of less than 10^9 cm. per second.

For many years it has been the custom, or convention, to speak of positive electricity and negative electricity; and from this standpoint the electron is considered *negative* electricity.

3. Charged Body.—From the electron viewpoint, the internal, or central, part of the structure of the atom itself, whatever it may be, is always charged electrically positive. In the normal state there are enough electrons grouped about the central part of the atom to just neutralize this positive charge. The normal atom, therefore, acts like an uncharged body, not because it has no electrical charge associated with it, but because it has just as much negative charge as it has positive charge, the two charges neutralizing one another, in so far as action of the atom on other atoms or bodies is concerned. In Fig. 1, the circle represents a molecule of hydrogen, the nucleus, or central portion of which, is positively charged; the small circles with the minus sign in them represent the electrons associated with the normal hydrogen molecule.

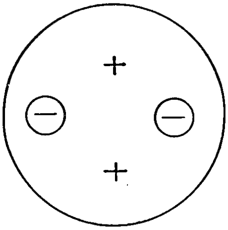


FIG. 1.

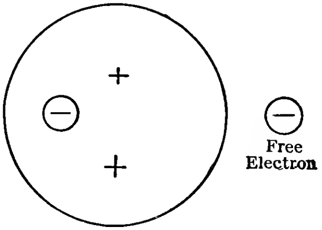


FIG. 2.

FIG. 1.—A crude conventional representation of a normal hydrogen molecule; the two electrons associated with the nucleus are undoubtedly in rapid orbital motion of some kind, not stationary as indicated in the diagram.

FIG. 2.—If one electron is removed from the normal hydrogen molecule, as indicated here, the removed electron is said to be "free," and what is left of the hydrogen molecule is necessarily charged now positively; such a gas molecule is said to be "ionized."

If one electron is removed from the molecule by some means or other (represented in Fig. 2) the balance between positive and negative charge is destroyed; an excess of positive charge exists on the molecule and the molecule is positively charged. The electron which has been removed from the molecule constitutes a negative charge. If the electron is allowed to go back to the molecule, the balance of charge is restored and the molecule is again uncharged, or neutral.

A positively charged body, therefore, is one that has been deprived of some of its normal number of electrons; a negatively charged body is one that has acquired more than its normal number of electrons. Thus, when a piece of sealing wax is rubbed with dry flannel, the wax becomes negatively charged and the flannel becomes positively charged. The friction between the wax and the flannel must have rubbed some of the electrons off the flannel molecules and left them on the surface of the wax.

The extra electrons on the wax are attracted by the deficient molecules of the flannel (positive and negative charges attract each other) and if the flannel and wax are left together after being rubbed they soon lose their charges; the molecules of the flannel regain their proper number of electrons.

4. Number of Electrons Removable from an Atom.—Although there may be a great number of electrons associated with an atom or molecule, it is generally difficult to remove more than one; in a body which is positively charged, most of the atoms are neutral, having their proper complement of electrons; others have had one electron removed. If but few of the atoms of a body have had an electron removed, the body has a small charge; the more highly the body is charged, the more deficient atoms there are in it.

From this viewpoint it seems that the amount of charge on a body should be counted; the charge consists of discreet things. Instead of saying that a body has a certain amount of negative electricity on it, we might more reasonably say that a certain number of electrons have been deposited on it.

5. Electric Fields.—If a light substance, such as a pith ball, is touched to a charged body, it will become similarly charged and, as like charges repel one another, the pith ball will be repelled from the charged body. If the body to which the pith ball is touched is positively charged, then a few electrons pass from the ball to the body, leaving both positively charged; if the body is negatively charged, a few of the excess electrons on the body pass to the pith ball. By experimenting, it will be found that the repulsive force between the pith ball and the original charge exists even when there is considerable distance between the two. The space surrounding a charged body is evidently under some kind of strain, which enables it to act upon another body with a force, attractive or repulsive, according to the relative polarities of the two charges. This space surrounding a charged body, in which another charged body is acted upon by a force tending to move it, constitutes an *electric*, or *electrostatic*, *field*.

Such an electric field surrounds every charged body. It really extends to infinity in all directions from the charged body; but as the force becomes very small as the distance is increased (force varies inversely as the square of the distance), it is generally considered that the electric field due to a charge extends but a short distance from the charge.

6. Electric Fields Represented by Lines.—In diagrams, the electric field surrounding a charge is most easily depicted by drawing lines from the charged body into surrounding space. The direction of the lines, if they are properly drawn, gives the direction of the electric force; and the relative closeness of the lines in various parts of the diagram shows the relative strengths of the field at these points, the closer the lines the more intense the field. A line of force originating on a positive charge is properly shown ending on an equal negative charge. In diagrams it is not always convenient to represent them so; they may be shown as discontinuous. It must not be supposed, however, that the electric force itself is discontinuous; it always continues from a positive charge to a negative charge.

Figure 3 shows how lines may be used to represent the electric field; it shows a positively charged metal ball supposedly far enough away from other bodies to be considered as by itself. The lines of force originate on the surface of the sphere and extend as radii in all directions. The arrow head on the lines indicates the direction in which a *positive* charge would be urged if placed in that part of the field.

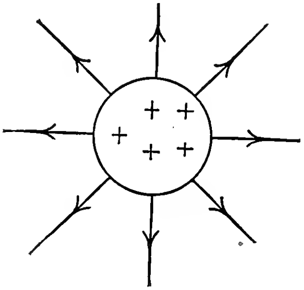


FIG. 3.

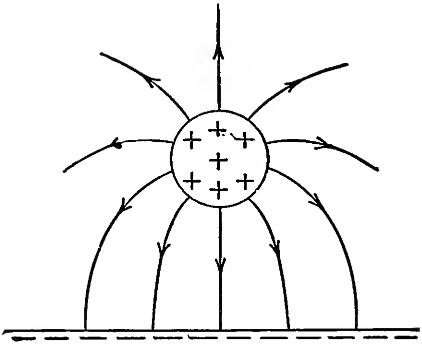


FIG. 4.

FIG. 3.—Around a positively charged sphere (one having a deficiency of electrons) the electric field is properly represented by radial lines which extend in all directions.

FIG. 4.—Actually the electric field around a charged sphere is not symmetrical as shown in Fig. 3 but, due to proximity of other bodies, is more or less unsymmetrical; if the sphere is near the earth's surface the lines emanating from it will all bend around and finally end on the earth's surface, as indicated here.

The lines are closest together at the surface of the sphere, indicating that the force is greatest at this point, a fact easily proved experimentally. Although the lines are shown as discontinuous, ending in uncharged space, each line really extends in some direction until it encounters a negative charge. In the case of a metallic sphere, suspended in the air distant from other bodies, the lines should all be shown as ending on the earth's surface as suggested in Fig. 4. Figure 5 represents the electric field between two parallel metallic plates, one of which has been charged positively and the other negatively. Moreover, as all the lines originating on the positive plate are shown as ending on the negative plate, it shows that the two plates have been given equal charges, that is, as many extra electrons have been deposited on one plate as have been taken away from the other. The field is properly shown as very intense between the two plates, weaker beyond the edges, and very weak in the space not included between the two plates.

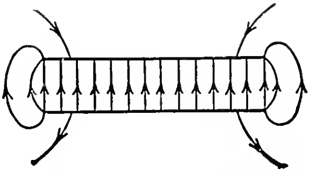


Fig. 5.—When two plates, having equal and opposite charges (as many extra electrons put on one as have been taken from the other) are placed close together the field directly between them is uniform and comparatively intense; there is a weak field outside, the lines of force here however being curved and non-uniform in distribution.

7. Induced Charges.—Suppose a charged metal ball is brought close to another conducting body, as a metal rod, the rod being uncharged. Experiment shows that as the rod is brought close to the brass ball, the rod itself becomes charged in a peculiar way. If the ball is positively charged, that end of the rod nearest to it becomes charged negatively and the farther end becomes positively charged, as indicated in Fig. 6. As a whole the rod is not charged, there being as much negative charge as there is positive charge. These charges which have been produced in the rod through the action of the charged ball are called *induced* charges.

Charges induced on a body are always double in kind; as much positive charge appears as does negative. However, if a wire having one end connected to the earth is touched to the end of the rod marked C, the positive charge which has been induced at this end of the rod will "run off to the earth," and when the wire is removed there will be left on the rod only the negative charge, as shown in Fig. 7. If the charged sphere is then taken away, the negative charge will distribute itself uniformly over the rod,

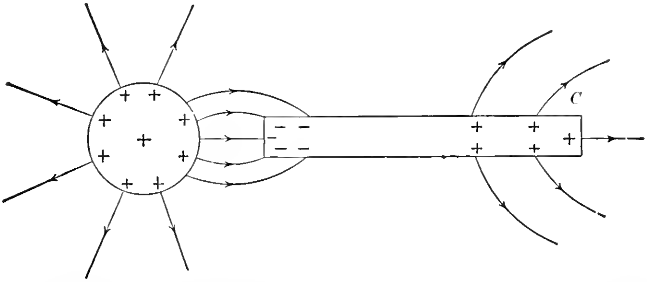


Fig. 6.—If an uncharged metal rod is brought in the vicinity of a positively charged sphere the rod will show what are called "induced charges"; although uncharged as a whole the rod exhibits a negative charge on that end near the sphere and an equal positive charge on the other end. In case the sphere were charged negatively the induced charges on the rod would be reversed in sign.

the lines of electric force from the rod now ending in the earth, as shown in Fig. 8.

8. Induced Charges from the Electron Viewpoint.—As will be explained later, the electrons in a metallic conductor are more or less free to pass from one atom of the substance to another; they are continually moving around in the complex aggregate of atoms comprising the metal. When the rod of Fig. 6 is brought into the neighborhood of the charged ball, the electric field due to the charge on the ball acts on the free electrons of the rod, attracting them. Hence, the free electrons of the rod tend to congregate at that end of the rod which is nearer the ball; they constitute the negative charge at this end of the rod.

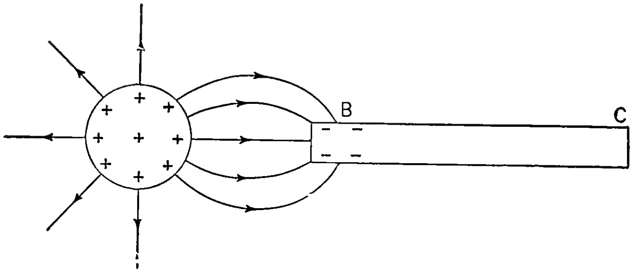


Fig. 7.—When a wire, connected to the earth on one end, has its other end connected to end *C* of the rod of Fig. 6 the free positive charge will "run off" to earth, in the ordinary explanation of induced charges, thus leaving on the rod the bound negative charge at the end nearer the charged sphere.

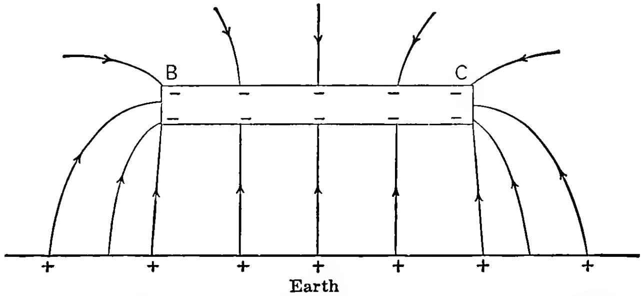


Fig. 8.—If the charged sphere is now removed the negative charge will spread itself over the rod, and lines of force will reach out from the rod to the earth. The maximum density of charge on the rod will occur at the corners of the rod, and here the electric field will be most intense.

But if the rod was uncharged before coming into the influence of the charged ball, there must have been just enough electrons on it to neutralize the positive charges of the atoms. If more than a proper portion of the electrons gather at one end of the rod there must necessarily be a shortage of them at the other end. *This shortage of electrons at the end C of the rod constitutes the positive charge at this end.*

When the end C is grounded, the positive atoms of the rod cannot leave the rod and go to the earth, but electrons from the earth can run up into the rod, and they do so, being attracted by the deficient atoms at C. These electrons from the earth appear in sufficient quantity to make the atoms at C neutral. When the wire connecting the rod to the earth is removed and the charged ball is also removed, the rod has on it a negative charge, the quantity of charge being equal to the number of electrons which came from the earth into the rod.

9. An Essential Difference between Positive and Negative Charge.—As before stated, the electrons from all substances are the same; the electrons have none of those qualities by which we distinguish and classify matter. It is possible to have electrons in space entirely devoid of matter; a negative charge can exist in a perfect vacuum.

The question may be raised: How can it be a perfect vacuum if there are electrons present? By a vacuum we mean a space in which there is no material substance, solids which can be bodily removed, liquids which can be poured out, or gases which can be pumped out. A glass vessel which has been evacuated as perfectly as is possible with modern pumping methods may nevertheless be filled with millions of millions of electrons.

From our conception of the positive charge, however, it is evident that a positive charge must always be associated with matter; in fact, the smallest available positive charge must be thought of as an atom from which an electron has been removed. If in a glass bulb, supposedly evacuated, it can be shown that under some circumstances positive charges exist, the vacuum is only partial; to the same extent that positive charges occur in the supposedly vacuous space, must matter of some kind (generally gas) be present.

CHAPTER II

CURRENT, CONDUCTORS, AND INSULATORS

10. The Electric Current.—The electric current is familiar to everyone. It manifests itself in various ways, by generating heat and light, by producing mechanical forces, as in magnets and motors, by producing chemical changes, etc.

Older conceptions of the electric current made it a peculiar fluid of some kind; others made it consist of two fluids with different properties, flowing in opposite directions. From the electron viewpoint, the conception of the electric current is easy to comprehend and enables one to give a fairly logical explanation of the various actions of the current.

11. Nature of the Electric Current—*An Electron in Motion Constitutes an Electric Current.*—The amount of electricity on one electron is so small that the current produced by one electron in motion would not be detectable by the finest and most sensitive current-measuring instrument. To produce currents of the magnitude occurring in everyday experience requires the motion of electrons measured in billions of billions per second.

An ordinary incandescent lamp requires a current of about one ampere; such a current, since an ampere is the passage of one coulomb per second, requires that about 10^{19} electrons flow past any point in the circuit each second. This large number per second might be brought about by a comparatively few electrons moving rapidly or by a great many moving more slowly. Contrary to what one would naturally think, the progressive movement of the electrons is very slow. To produce a current of one ampere in a copper wire one millimeter in diameter requires that the average velocity of the electrons along the wire be only about 0.01 cm. per second.

Although the progressive motion of the electrons is very slow, as indicated above, it must not be thought that the actual velocity of the electrons is small. When no current is flowing in a wire, the electrons have a haphazard motion, due to the thermal agitation of the atoms (or molecules), which gives them a velocity of about 1×10^7 cm. per second. When current flows, the required progressive velocity of the electrons is only a fraction of a centimeter per second. Thus an accurate concept of the electric current shows it to be an inappreciable "drift" of the electrons which have, due to temperature effects, heterogeneous velocities millions of times as great as the velocity of drift which constitutes the current.

The reason for the slow progressive motion of the electrons is to be seen in the tremendous number of collisions they have with the molecules of the substance. A given electron, acted upon by a potential gradient in the wire carrying current, accelerates very rapidly, and would acquire tremendous velocities if it did not continually collide with the more massive molecules. The mean free path of the free electrons in a copper wire is so small between successive collisions, and the potential difference over this short path is so small, that the electron gains in the average only extremely small velocity along the conductor.

Suppose that we wanted to measure the rate of flow of people past a given point in a large city; the unit of flow might be 100,000 persons per hour. At any time there will be people going in all directions, some uptown, some downtown, and some crosstown. In the morning a million people pass a certain point where the flow is to be ascertained. If 200,000 move uptown, and 800,000 move downtown, the net flow is 600,000 people, or at a rate of 6 units downtown. At noon time, a million people pass the same place, let us suppose; 300,000 move uptown, 400,000 move downtown, and 150,000 move crosstown west and 150,000 move crosstown east. The net flow is now 100,000 people downtown, or at a rate of 1 unit downtown. Some of the people would be moving rapidly, others going more slowly and some might, at times, be standing still.

Or consider a crowd through which a number of fleet runners are attempting to force their way in a given direction. Continually colliding with the members of the crowd, they dodge back and forth so that their *progressive velocity* is relatively low, although they are actually running very rapidly.

The pictures suggested probably give one a reasonable idea of the motion of electrons in a conductor carrying current; it is, of course, too simple, because of the immense number of electrons in a conductor and the tremendous number of collisions occurring between the electrons and molecules. When a conductor is carrying no current, the motion of the electrons resembles that of the individuals in a stationary crowd; there is a deal of agitation among the electrons, but they, on the whole, show no progress along the conductor.

12. Electromotive Force.—Suppose a copper rod, having in itself the heterogeneously moving electrons suggested above, is connected at its ends to a battery, as shown in Fig. 9. The end A of the rod becomes positive with respect to end B, and the electrons, instead of moving backwards and forwards to the same extent, progress slowly towards A. When they arrive at A they leave the copper rod, move down the connecting wire, through the battery, through the other connecting wire, and so back to the rod. As long as the circuit remains closed, as shown, the electrons will continue to move around the circuit, bounding backward, forward, and across the conductor, but on the whole, progressing gradually around the circuit; this progression of the electrons constitutes the electric current. The cause of the flow is the battery; it holds one end of the rod positive with respect to the other, and so maintains the flow of electrons. The maintenance of this difference of electric pressure (or difference of potential) across the rod is due to chemical changes going on inside the battery.

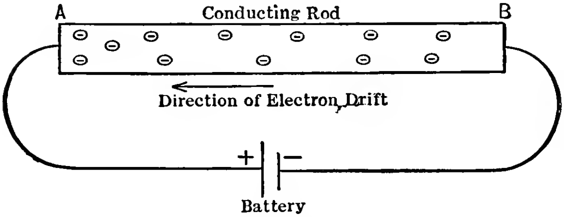


FIG. 9.—If one end of a conductor is made positive with respect to the other, by connecting a battery or generator, the heterogeneously moving electrons will begin to drift towards the positive end of the rod, this constituting an electric current through the rod.

A piece of apparatus which has the ability to maintain one of its terminals at a higher potential than the other, even though current is allowed to flow through it, is said to develop an *electromotive force.* As sources of electromotive force for the production of currents on a commercial scale, we have only the thermo-couple, the ordinary battery and the electric generator. The battery depends upon chemical action for maintaining its difference of potential, and the generator depends upon the conductors of its armature being driven through the magnetic field produced by its field poles.

13. Electromotive Force and Difference of Potential.—It is well to distinguish between electromotive force and difference of potential. Thus, two brass balls, one charged positively and the other negatively, have a difference of potential between them, and they will, if connected by a wire, cause a momentary flow of current through the connecting wire; when a sufficient number of electrons have passed from the negatively charged ball to neutralize the positive charge on the other, the current will cease. There is no action taking place which tends to maintain the difference of potential between the balls; such a combination does not generate an electromotive force (hereafter abbreviated e.m.f.).

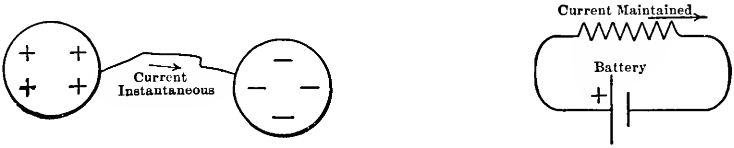


FIG. 10.—Indicating a difference between electro-motive force and potential difference. The two charged balls show no tendency to keep the current flowing after the discharge has passed, but the battery, by virtue of internal chemical action, will maintain its current for a long time.

In the case of the battery or generator, however, when the two terminals are connected by a wire, a current flows and continues to flow until the battery is worn out or the generator is stopped. Such devices develop, or generate, an e.m.f. These ideas are depicted in Fig. 10. We may say, in general, that any device which is capable of sustaining an electric current through a closed circuit (of which circuit the device is itself a part) generates an e.m.f. With our present limited knowledge of electricity, it is useless to try to give an exact explanation of electromotive force, because we know nothing about the nature of the force. In the same way, we cannot explain in simple terms the force of gravitation.

An analysis of the above distinction between electromotive force and potential difference shows that, to develop an e.m.f., a device must be capable of absorbing energy in some form and changing it over into electrical energy; the device must be capable of energy transformation and have available, energy in some form other than electrical. Of course, in making this statement, we realize that the so-called chemical energy of the battery is probably electrical in its ultimate nature, but the statement supposes a difference between chemical and electrical energy.

14. Direction of Flow of Current.—It has been accepted *as convention* that, in a wire connecting the poles of a battery, the current flow is from the positive pole of the battery to the negative; but, by reference to Fig. 9, it is evident that in the connecting wire the electrons flow from the negative pole of the battery to the positive. Hence, it must be remembered that although we shall, *for the sake of convention,* talk of the current flowing from the positive terminal to the negative terminal of a battery or generator, the electrons (which really are the current) *are flowing in the opposite direction, and there is nothing flowing in the direction assumed for the current.*

15. Conductors and Insulators.—Roughly speaking, a conductor is a body that readily permits the passage of an electric current, and an insulator is a body that offers a very high resistance to its passage. There is, however, no sharp distinction between conductors and insulators; a material which, for some cases, would be regarded as an insulator would, in other circumstances, be regarded as a conductor.

For example, if strips of various substances are made up, all of equal cross-section and length, and connected to the terminals of a generator, results of current measurements might be as follows:

Strip made of copper.	100 units of current
Strip made of iron.	16 units of current
Strip made of mercury.	1.6 units of current
Strip made of carbon.	$\frac{1}{2500}$ unit of current
Strip made of selenium.	$\frac{1}{40.000.000.000}$ unit of current

All of these substances are called conductors. If other strips, made of mica, glass, gutta percha, etc., are used, some current would flow, but it would be so small that all ordinary measuring instruments would fail to detect it; these strips would be called insulators.

Also, some substances which are good insulators at low temperatures may be fair conductors at high temperatures. Glass is the most striking illustration of this change of character with change of temperature; at ordinary temperatures it ranks high among the very best insulators, but if it is heated in some way to a red heat it becomes a fair conductor and will permit the passage of enough current to cause it to melt.

16. Difference between Conductors and Insulators from the Electron Viewpoint.—When a conductor is carrying an electric current, the electrons throughout the substance of the conductor are moving gradually along through the substance of the conductor. Now, in a solid body, such as a metallic conductor, the atoms or molecules comprising the substance are practically fixed in position. They are not actually stationary in space at ordinary temperatures, of course; as a matter of fact, the atoms have an irregular to-and-fro motion similar to that of the electrons. *However, there cannot be a progressive motion of the atoms as there may be of the electrons.* The reason for this statement is more or less evident. Suppose that a copper wire is fastened to the terminals of a battery and that current is flowing, as indicated in Fig. 9. The electrons move all the way around the circuit, through the wire, connections, solution in the battery, etc.

As the atoms of copper are charged positively after an electron has left them, it might seem that as the electrons move from B to A through the wire, the atoms would move from A to B, then into and through the battery and so back to the wire. But the atoms are the real substance of the wire, and hence if the atoms should progress one way or the other, it would result in *the copper itself* being carried from one end of the wire to the other and then through the battery. This state of affairs is not possible in solid bodies like metals; it would result in the mixing of metals wherever a current left one metal and went into another.

In chemical solutions, e.g., copper sulphate in water, the salt molecule breaks up into two parts, one of which has one electron more than its proper number, the other part lacking one electron. The two parts of the molecule are called ions; the metallic ion (in the above case, copper) lacks one electron and so is charged positively. If a current is passed through such a solution, the metallic ion does move through the solution and is carried from the solution to one of the wires by which the current is led into the solution. Here the copper itself is transported by the current, and we have the process of electro-plating.

From what has been said, it follows that if the molecules of a body cling to the electrons so tightly that none of them are free to move away from the molecule there can be no current in such a substance. As long as the molecule keeps all its electrons, it remains electrically neutral, and so has no tendency to move when in an electric field. *This is the essential difference between insulators and conductors;* in the one the electrons cannot move from the atom or molecule, and in the other the electrons are perfectly free to leave the atom.

17. Disruptive Strength of an Insulator.—If the above idea is kept in mind, the possibility of break-down of an insulator, due to high voltage, becomes apparent. For low voltage, the force tending to move the electron is not sufficient to break it loose from its atom. But it is reasonable to believe that, if the voltage gradient is made sufficiently high, any atom can be forced to let go of one electron, and such is the case. Such excellent insulators as glass and mica break down and carry current when a great enough voltage is employed. The only real insulator we know of is a perfect vacuum in which there are no free electrons; in such a vacuum, since there are no atoms or molecules to break up and yield current-carrying portions, evidently no current can flow. Experiment proves the point very well; a 100,000-volt pressure will break down 10 inches of air at ordinary pressure and temperature, but can cause no breakdown, or passage of current, through a gap of a fraction of an inch in vacuum.

18. Effect of Temperature on the Disruptive Strength of an Insulator.—Imagine a good insulator heated by some outside source of power. The rise in temperature increases the to-and-fro motion of its molecules, with the result that the collisions between the various molecules become more frequent and violent as the temperature is raised. As these collisions occur, the resulting disturbances in the molecular structure tend to weaken the hold of the molecule on its electrons. Hence, if an electric force is impressed and maintained as an insulator is heated, the combination of electric force and weakening of the molecular holding power will finally result in some electrons leaving their molecules; the electric force will then urge them along through the substance of the insulator, resulting in a small current. This would be interpreted by the man testing the insulator as a weakening of the insulating power of the substance; such current would be called a *leakage* current through the insulator.

Generally, the partial breakdown of an insulator, as described above, is rapidly followed by its complete breakdown; as current, even though small, flows through the insulator, it generates more heat, thus still further decreasing the disruptive strength.

This effect of temperature upon the disruptive strength of an insulator is very noticeable in the case of condensers. A condenser may stand 500 volts continuous current for an indefinite period, but when operated on a 60-cycle, 330-volt, alternating current line (maximum voltage 500) it may fail after a short time. Condensers operated on alternating current heat up more or less; and if the heat is generated rapidly enough to cause the temperature of the dielectric to increase appreciably, the condenser will generally fail in a short time.

19. Resistance.—In a conductor in which the electrons are free to leave the atom, their progressive motion is hindered by collisions with the atoms of the substance. *This hindrance to their free progress constitutes the electrical resistance of the conductor.* It differs, as might well be expected, in different metals, and it varies with the temperature. As the temperature of a metal increases, the agitation of its atoms or molecules increases, and this results in more hindrance to the progressive motion of the electrons because of the more frequent collisions between the electrons and the atoms.

That resistance increases with length of conductor is easily accounted for, the number of collisions per unit length being constant for a given size and current; the greater the length the greater the total number of collisions or hindrance to the passage of the electrons. If the area of the conductor is increased, for a given current the number of electrons progressing per unit area decreases inversely as the area of the conductor, and therefore fewer collisions with atoms take place.

With increase of current, more electrons are progressing, and hence the number of collisions with atoms is increased. This results in increased agitation of the atoms and molecules, and this increased agitation is itself an increase of temperature of the conductor. As the molecules are more violently agitated, it becomes harder for the electrons to progress or in other words, the resistance of the conductor has increased.

This elementary presentation accounts for the well-known fact that when a conductor carries current it always heats to some extent, and heats more with larger than with smaller currents. It does not, however, account for the cases of some few substances, such as carbon, which, upon increase in temperature, act in a manner that does not fit in with the foregoing theory.

Our analogy of fleet runners (corresponding to the electrons) trying to get through a crowd of people (corresponding to the positively charged atoms of the conductor) holds good here also. It seems at once evident that it will be easier for the runners to progress through the crowd if the members thereof are stationary than if they are rapidly moving back and forth; in fact, it will be practically impossible for the runners to force their way through the crowd if they are bumped around as much as would be the case if the individuals of the crowd themselves were rapidly running about.

PROBLEMS

1. A galvanometer requires a current of 4×10^{-10} amperes to give a deflection of 1 mm. How many electrons flow through it per second when deflecting 2.3. mm.?

2. During one hour of operation, how many electrons pass through a 10-ampere lamp?

3. An electron, starting from rest, falls unimpeded through a potential difference of 1 volt. How fast is it going?

4. An electron, moving with a velocity of 10^7 cm. per second, falls through a difference of potential of 0.001 volt in the direction in which it is already moving, thus gaining additional velocity. What is its final velocity?

5. Assuming 10^{24} free electrons per cubic centimeter of copper, how long does it take, on the average, for an electron to complete the round of a circuit composed of one-million-circular-mil cable, the circuit being 1 mile long and carrying an average current of 100 amperes?

CHAPTER III

ELECTROMAGNETISM

20. Fundamental Facts.—The action of the magnet has been familiar for many years, the earliest form being the lodestone, an oxide of iron, now known as magnetite. That lodestones had the property of attracting bits of iron with a readily perceptible force was known to the Greeks, and that this property could be imparted to other pieces of iron, by rubbing with a lodestone, has also long been known. But it was not until after Oersted announced the connection between magnetism and currents of electricity, in 1819, that it was found that iron could be magnetized by current through a surrounding coil.

Whenever a soft iron bar is magnetized, it retains its magnetic properties only so long as it is in the presence of a magnetizing agent; the iron is thus only a temporary magnet. When, as in the case of certain alloys of iron, properly heat-treated, the bar retains its magnetic properties, it is said to be a permanent magnet.

From the action of a freely suspended magnet and the field of the earth, recognized as a magnet by Gilbert, we obtain the compass and a means of distinguishing between the two ends of a magnet; by convention, the north-seeking end, or that pointing to the geographical north, has been called the north pole of the magnet, and its south-seeking end, the south pole.

Early experiments showed that if two freely suspended magnets were brought close together, there was a force of attraction between a north and a south pole and a force of repulsion between two north or two south poles. This action, resulting in the law that like poles repel each other and that unlike poles attract, is known to everyone.

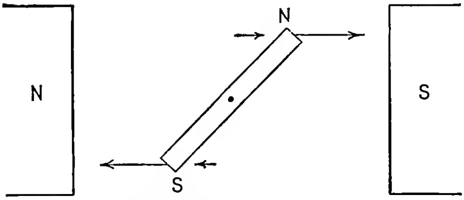


Fig. 11.—Forces acting on a slim magnet pivoted in a magnetic field.

In Fig. 11 is shown a slender magnet, pivoted at its center and placed between two large magnet poles, the forces which are exerted on the poles of the small magnet by the large poles being represented by the arrows. This figure really indicates all that our senses can realize, namely, that between the poles *forces exist which have direction and intensity.* This fact must carefully be borne in mind, for all that we shall say regarding magnetism must necessarily be based on this idea, together with certain other experimental facts which have been determined. Direction and intensity of force acting on unit magnetic pole—this is all we really know about the magnetic field; its exact nature leads to discussions without the scope of this text.

21. Unit Pole.—All measurement being a matter of comparison, it is necessary to assume and define a unit magnet pole, and this is generally defined as a point pole of such strength that when placed in air a distance of one centimeter from an equal point pole, it attracts or repels this other pole with a force of one dyne.

The idea of a point pole is, of course, an artificial one, the magnetism never being concentrated at a single point. It is, however, nearly approximated in the case of long slender magnets. In speaking of unit poles, it is always assumed that the accompanying south pole is so far away as to exert negligible force; this south pole is always required, for evidently there cannot be a north pole without its companion south pole.

By comparison with the unit pole, other poles of unknown strength are measured; for, if such an unknown pole is placed a distance of one centimeter in air from a unit pole, and the force between them is m dynes, the unknown pole has a strength of m units. If two poles, of strength m and m' respectively, are placed one centimeter apart in air, they will attract or repel each other with a force of mm' dynes.

22. Coulomb's Law.—Coulomb, in 1800, discovered that the force exerted between two magnet poles varies inversely as the square of the distance between them. As the force also varies with the medium surrounding the magnets, the force between two magnet poles m and m', separated a distance of r centimeters, may be represented by

$$f = k\frac{mm'}{r^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where k is a constant depending on the medium. For air, k is essentially unity.

All that was said in the previous paragraph is involved in this equation.

23. Direction of the Field of a Magnet.—The space surrounding a magnet pole, or in which magnetic forces act, is called a *magnetic field.* The direction of the field at any point is the direction in which a unit north pole, placed at that point, is urged to move. If a unit north pole is introduced at some point of the field of a bar magnet, as in Fig. 12, the direction of the force acting on the unit pole may easily be determined. The force of repulsion between N and n, along the line a, is represented by the line nR, and the force of attraction between n and S, along b, is represented by the line nA. From Coulomb's law,

$$R : A = b^2 : a^2.$$

The resultant force, nF, acting on n, is the vector sum of R and A.

After the direction of the force at all points in the field has thus been determined, the results may best be represented by curved lines drawn from the north to the south pole, as in Fig. 13; the direction of the force at any point in the field is obtained by drawing a tangent to the curved line drawn through the point. The curved

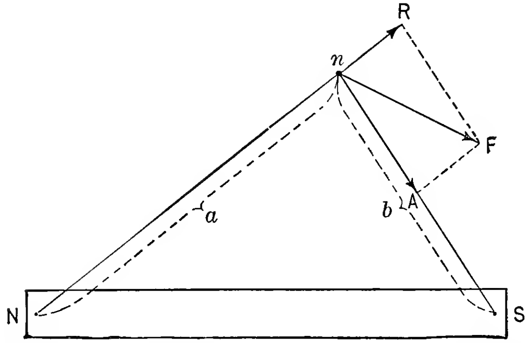


FIG. 12.—Resultant force on unit north pole placed in the field of a bar magnet, a distance a from the north pole and a distance b from the south pole.

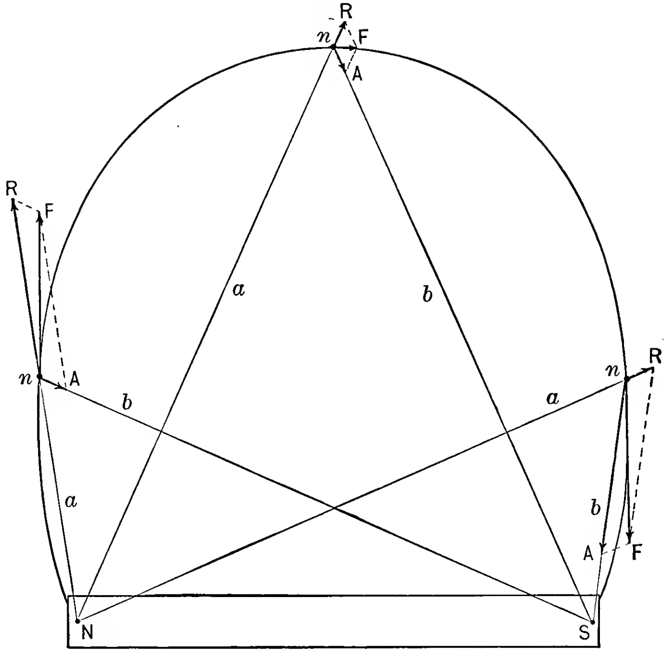


FIG. 13.—Path of a slowly moving unit north pole in the field of a bar magnet. The direction of the resultant force acting on the unit pole is tangent to the curve at any point. If the small north pole was started from another part of the pole of the bar magnet it would describe another curved path, somewhat similar to that shown here, which has been constructed accurately, to scale.

line also, of course, represents the path that a freely moving north pole would follow if allowed to move slowly.

That the direction of the magnetic force is as has just been shown, is proved by the familiar experiment in which iron filings are sprinkled over a paper placed over a bar magnet. When the paper is tapped, the iron filings arrange themselves in filaments or lines similar to those shown above in Fig. 13.

24. Unit Field.—We have defined a magnetic field as the region surrounding a magnet, or any region in which magnetic forces act. It follows from this that whenever a unit pole is introduced into a magnetic field it will be acted upon by a force. We can then define the intensity of a magnetic field at any point as the *force in dynes exerted on a unit pole placed at that point,* provided, however, that the introduction of the unit pole does not alter the original intensity or distribution. When the force acting on the unit pole is 1 dyne, then the pole is situated in unit field intensity.

By this definition, when two unit poles are placed 1 cm. apart, the force acting between them being 1 dyne, each unit pole is situated in unit field produced by the other pole.

25. Representation of Magnetic Fields.—Magnetic forces, having direction and intensity, are vector quantities and may therefore be represented by vectors.

A vector quantity, such as a force, is ordinarily represented by a line the direction of which indicates the direction of the force and the length of which (to some arbitrary scale) measures the intensity of the force. Thus, a square piston, with an area of 16 square inches and with a force of 2 pounds per square inch acting on it, bears a total force of 32 pounds. This might be represented, as in Fig. 14, by a single vector drawn to the center of the piston and 32 units long, one unit representing one pound.

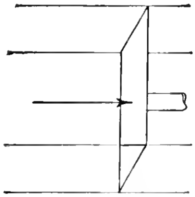


FIG. 14.—Representation of a uniform force over the area of a square piston by a single vector, the length of which corresponds to the total force on the piston.

Another way of representing the same force is to draw as many arrows per unit area as the force per unit area. In the case above, there would be two arrows per square inch over the entire area of the piston, as in Fig. 15. Or, if the force dropped to half a pound per square inch, and the same units were used, there would be one arrow in each two square inches.

In this method, direction of the force is shown as before, but intensity of the force is represented by the number of arrows per unit area; the length of the arrows has no significance in this method of representation.

The latter method of vector representation is of special value where both the direction and intensity of the force vary from point to point.

Imagine a stream of air passing out of a pipe through a funnel, with a number of obstacles in the small end, which cause the air to swirl and eddy as it passes through the funnel. Let us, however, assume that there are no pressure losses, due either to these eddies or to friction on the sides of the funnel.

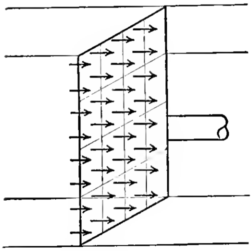


FIG. 15.—Another scheme of vector representation of uniform force over the area of a piston, the number of arrows per unit area indicating the intensity of the force.

If we wish to map out the pressure of the air currents over some test area *aa′*, we may conveniently do so by having one arrow represent a force of one ounce per square inch. If the force at certain regions were only half an ounce per square inch, we would draw one arrow in each two square inches, and so on. In Fig. 16, this has been done for sections *aa′* and *bb′* of the funnel, and it follows that, if the test area is gradually moved over the entire length of the funnel, the arrows become lines, the direction of which at any point indicates the direction of the pressure, while the number of lines per square inch indicates the intensity of the pressure.

From our assumption of no pressure losses due to eddies or wall friction, it follows that the lines are continuous, the product of average pressure per square inch and area remaining constant. It so happens that in this particular case the continuous lines also show the stream lines of the air, but this is only because there is something actually moving.

26. Lines of Force.—We may use for magnetic fields the same method of vector representation as that given in Fig. 16. Having defined unit field intensity as that field strength which will exert a force of one dyne on a unit pole, we might map out the field of a bar magnet by placing a unit pole at every point in the field and noting the force exerted on it. We then draw as many arrows per square centimeter as there are dynes of force at that point on the unit pole, as is represented in Fig. 17. In this case also, as the positions mapped out come closer and closer together, the arrows tend to form continuous lines.

Unit field is by definition represented by one line per square centimeter. Having a field mapped out according to these ideas, if we desire to determine the force at any point, we need only insert at that point a test area of one square centimeter at right angles to the direction of the lines and count the number of lines traversing it.

It must, however, be carefully borne in mind that the lines of our diagram, which in the case of magnetic fields are called *lines of force*, are but a vector representation of the direction and intensity of the force of the field on a unit pole, and nothing more. They have no physical existence and can therefore have no properties of their own; nor do they represent the flow of anything, any more than they would if used to represent intensity and direction of gravitational forces.

However, lines of force serve as a means of determining or measuring field intensities and are universally used; we speak of a field of 100,000 lines of force per square centimeter, really meaning a field of such strength as would exert a force of 100,000 dynes on a unit pole.

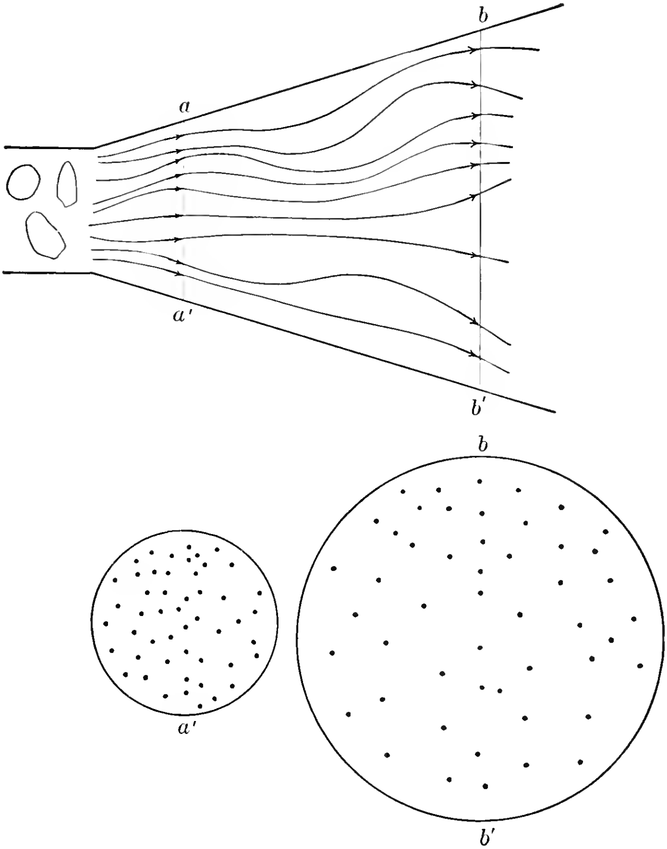


FIG. 16.—Representation of the air pressure gradient in a funnel carrying a stream of air; by using the scheme of Fig. 15, for closely adjacent sections across the funnel, at right angles to its axis, and connecting the force arrows in the adjacent sections, the stream line diagram in the upper part of the figure is obtained. The lower diagram shows the density of the force arrows over the two sections a–a' and b–b', above.

27. Flux and Flux Density—The Gauss.—The total number of lines of force leaving or entering a magnetic pole is called its flux, or total flux. It is generally spoken of as so many lines of force, or maxwells, a *maxwell* being one line of force. The symbol ϕ (Greek letter *phi*) is generally used to represent total flux.

The flux density of a field is the number of lines of force per square centimeter, and is thus a measure of field intensity. A recognized unit of flux density is the gauss, which is equal to one line of force per square centimeter. From this we get the kilogauss, or 1000 lines per square centimeter. Flux densities may also be expressed in lines per square inch or other unit of area. The symbol B is generally used to represent flux densities.

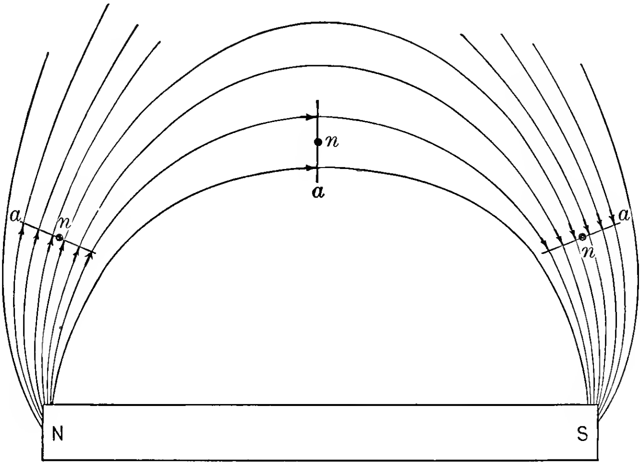


FIG. 17.—Vector representation of the force on a unit north pole n, at the center of a square centimeter, a. The number of arrows normal to the square centimeter represents the force, in dynes, acting on the unit pole. As the points mapped out in the field come closer together the arrows become lines of force.

28. Flux from Unit Pole.—If a sphere of one centimeter radius is drawn about a unit pole as center, as in Fig. 18, a similar unit pole placed at any point on the surface of this sphere would be repelled by a force of one dyne. The second pole is, therefore, by definition, situated in unit field and there must be one line of force per square centimeter of surface of the sphere, the area of which is $4\pi r^2$ sq. cm. Accordingly, since $r = 1$, the total flux from the unit pole is 4π lines. At a distance $r/2$ cm. from the pole, the force, from Coulomb's law, is 4 dynes, and the area of the sphere is $\frac{4\pi r^2}{4}$, and again there are 4π lines; the lines are thus continuous as has been previously stated. For a pole of strength m units, the flux emanating is $4\pi m$ lines. In general, the flux density due to a unit pole, in a homogeneous medium, is equal to $1/r^2$, and that due to any pole of strength m, is m/r^2.

It must be remembered that a unit south pole must accompany every unit north pole, and if the unit north pole is considered at the end of a long slender magnet, the 4π lines leaving it must enter the sphere through the substance of the magnet, as shown in Fig. 18.

29. Applications.—The dyne being a very small unit and commercial magnetic fields being generally very powerful, we find such fields reaching flux densities of thousands of lines per square centimeter. The air-gap flux densities of modern continuous current generators and motors may reach 60,000 lines per square inch or 9300 lines per square centimeter; a unit pole in such a field would be acted upon by a force of 9300 dynes or 9.5 grams.

The field of the ordinary direct current ammeter or voltmeter (discussed in Chapter VI) of the permanent magnet type, as shown simply in Fig. 114, has a flux density of about 1,200 lines per square centimeter.

The earth's magnetic field may be considered in terms of its horizontal and a vertical components, the values of which vary, particularly with latitude. At Washington, D. C., the horizontal and vertical components are roughly 0.2 and 0.55 line per square centimeter, respectively.

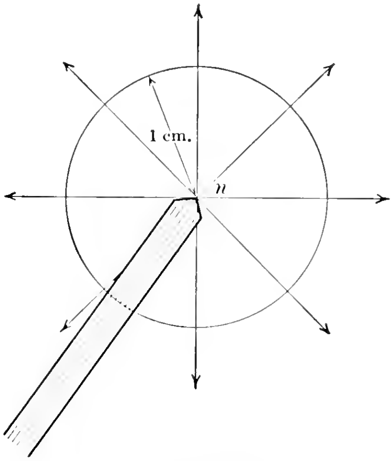


FIG. 18.—Flux emanating from unit north pole. According to the conventional method of representing the field strength by the number of lines per square centimeter, and the definition of unit pole strength, there are 4π lines emanating from a pole of unit strength.

30. Field Intensity or Magnetizing Force.—We have defined the intensity of a magnetic field at a point as the force in dynes acting on a unit pole. This is also called the magnetizing force at that point and, in air, is usually also represented by the symbol H.

Consider a unit pole at a distance r from a pole of strength m. From Coulomb's law, the force due to pole m, acting on unit pole, is

$$F=\frac{1\times m}{r^2}.$$

Hence,

$$H=\frac{m}{r^2}. \qquad (2)$$

If a pole of strength m' were substituted for the unit pole, the force acting on pole m' would be

$$F=\frac{m'\times m}{r^2}=m'H. \qquad (3)$$

From this it follows that the force on m' can be computed either by considering it in proximity to another pole m, or placed in a certain field of intensity H, the origin of which does not concern us. It is thus possible to consider either the pole or its field; in general electrical engineering, the field, and never the pole itself, is considered.

The definition given for field intensity suffices also for the flux density in air, as they are here equal, that is $B = H$. We say that unit field intensity or unit magnetizing force produces one line of magnetic flux per square centimeter, in air.

Another view of field intensity, and more particularly of magnetizing force H, is that it is the work done in moving a unit pole a distance of one centimeter along the path of the lines of force. The force due to a field of intensity H being H dynes, the work done per centimeter becomes H ergs. The conception that magnetizing force is the work done per centimeter per unit pole is, in general, the most convenient one in considering the various relations existing between the different magnetic quantities.

31. Magnetic Field Surrounding a Conductor Carrying Current.—In 1819, Oersted discovered the fact that a magnetic needle is disturbed when a conductor carrying electric current is brought into its vicinity. Further experiment showed that there was a magnetic field around such a conductor, that the direction of the field was circular about the conductor, as in Fig. 19, and its intensity was proportional to the current.

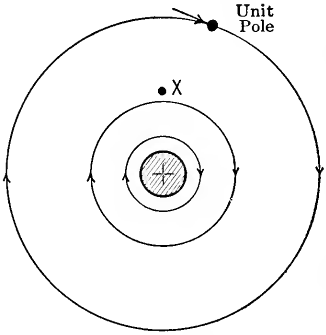


Fig. 19.—Map of the magnetic field surrounding a conductor which is carrying current. Using the conventional ideas as to direction of current and magnetic field, the current in this conductor must be flowing into the paper. The direction shown for the magnetic field is that in which a north magnetic pole would be urged if placed in the vicinity of the conductor.

Using the conventional directions of current, as spoken of in paragraph 13, when current flows into the paper the direction of the field is found to be clockwise, a unit pole being so urged, and vice versa when current flows out. Two simple rules are available for determining this relation: If a right-handed screw is screwed into the conductor in the direction of the current flow, then the screw is turned in the direction of the lines of force; or if the conductor is grasped by the right hand, with the thumb extending in the direction of the current, the direction of the fingers indicates the direction of the lines of force.

In Figs. 19 and 20, the field is correctly mapped out for the current the conductor is carrying, the number of lines per square centimeter indicating the intensity of the field. In Fig. 20, the current is supposed to be double that in Fig. 19, and the field intensity at any point X must then be doubled; there are therefore twice as many lines per square centimeter.

If the increase in current were a gradual one, the lines would appear to be spreading out from the conductor as the current increases. The lines, having no physical existence, are not really doing anything of the sort; what actually happens is that the field intensity at any point, X, is increasing. It is, however, convenient to speak of the lines as spreading out, and we often conceive the growing magnetic field to be established in this fashion.

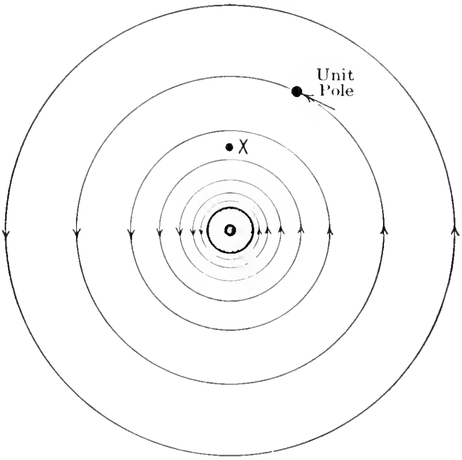


FIG. 20.—If the current in the conductor of Fig. 19 is doubled the intensity of the magnetic field around the conductor will everywhere be doubled, so that the field is now shown with the lines of force everywhere twice as dense. Current is assumed as flowing out of the paper so that the direction of the field is opposite to what it was in Fig. 19.

32. Field at Center of a Circular Loop Carrying Current.—In Fig. 21 is shown a circular loop of wire of radius r centimeters, and carrying a current i; at the center of the loop there is placed a unit north pole. From the ideas presented in the previous section, it is evident that there will be a force acting upward on the unit north pole, due to any element of the wire ab. It is found by experiment that the force on the pole varies directly as the length of the element of the wire and the strength of the current, and inversely as the square of the distance between the element of wire and the pole, or

$$F \propto \frac{ab \cdot i}{r^2},$$

and the total force due to the entire loop,

$$F \propto \frac{2\pi r \cdot i}{r^2} \propto \frac{2\pi \cdot i}{r}.$$

Evidently, by proper choice of units, this proportionality may be written in the form of an equation

$$F = \frac{2\pi r i}{r^2} = \frac{2\pi i}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

33. Unit Current.—If the force on the unit north pole placed at the center of the loop is 2π dynes, then the current value is taken as unity. Accordingly, unit current will exert a force of 2π dynes on a unit pole placed at the center of a loop of 1 cm. radius. Such unit current is called the c.g.s. ampere, or absolute ampere, or abampere; but as this is a fairly large unit, the practical unit, the ampere, has been taken as one-tenth of the abampere.

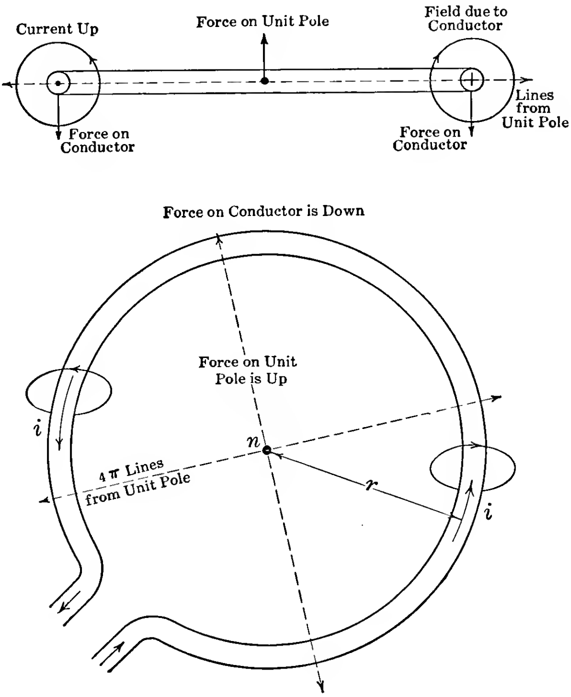


Fig. 21.—Force acting on a unit north pole at the center of a circular loop of wire carrying current. If the loop is one centimeter in radius, unit current (one abampere) will produce on the pole a force of 2π dynes.

The determination of the ampere by the method just suggested is difficult and subject to error. A much easier and more accurate way is to determine the ampere in terms of its electrolytic effects; according to standard specifications, the ampere is that unvarying current, which, when passed through a standard silver nitrate solution, under specified conditions, deposits silver at the rate of 0.001118 gram per second

34. Force Acting on a Conductor Carrying Current in a Magnetic Field.—In the conditions shown by Fig. 21, it is evident that there must be a force acting on the loop of wire, due to the pole. Or, by considering the field of the pole instead of the pole itself, we get the idea that a conductor placed in a magnetic field has a force acting on it.

In the expression $F=\frac{2\pi ri}{r^2}$, it will be noticed that $2\pi r$ is the length, l, of the conductor, and if we say that the intensity of the field at the conductor, due to the pole, is H, then as

$$H=\frac{1}{r^2},$$

it follows, by substituting these values in the above equation, that

$$F=Hli \text{ dynes.} \qquad (5)$$

The expression better serves our purpose if, instead of field intensity, we express this value as flux density, which, in air is the same thing, as is seen from the relation

$$B(\text{at distance of } r \text{ cm.})=\frac{\text{Flux emanating from the unit pole}}{\text{Area of surrounding sphere of } r \text{ cm. radius}},$$

$$=\frac{4\pi}{4\pi r^2}=\frac{1}{r^2},$$

so that

$$F=Bli. \qquad (6)$$

If, as in Fig. 22, the current-carrying conductor is a straight wire placed at right angles to the direction of a field of uniform flux density, B, the force is dependent on the same quantities.

The equation, $F=Bli$, also serves as a means of defining the abampere, for if the conductor is placed in unit field and the force on the conductor is 1 dyne per centimeter length of conductor, then the current flowing is 1 abampere.

The relation between the directions of the current, field, and force are given by Fleming's left-hand rule. When the thumb, forefinger, and middle finger of the left hand are held mutually perpendicular to one another, the forefinger pointing in the direction of the field and the middle finger in the direction of the flow of the current, then the thumb will point in the

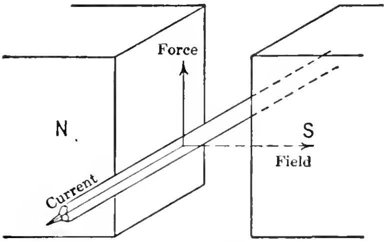


FIG. 22.—The force acting on a conductor, which is carrying current and is placed in a magnetic field, other than its own, is at right angles to the length of the conductor and to the direction of the field.

direction of the force on the conductor. If the conductor is not perpendicular to the direction of the field, but makes some angle θ with it, the force will vary as the cosine of the angle θ.

35. Magnetic Field of a Solenoid, Carrying Current.—A solenoid is a conductor wound in the form of a long cylindrical helix of constant cross-section. When carrying current, a solenoid has a magnetic field and, generally speaking, has all of the properties of a bar magnet. When mapped out with a unit pole, the magnetic field of a solenoid with an air core appears as in Fig. 23*a*. The flux lines are found to be closed, leaving at one end, passing around outside and entering at the other end. The field at the center is found to be quite uniform, but all of the lines do not leave at the ends, some leaking out before they reach the ends. If the turns comprising the solenoid are not close together, there will be much more flux leaking out between turns, and if the turns are far enough apart, closed lines will exist to an appreciable extent around the individual turns, as shown in Fig. 23*b*.

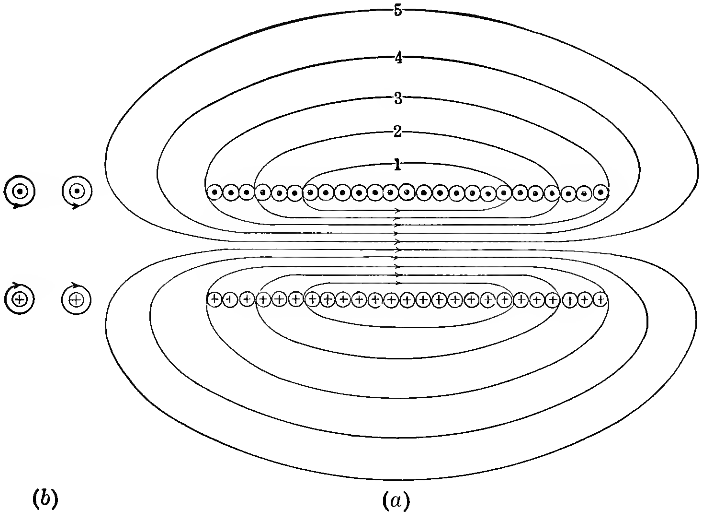


FIG. 23.—Map of the field set up in a solenoid, having an air core; the actual field is caused by the resultant effect of all the circular loops, each acting as though the others were not present, as suggested by the two extra turns shown at *b*.

If the solenoid is long in comparison to its diameter (say length = 10 times diameter), the strength of field at the ends of the solenoid will be practically half as much as it is at the central portion of the solenoid.

36. Action of a Conductor Carrying Current in a Magnetic Field, from the Standpoint of Interlinkages.—Every electric circuit carrying current forms one or more closed loops, and every line of force also constitutes a closed loop. When a line of force links with a closed loop in the same way that two links of a chain link with each other, one *interlinkage* results. The flux lines may be set up by the coil itself, or by some neighboring coil or magnet. If several lines link with a number of turns, the number of interlinkages is equal to their product, as in Fig. 24, where 3 lines link with 2 turns, producing 6 interlinkages. In Fig. 23 we have 2 lines linking with 13 turns, 2 lines linking with 19 turns, and 6 lines linking with 25 turns, resulting in a total of 214 interlinkages.

As stated in the previous paragraph, a coil of wire carrying current has all the properties of a bar magnet, and it is therefore always possible to replace a permanent magnet by one or more coils. In Fig. 25 is shown a field between the poles of a permanent magnet.

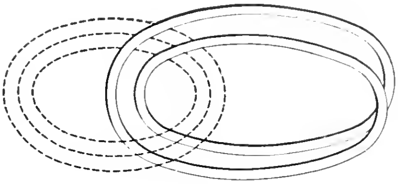


Fig. 24

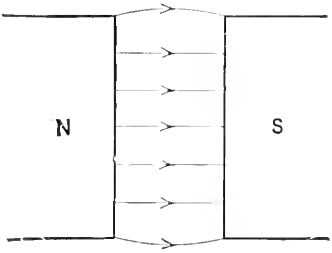


Fig. 25

Fig. 24.—This figure gives the idea of interlinkages; there are shown three lines of magnetic flux each of them linking with the two turns of wire shown. One line of flux linking with one turn of a circuit constitutes one interlinkage, so in this diagram there are six interlinkages.

Fig. 25.—Field between the poles of a permanent magnet. If the pole faces are plain, as shown here, the lines of flux are parallel to each other except at the edges, where they curve out. The lines leave the pole faces perpendicular to the surface.

Now, two coils separated from each other as in Fig. 26 can be made to produce a field of the same intensity and distribution as that given by the above magnet, by properly adjusting the current passing through them, and coils can therefore be used to replace the magnet of Fig. 25.

Consider a system as in Fig. 27, made up of a movable rectangular coil of, say 3 turns, with its magnetic field, and a field set up by a permanent magnet, the coil and the magnet being far enough away not to influence each other. The current flowing through the coil is of such value that 6 lines of force are set up, giving 18 interlinkages. If we replace the permanent magnet by 2 fixed coils, each of 2 turns, and pass through them a current of such value as to produce a field between the coils of the same density as between the poles of the magnets of Fig. 27, we find conditions as in Fig. 28, assuming that all the lines pass completely through all the turns of both coils. If the current sets up 8 lines, we have 32 interlinkages for the fixed coil and 18 interlinkages for the movable coil, or 50 in the entire system.

The action of a current-carrying conductor in a magnetic field may now be regarded from the standpoint of interlinkages, for every such conductor is but part of a closed loop. If the rectangular movable coil of Fig. 28 is moved into the field of the fixed coils, as indicated simply in Fig. 29, application of Fleming's left-hand rule indicates that the lower coil-side L, in which we are imagining current to be flowing into the paper, will develop a force tending to move the coil down into the field of the fixed coils. It will be seen that the direction of the field of the fixed coils and that set up by the movable coil across its own plane are both from left to right (as shown by one dotted line for each of the coils); by moving into the field of the fixed coils (or the magnet) the movable coil acts to include within itself the greatest possible number of lines.

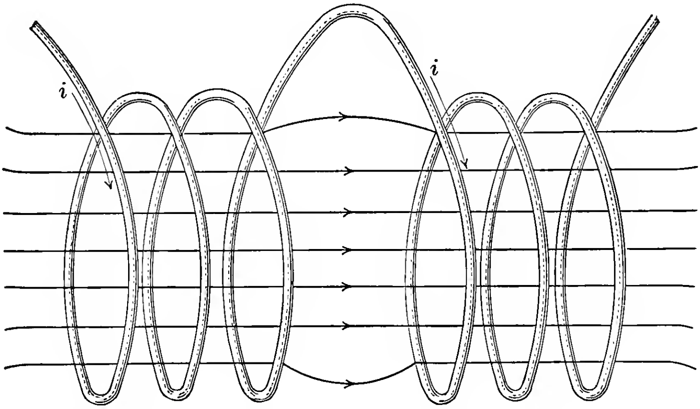


FIG. 26.—The field of the permanent magnet shown in Fig. 25 may be replaced by the field between two coils; if the coils are properly placed, and have the proper number of turns and current the field set up between them will be exactly the same as that shown in Figure 25.

When the movable coil has moved so as to include as much of the fixed field as possible, as in Fig. 30, the number of interlinkages of the system is a maximum. Assuming that the medium is air, the lines threading both coils will now be the sum of those individually set up, or $6+8=14$. Since there are altogether 7 turns in the system, $7\times14=98$ interlinkages result. Thus, the action of a coil carrying a current (which is maintained constant), placed in a magnetic field, is to move so *as to increase the total number of interlinkages of the system.*

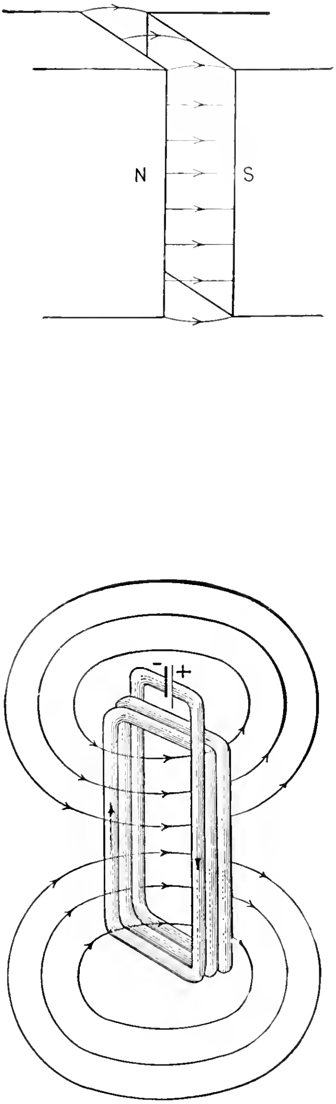


Fig. 27

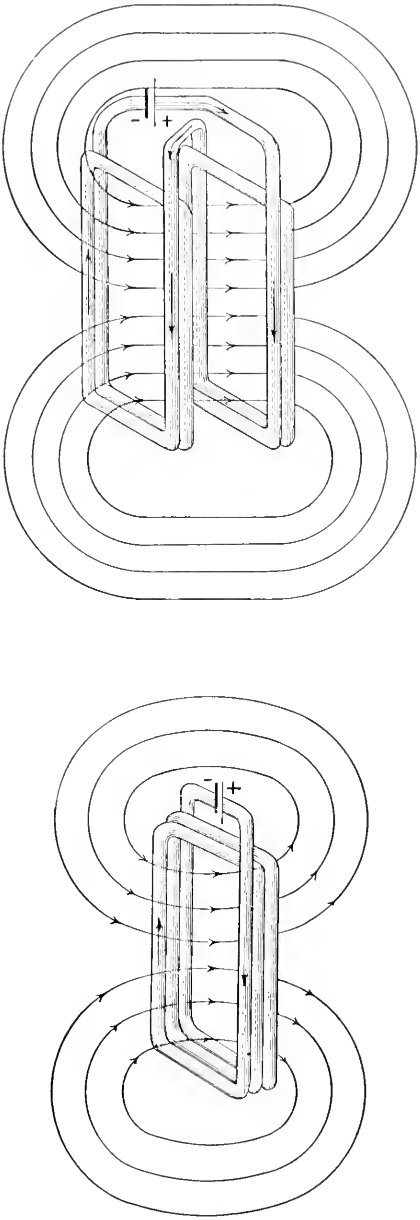


Fig. 28

Fig. 27.—Coil carrying current approaching the field due to a permanent magnet. The two magnetic fields are not, as yet, appreciably interlinked.

Fig. 28.—The permanent magnet of Fig. 27 replaced by two coils. The system, composed of the two fixed coils (upper) and the single movable coil (lower) has a total of 50 interlinkages, 32 for the upper coil and 18 for the lower coil.

The assumption that when the movable coil has moved completely into the field, a total of 14 lines will be set up, or that all of the lines set up will completely thread all of the turns of the system, is not necessarily correct; but it is evident that the total number of interlinkages of the system is increased when the movable coil comes into the field, as in Fig. 30.

When the coil is in the position shown in Fig. 30, where the total number of interlinkages of the system is a maximum, it is in a position of stable equilibrium, a force tending to move the coil up being developed by the upper coil-side *U*, and an equal downward force by the lower coil-side, *L*.

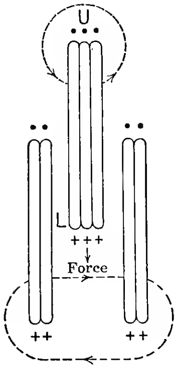


Fig. 29.

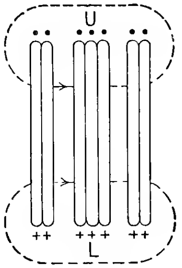


Fig. 30.

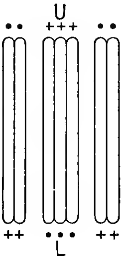


Fig. 31.

Fig. 29.—The movable coil of Fig. 28 is here shown, in simplified form, partly in the field of the two fixed coils of Fig. 28. The force exerted between movable coil and the field of the fixed coils, tends to draw the movable coil further into the field of the fixed coils, thereby increasing the total interlinkages of the system.

Fig. 30.—The movable coil of Fig. 29 is now entirely in the field of the fixed coils. The interlinkages of the combination are now a maximum, and the movable coil is therefore in a position of stable equilibrium.

Fig. 31.—The current in the movable coil has been reversed, so that the magnetizing effects of the two coils tend to neutralize each other. The interlinkages of the system is now a minimum and any motion of the movable coil will result in an increase in the total interlinkages; the movable coil is therefore in a position of unstable equilibrium.

Any dislodgment of the movable coil, tending to reduce the total number of interlinkages of the system, results in unbalancing the forces developed by the coil-sides, thus tending to restore the coil to its position in Fig. 30. This follows because whichever coil-side of the movable coil moves out of the fixed field will evidently develop less force than the other coil-side, which has moved up into a position of more intense field.

However, if the direction of the current in the movable coil is reversed, as in Fig. 31, the coil is in a position of unstable equilibrium and tends to move out of the field, as application of Fleming's left-hand rule will show.

It may move out of the fixed field in either an upward or a downward direction, depending upon which way it is started, as indicated in Fig. 32.

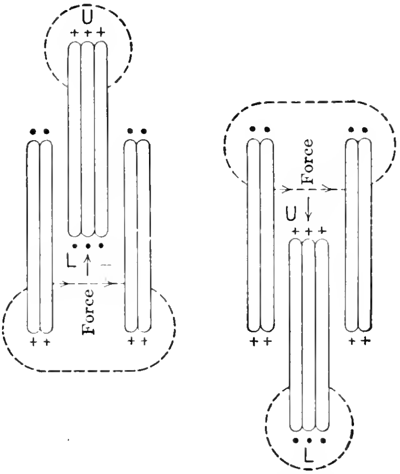


FIG. 32.—If the movable coil of Fig. 31 is dislodged in either direction, a force is developed which tends to make it move farther in the same direction, until it is entirely outside the field of the fixed coils. Increase of interlinkages results.

With the current reversed, as in Fig. 31, and with the same assumptions as to the lines threading all of the coils, the total lines set up through both coils will now be $8-6=2$, since the movable coil now tends to set up its lines in a direction opposite to that of the fixed coils. There are now 2×7, or 14, interlinkages in the system. When the movable coil has moved completely out of the fixed field, *the total number of interlinkages of the system will have increased* to 50, the conditions being as shown in Fig. 28.

Thus, the action of the coil has again been to move so as to increase the total number of interlinkages of the entire system, and we can conclude that whenever a coil-carrying current is situated in a magnetic field other than its own, it will always tend to move so as to make the total interlinkages of the system a maximum.

37. Applications of the Principle of a Current-carrying Conductor in a Magnetic Field.—Applications of this principle are numerous. The most important of the numerous applications of this principle is the motor, whose armature carries many coils. Suppose a pivoted coil, carrying current out along the lower conductor and in along the upper, is placed in a magnetic field, as in Fig. 33*a*. Applying our first line of reasoning, we see that forces will act on both conductors, tending to rotate the coil in a clockwise direction until the coil reaches approximately its position in Fig. 33*b*. The action has also been to make the coil include a greater number of flux interlinkages than before. It is interesting to note that if the coil has sufficient momentum to carry it from its position in Fig. 33*b* about to the dotted line, and the current in the coil were reversed at about the same time, the coil would continue to rotate. This is the principle of the motor, the reversal of the current being effected by the commutator.

In the D'Arsonval, or moving-coil, type of meters, the same idea is utilized by attaching a pointer to the coil and making the coil work against a spring. The two limiting positions of the coil are shown in Fig. 34.

As the simplest illustration of "motion such that flux interlinkages increase" it may be found by experiment that if a single turn of flexible conductor, such as lamp cord, is made to have the form of a narrow rectangle, and a heavy current is passed through it, the sides of the rectangle will at once jump apart, so as to give a turn of circular form, this being the shape of turn which gives the most flux for a given current and length of wire.

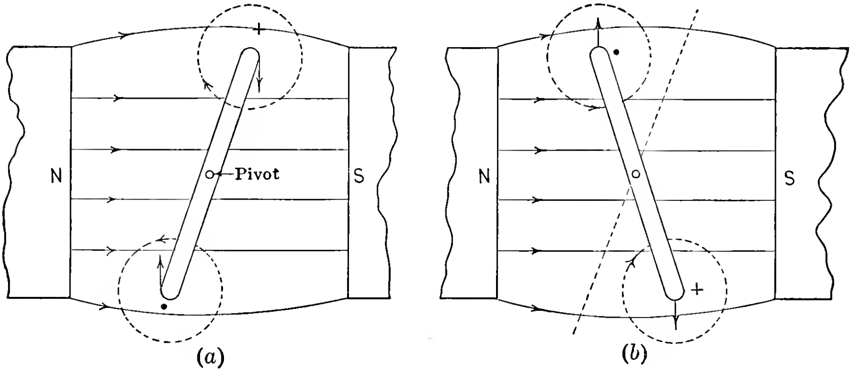


FIG. 33.—In a coil carrying current, pivoted in a magnetic field, the forces developed tend to rotate the coil. Evidently in position *b* the total interlinkages is greater than in position *a* because in *a* the current in the coil tends to partly neutralize the magnetic field of the magnet.

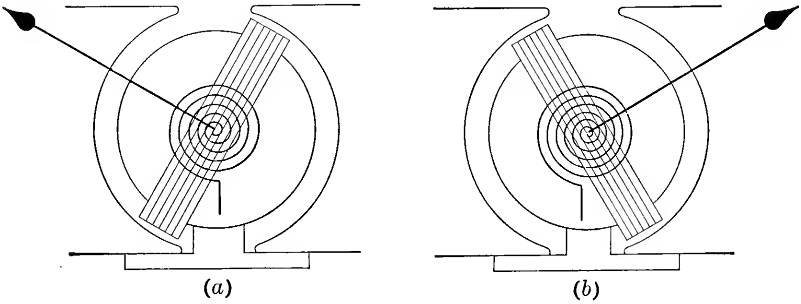


FIG. 34.—A pivoted coil in the magnetic field of a permanent magnet is used in the D'Arsonval, or moving coil, type of continuous current meter. The limiting positions of the coil are shown above, as the coil evidently cannot pull itself farther than position *b*; the force acting on the coil becomes zero when it moves past the pole tip of the magnet.

If, in Figs. 33 and 34, we had used, instead of a coil of a single turn, one having two turns very close together, each turn carrying the same current, the forces would have been doubled in each case. But in the case of the one-turn coil the force would also have been doubled if we had doubled the current; or, with two turns and doubled current, the forces would have been quadrupled. In other words, the effects are proportional to the product of amperes and turns, or *ampere-turns*, for a given value of flux density; the same product of amperes and turns gives the same effect, no matter what the separate values of current and turns may be.

38. Laws of Induction.—The simplest method of generating a voltage in a circuit is by means of chemical action, such as that which takes place in the dry cell or storage battery, but in the method much more frequently employed, voltage is generated by relative motion between a magnetic field and a conductor (the motion being in such a direction that the conductor "cuts" flux); such voltage is called an induced voltage.

The general laws of induced voltages, first discovered by Faraday in 1831, and more completely summed up by Lenz about 1834, are as follows:

First Law.—Whenever a conductor moves so as to cut flux, or whenever there is a change in the number of lines of force passing through or linking with a closed circuit, voltage is induced, which tends to set up current in such a direction as to oppose the *change* in the flux threading the circuit.

Second Law.—The voltage induced in a single turn of wire is equal to the rate of change of the lines of force passing through, or linking with, that turn.

39. Unit E.M.F.—The absolute (c.g.s.) unit of electromotive force is the electromotive force (e.m.f.) induced when an inductor is cutting flux at a rate of one line of force per second; or when the flux threading a coil of one turn is changing at a rate of one line per second. This unit is called the absolute volt, or abvolt, but is, for ordinary purposes, an exceedingly small unit. The practical unit, the volt, is taken as 10^8 absolute volts, and is then the e.m.f. generated when the flux threading a coil of one turn is changing at a rate of 10^8 lines of force per second.

We have then,

$$e=\frac{\phi}{t}\text{ abvolts,} \quad \text{or} \quad E=\frac{\phi}{t\cdot 10^8}\text{ volts.} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Working standards of e.m.f. are provided by certain standard cells, the voltages of which are very definite and constant, on open circuit. The Clark cell has an open-circuit e.m.f. of 1.4328 volts at 15° C., and the Weston cell, the one principally used, has an e.m.f. of 1.01830 volts on open circuit at 20° C.

40. Conductor Moving in a Magnetic Field.—Experiment shows that when a conductor moves in a magnetic field at right angles to both the direction of the magnetic field and itself, so as to "cut" the lines of force, an electromotive force is induced in the wire as long as the "cutting" of the lines continues. The voltage generated is found to be equal to the product of the flux density in which the conductor is moving, the length of the wire cutting flux, and the velocity of the conductor in a direction at right angles to the field. This follows from the definition of unit e.m.f., as may be seen from Fig. 35.

Let B = flux density in which conductor is moving;
l = the length of conductor cutting flux;
d = the distance the conductor moves at right angles to the direction of the field, in time t.

The velocity of the conductor, v, is then d/t, and the product $l \cdot d$ is evidently the area passed over by the wire. Hence, the total flux cut is

$$Bld,$$

and the flux cut per second,

$$Blv.$$

The voltage generated is then

$$e = Blv \text{ abvolts, or } E = \frac{Blv}{10^8} \text{ volts. . . . . } \quad (8)$$

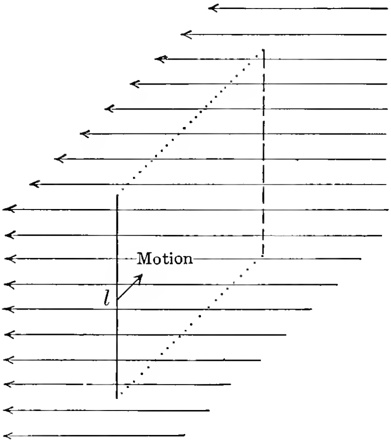


FIG. 35.

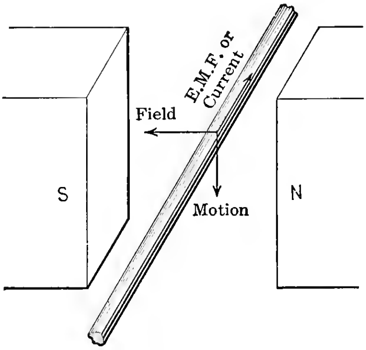


FIG. 36.

FIG. 35.—A conductor moving through a magnetic field so as to "cut" lines of force, generates an electromotive force.

FIG. 36.—This diagram gives the relative directions of the motion, field, and induced voltage set up by the conductor's motion.

If the conductor does not move at right angles either to itself or to the field, the voltage generated is proportional to the cosine of the angle it makes with either. It will be remembered that the force acting on a conductor carrying current in a magnetic field involves the same angle in a similar manner.

The relation between the directions of voltage, field, and motion is given by Fleming's right-hand rule, which is similar to his left-hand rule; the thumb and two fingers held mutually at right angles to each other, the thumb in the direction of motion, the forefinger in the direction of the field, then the second finger points in the direction of the induced voltage. (Fig. 36.)

41. Coil Moving in a Magnetic Field.—Considering the conductor as part of a closed circuit, or coil, we find in Fig. 37 that the coil is moving to enclose the flux from the poles or "fill itself" with flux. The voltage induced in the lower coil-side, *L*, in Fig. 37*a*, as it moves downward, is, by the application of Fleming's right-hand rule, towards us. In Fig. 37*b*, the coil is moving upwards and the voltage in the upper coil-side, *U*, is away from us. Thus, as the flux passing through the coil is changed, e.m.f. is generated. The current induced will be in the same direction as the voltage generated, and in both cases considered tends to set up a magnetic field (as shown in dotted lines) which is in *the opposite direction* to that of the main field and therefore tends to prevent the flux through the coil from increasing.

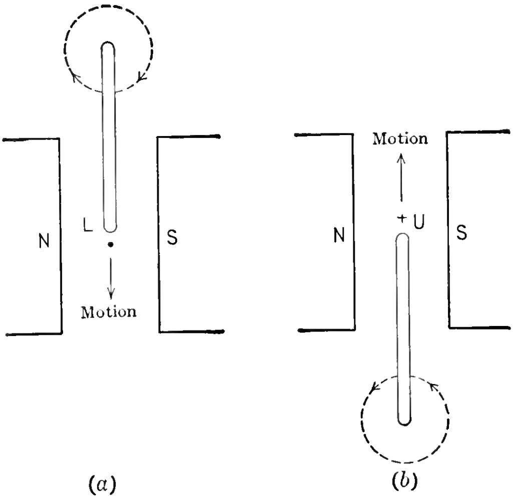


FIG. 37.—A coil moved into a magnetic field; current is set up in the coil which tends to set up a magnetic field which is in the opposite direction to that of the field into which the coil is moving, that is, the current set up in the coil acts to prevent the flux linking the coil from increasing.

If the coil starts to move from a position in which it encloses all the flux from the field, as represented in Fig. 38, and so tends to empty itself of flux, the direction of the current induced is opposite to what it was when the coil was moving into the field. In Fig. 39*a*, the voltage in the upper coil-side, *U*, is now towards us, and in Fig. 39*b*, in the lower coil-side *L*, it is away from us. The direction of the field which the induced current tends to set up is in the same direction as that of the main field; the action of the coil thus tends to keep the flux through itself from decreasing.

The conclusion to be reached from the above analysis is that when the flux threading a coil is changing, e.m.f. is induced in amount proportional to the rate of change of the flux threading the circuit or coil, and that the action of the resulting induced current is always such as to prevent the change of flux which is causing this induced current in the circuit, or, in other words, to maintain the flux linking the circuit constant.

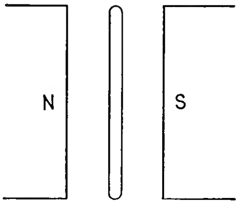


FIG. 38.—The coil is now in such a position that it links with all of the flux from the magnet.

If the coil comprises more than one turn, so as to have a number of inductors in series, the voltage induced will be equal to the product of the voltage induced per turn and the number of turns. Thus, if n turns cut a total flux of ϕ lines in t seconds, the average voltage induced is

$$e=\frac{n\phi}{t} \text{ abvolts, or, } E=\frac{n\phi}{t\cdot 10^8} \text{ volts.} \quad . \quad . \quad . \quad (9)$$

The product $n\phi$ is evidently the number of interlinkages between flux and turns, and the voltage generated is therefore equal to the rate of change of interlinkages.

In addition to the voltage and current effects just analyzed, it is to be noted that if current is induced by the motion of the conductor in a magnetic field, it is also to be regarded as a current-carrying conductor in a magnetic field and therefore there are mechanical forces to be considered. The application of Fleming's left-hand rule then shows that the force developed by the conductor, due to currents set up by its motion, is always such as to oppose the motion to which the induced current is due. Therefore, when a conductor which is part of a closed circuit moves so as to cut a magnetic field, there are two separate reactions to be considered, one electrical and the other mechanical. The electrical reaction tends to oppose any change in the magnetic field linking the circuit, by tending to set up a field which opposes this change; and the mechanical reaction tends to stop the motion by the development of a resisting mechanical force.

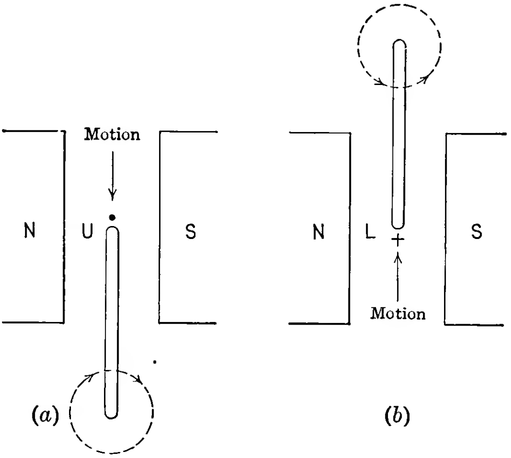


FIG. 39.—Coil moving out from the field of the magnet; the current set up by the motion now acts to prevent the flux threading the coil from decreasing, so the current is in the opposite direction to that it had when the coil was being moved into the field of the magnet.

Considering the case as that of a current-carrying conductor in a magnetic field, we find that as the conductor or coil of which it is a part is allowed to move into the magnetic field, reactions are set up which tend to stop this action. In Fig. 40, the movable coil of Fig. 29 is reproduced, carrying current towards us in the upper coil-sides and away from us in the lower coil-sides, as indicated by the dots and crosses outside the wires. As the coil moves down into the field, we must consider that its lower coil-side L, is cutting the flux of the poles, or that the coil is filling itself with flux. As we have shown in Fig. 37, such motion, or flux change through the coil, results in the generation of voltage which, as indicated by the dots and crosses inside the wires, will be in a direction opposite, or counter to, that of the current. We then have two e.m.fs. acting in the coil, the impressed e.m.f., E, which is necessary to maintain the current which we have supposed flowing in the coil, and the induced, or counter, e.m.f., e, which opposes the impressed e.m.f. and *lasts as long as the flux through the movable coil is changing.* The current, I, through the coil will then be

$$I=\frac{E-e}{R}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where R is the resistance of the coil.

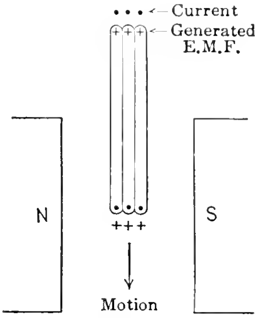


FIG. 40.—Coil carrying current, being moved into a magnetic field; the motion induces a voltage in such a direction that the current flowing in the coil is decreased. This evidently decreases the force which is pulling the coil into the magnet's field, so that the induced current thus tends to prevent the interlinkages from increasing, as before.

Thus, by reducing the current, the generation of a counter e.m.f. acts to decrease the mechanical force causing the motion of the coil. Furthermore, as this decreased current will reduce the flux caused by the coil itself, it follows that the c.e.m.f. tends to prevent an increase of the interlinkages of the system.

When the coil is forcibly removed from its position of stable equilibrium in Fig. 30, being moved out of the field as in Fig. 41, and thus cutting flux, the voltage induced will be in the same direction as the current, so that the latter will be momentarily increased. The induced e.m.f. therefore acts to cause an increase in the force developed by the coil, which force must be overcome in removing it from the field. Furthermore, as this increased current will increase the flux set up by the coil, it follows that the c.e.m.f. tends to prevent a decrease in the interlinkages of the system.

In general, we may then conclude that whenever a coil moves so as to cause the flux interlinkages to change, reactions at once result which tend to oppose this change.

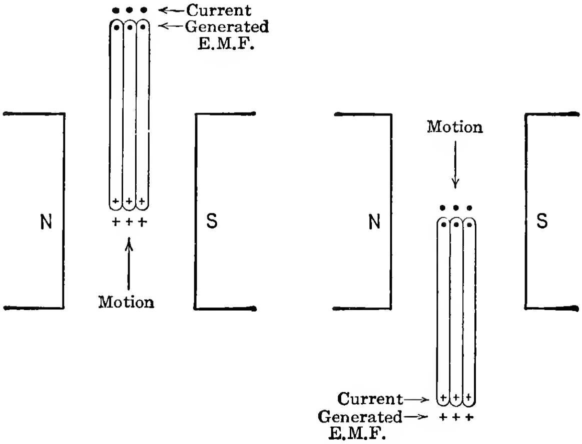


FIG. 41.—Coil carrying current as before, now being moved out of the magnetic field; the generation of e.m.f. by the cutting of the flux now causes increase of current, thus increasing the force by virtue of which the coil tends to retain its position. This action evidently tends to maintain the interlinkages constant, as before.

42. Eddy Currents.—In the construction of electrical machinery, iron is extensively used as a path for the flux and a support for the copper conductors. Whenever the flux through iron changes, voltages are induced, and, as iron itself is a conductor, currents flow in the volume of the iron. The action of these currents (generally called eddy, or Foucault, currents) is exactly the same as that of those just discussed, although their presence is usually objectionable, and every effort is made to reduce them to reasonable values.

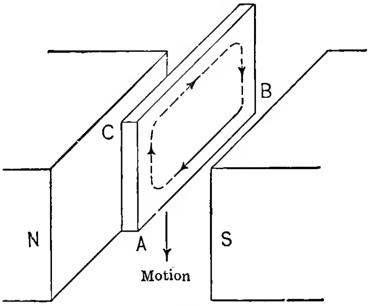


FIG. 42.—So-called "eddy currents" generated in a sheet of iron moving into a magnetic field; because of this effect all iron cores in which the magnetic flux changes when the apparatus is in operation, are made laminated, being built up of thin sheets of iron, insulated from one another.

Suppose a sheet of iron to be pushed edgewise into a magnetic field, as in Fig. 42. As the edge moves into the field, a voltage will be induced from *B* to *A* and, there being no voltage induced elsewhere, current will circulate or eddy in the iron, as shown by the dotted lines.

To reduce the eddy currents, it is necessary to build up the volume of iron with thin sheets or laminations parallel to face *CA*, and to provide some degree of insulation between them, so that current cannot pass freely from one lamination to another. Eddy currents will be further discussed later on in the text.

43. Work Done in Moving a Conductor Carrying Current in a Magnetic Field.—We have already shown that when a conductor carrying current is placed in a magnetic field (Fig. 22), the force exerted on the conductor is

$$F = Bli,$$

where F = force in dynes;
B = flux density in lines per square centimeter;
l = length of conductor in centimeters;
i = current in abamperes.

If the conductor is moved a distance X centimeters (Fig. 43), at right angles to both itself and the field, the work done in ergs is

$$w = FX = BliX. \qquad (11)$$

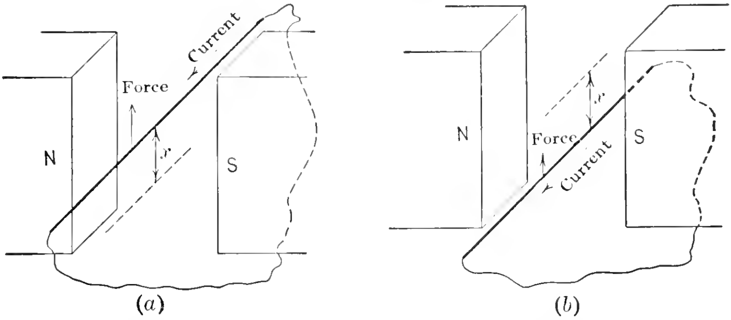


FIG. 43.—Conductor carrying current, moved in a magnetic field; the work done is proportional to the product of current and flux cut by the conductor.

The quantity of BlX is evidently the total flux cut in moving the conductor over the distance X and may be represented by ϕ. Hence,

$$w = i\phi. \qquad (12)$$

This expression is an important one, indicating that whenever a current-carrying conductor moves so as to cut flux, work is done in cutting this flux. Whatever work has been done in setting up either the current in the coil or the magnetic flux through which the coil is being moved, is not here considered, but merely the work done in moving the current-carrying conductor through the constant flux ϕ.

If the conductor is forced through the field, the work is done *by the agency* pushing the conductor, as in the electric generator. But if the conductor is allowed to move by itself (in the opposite direction), the work is done *by the conductor* as in the electric motor.

The above expression for work was derived for a single conductor. If there were n conductors in series, the work done would be

$$w = \phi ni, \qquad (13)$$

and we find that the work is equal to the change in the product of current and flux interlinkages.

If we imagine the conductor in the position shown in Fig. 43*a*, but with the field not set up, then if the field is built up to the same value it had in the foregoing discussion, the work that would have to be done *in building up the field* in the circuit of which the conductor forms a part would be the same as is done *in moving the conductor into the field* so as to enclose all the lines of force. This becomes the more apparent if we look at it from the standpoint of flux interlinkages. Work must be done in changing their number, whether it be by moving the coil or building up the interlinkages within the coil.

From section 40, it is evident that whenever the current-carrying conductor moves, voltage is generated. By the use of equation (12) we can evidently derive the relation

$$w=i\phi=(it)\frac{\phi}{t}=qe. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The expression $\phi/t=e$ is evidently the voltage generated by the moving conductor, and $it=q$ (current times time), is the quantity of electricity. Hence, the work done is that necessary to raise a quantity of electricity through a certain potential difference.

Thus, when a conductor carrying current is moved through a magnetic field, the mechanical energy supplied must be used in doing the electrical work represented by raising a quantity of electricity through difference of potential. This is the action of the generator.

44. Power—Unit of Power.—If the work done in moving a conductor carrying current through a magnetic field is represented, as before, by

$$w=i\phi,$$

then the rate of work done or power is

$$p=\frac{w}{t}=i\frac{\phi}{t} \text{ ergs per second.}$$

But $\phi/t=e$, the voltage generated by the moving conductor. Hence

$$p=ie \text{ ergs per second.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Expressed in amperes and volts we have,

$$p=I\cdot 10^{-1}\cdot E\cdot 10^{8}=EI\cdot 10^{7} \text{ ergs per second.}$$

The *watt* being taken as 10^7 ergs per second, electrical power in watts is then the product of amperes and volts, or,

$$P=EI. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The unit of electrical power used probably more than any other is the *kilowatt*, which is equal to one thousand watts.

The *joule*, a unit of work much used, is taken as 10^7 ergs. A watt is thus a joule per second, or a joule is one watt-second, i.e., a watt for a second (not a watt per second). (It is convenient to remember that a joule of work is about three-quarters of one foot-pound of work.) The unit of energy most frequently used in electrical measurements is the kilowatt-hour, which is equal to the energy supplied by one kilowatt for one hour; it is equal to 3.6×10^6 joules.

45. Field Intensity Produced by Current Flowing in a Straight Wire.—If a unit north pole is moved around the conductor as in Fig. 44, describing a circle of radius $=r$ centimeters, against the field intensity H (r centimeters from the wire), the amount of work done is

$$w = H2\pi r.$$

The work done is also equal to the product of current flowing in the conductor and the flux cut by the conductor as the pole is moved around. There being 4π lines emanating from the unit pole, we have

$$w = \phi i = 4\pi i.$$

Equating, we have

$$H2\pi r = 4\pi i,$$

or

$$H = \frac{2i}{r}. \qquad (17)$$

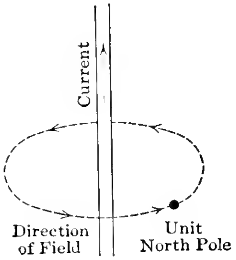


FIG. 44.—Field about a straight wire carrying current can be calculated from the force which must act on a unit pole in the vicinity of the conductor.

If the force on the unit pole is $H = \frac{2i}{r}$ dynes, there must be this number of lines of force per square centimeter a distance of r cm. from the wire, due to the current in the wire, or,

$$B = \frac{2i}{r}.$$

If current is expressed in amperes, then

$$H = \frac{2I}{10r} \quad \text{and} \quad B = \frac{2I}{10r} \quad \text{or} \quad \frac{0.2I}{r}. \qquad (18)$$

46. Force between Two Parallel Conductors Carrying Current.—In Fig. 45, either conductor may be considered as lying in a field of flux density $\frac{2i}{r}$ due to the current i in the other. Hence, there will be a force acting between them, which, according to Fleming's left-hand rule, is one of attraction when the currents are flowing in the same direction in the two conductors (Fig. 45*a*), and one of repulsion when the currents are in opposite directions (Fig. 45*b*).

The force on either conductor, due to the field of the other, is

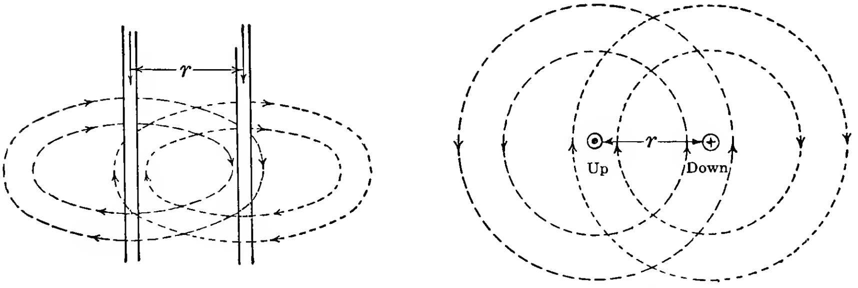
$$F = Bli \text{ dynes.}$$

Since $B = \frac{2i}{r}$ we have

$$F = \frac{2i}{r} \cdot li = \frac{2i^2 l}{r}.$$

The force per centimeter length of conductor is then

$$F = \frac{2i^2}{r}. \qquad \text{. . . . .} \qquad (19)$$



(*a*)
Attraction

(*b*)
Repulsion

FIG. 45.—The force acting between parallel conductors carrying current, depends upon their proximity, and magnitude of current; the direction of the force, attractive or repulsive, depends upon the relative directions of the currents.

An application of this principle is found in the Siemens dynamometer, in which two coils at right angles to each other, one stationary and the other movable, are used to measure current. A meter which operates on the principle of the dynamometer is shown in Fig. 121 p. 133.

It is convenient to remember that when two conductors, carrying current, act on one another, and they are free to move, they will always so move as to increase the total magnetic field or interlinkages set up by the combination of turns of which the conductors are but a part. If the two conductors are carrying current in the same direction, more field will be produced if they come close together; if in the opposite direction, it is evident that the two currents are tending to neutralize each other magnetically, so that the force between them must tend to push them apart, thereby increasing the sum of the magnetic effects of the two.

It is interesting to note that if we have a circuit consisting of two coils (or conductors) in series, and these coils are carrying current, they will so act on each other that, if allowed to move, the total interlinkages of the system will increase. Now, the increase in interlinkages really represents an increase in the stored magnetic energy of the system, as will be shown in section 54, and, of course, the increase in the stored energy must come from the battery which holds the current in the circuit constant as the coils move together. But if the extra energy drawn from the battery, as the coils (or conductors) move together is measured, it will be found to be *twice as much as the increase in stored magnetic energy.*

As the two coils move together they will overcome whatever mechanical forces are tending to hold them apart, and the amount of work (mechanical) done in actually moving the coils against these resisting forces can be shown to be just equal to that by which the magnetic energy of the system has been increased.

47. Permeability.—In section (27) it was stated that, in air, unit field intensity or unit magnetizing force produced one line of force per square centimeter or unit flux density, and from this it followed that $H=B$.

In other mediums, unit field intensity or unit magnetizing force sometimes produces more than one line per square centimeter; the ratio of the number of lines in the medium to the number which would be produced in air by the same magnetizing force is termed the *permeability* of the medium, and is designated by the symbol μ. We have then,

$$\mu=B/H \quad \text{and} \quad B=\mu H. \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

For such mediums as iron and its compounds, and to a lesser degree cobalt and nickel, μ is considerably greater than unity, which means that for a given magnetizing force, a greater flux density will result through a volume of iron than through an equal volume of air. For all other mediums, permeability is practically equal to unity.

Permeability is then the ratio of the flux density produced in a volume of any medium, to the flux density produced in an equivalent volume of air, by the same magnetizing force.

For iron, permeability is not a constant, but varies greatly with the flux density, from perhaps 50 for very low densities, say one gauss, to perhaps 3000 at densities of several thousand gausses. In specially treated iron (refined in vacuum) the permeability may be five or ten times that of ordinary electric steel, but so far the processes of manufacture do not make it commercially available.

48. Magnetomotive Force.—Consider a toroid (solenoid bent into the form of a ring) of N turns, with an air core (permeability$=1$) of mean radius r, and carrying a current of i abamperes, as in Fig. 46. The field intensity in the core is H lines per square centimeter. This field density H will be nearly constant over the cross-section of the core, providing the radius of the cross-section of the ring is small compared to r, the average radius of the ring; this condition holds good, for example, for a toroid in the form of a bicycle tire, but would not hold good for a toroid of the proportion of a doughnut.

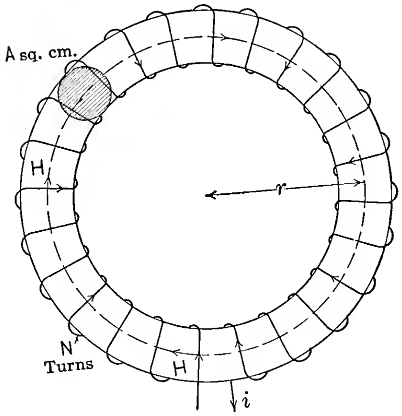


FIG. 46.—Toroidal coil carrying current; if the inner and outer radii are nearly the same this is a very simple magnetic circuit to analyze.

If a unit north pole is carried around the magnetic circuit, the work done will be equal to

$$w = 2\pi r H \text{ ergs.}$$

The 4π lines emanating from the unit pole, however, in moving around the circuit, in a time we take as t seconds, and cutting the N turns, will generate in the toroid an e.m.f.

$$e = \frac{4\pi N}{t} \text{ abvolts.}$$

For t seconds, then, the toroid acts as a generator, inducing e abvolts and supplying i abamperes. It thus furnishes an amount of power ei ergs per second, or a total amount of work of eit ergs.

The mechanical work done in moving the pole is thus transformed into electrical work, and hence these must be equal. Hence

$$2\pi r H = eit = \frac{4\pi N i t}{t} = 4\pi N i.$$

The length of the magnetic path is $2\pi r = l$. Hence,

$$lH = 4\pi N i,$$

or

$$H = \frac{4\pi N i}{l}. \qquad (21)$$

The quantity $lH = 4\pi N i$, which represents the work needed to move the unit pole around the toroid, is called the *magnetomotive force* (abbreviated m.m.f.) of the coil. H is then the m.m.f. per centimeter length (or work required to move unit pole one centimeter against the field); which agrees with the statement in section 30.

If the current is expressed in amperes, then

$$\text{m.m.f} = lH = 0.4\pi N I, \qquad (22)$$

and

$$H = \frac{0.4\pi N I}{l}. \qquad (23)$$

Magnetomotive force is thus proportional to the product of amperes and turns, or ampere turns. Nor is it necessary that the turns be evenly distributed over the whole magnetic path; they may be bunched in one or more places, but the m.m.f. for the complete magnetic circuit is the same as though they were distributed uniformly.

It is to be noticed that the term force, when used in the word magnetomotive force, is really a unit of work, not force. The same comment holds for the term electromotive force.

49. Reluctance and the Law of the Magnetic Circuit.—The reluctance of a magnetic circuit may be defined as the resistance offered by the circuit to the setting up of a magnetic field, and we may write that

$$\text{Flux} = \frac{\text{m.m.f.}}{\text{Reluctance}} = \frac{\text{m.m.f.}}{\mathcal{R}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Unit reluctance, called the *oersted*, is that of a centimeter cube of air, for when unit m.m.f. is applied across a centimeter cube of air, one line per square centimeter is produced through a length of one cm. But if unit m.m.f is applied to a centimeter cube of a magnetic material of permeability μ, then μ lines per square centimeter will be set up and the reluctance will be $1/\mu$.

It is reasonably evident that the reluctance of a path will increase with the length, and decrease as the area of the path increases. Thus, the reluctance of a path of l centimeters length and uniform area of cross-section, A, will be

$$\mathcal{R} = \frac{l}{\mu A}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

and hence

$$\phi = \frac{\text{m.m.f.}}{l \ \ \mu A} = \frac{0.4\pi N I \mu A}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

If the magnetic path is made up of several parts in series, as is generally the case, the expression for reluctance of the entire path will comprise several terms. We have then

$$\mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 +,$$

and

$$\mathcal{R} = \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \frac{l_3}{\mu_3 A_3} + \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

where $\mathcal{R}$ is the reluctance of the entire circuit.

50. Reluctances in Parallel.—Consider a ring built up of three materials, with permeabilities, μ_1, μ_2, μ_3, as in Fig. 47. With a given m.m.f. impressed on the entire circuit the flux produced in each section will be,

$$\phi_1=\frac{\text{m.m.f.}}{\mathcal{R}_1},\quad \phi_2=\frac{\text{m.m.f.}}{\mathcal{R}_2},\quad \phi_3=\frac{\text{m.m.f.}}{\mathcal{R}_3}.$$

Since the total flux $\phi=\phi_1+\phi_2+\phi_3$,

$$\phi=\frac{\text{m.m.f.},}{\mathcal{R}}=\text{m.m.f.}\left(\frac{1}{\mathcal{R}_1}+\frac{1}{\mathcal{R}_2}+\frac{1}{\mathcal{R}_3}\right);$$

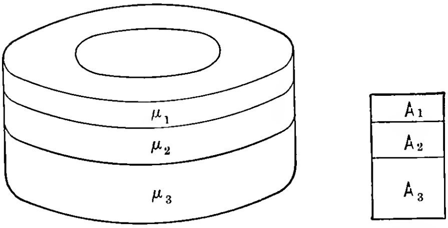


FIG. 47.—Magnetic circuits in parallel; the reluctance of the whole path is evidently less than that of any one of its sections, so that the addition of sections permits more flux with a given magneto-motive force.

the reluctance of the entire circuit will then be determined by the expression

$$\frac{1}{\mathcal{R}}=\frac{1}{\mathcal{R}_1}+\frac{1}{\mathcal{R}_2}+\frac{1}{\mathcal{R}_3},$$

or

$$\mathcal{R}=\frac{1}{\frac{1}{\mathcal{R}_1}+\frac{1}{\mathcal{R}_2}+\frac{1}{\mathcal{R}_3}+}. \qquad (28)$$

51. Permeability not Constant.—Not only is there a difference in permeability between various irons and steels and other magnetic materials, but it is found that the permeability of even one particular sample of magnetic material is dependent upon the degree to which the material is magnetized, or upon the flux density.

If a completely demagnetized sample, say in the form of a ring, is gradually magnetized, and the corresponding values of m.m.f's. per centimeter length and flux densities are plotted, a so-called *magnetization curve* is obtained, as in Fig. 48.

Since permeability is the ratio of flux density to m.m.f. per centimeter length, or $\mu=B/H$, the values of permeability corresponding to the magnetization curve can readily be determined and plotted, as in Fig. 48. It will be seen that as the flux density increases, the value of μ first increases to a maximum and subsequently decreases.

The relation between permeability and flux density, for an unknown sample, *can only be determined experimentally*, and it becomes at once evident that the use of formulæ involving reluctance in dealing with magnetic problems is impracticable. If we deal with reluctance to get flux, and hence flux densities, we must assume values for permeability, but as permeability depends upon flux density, we can only arrive at a solution after several trials.

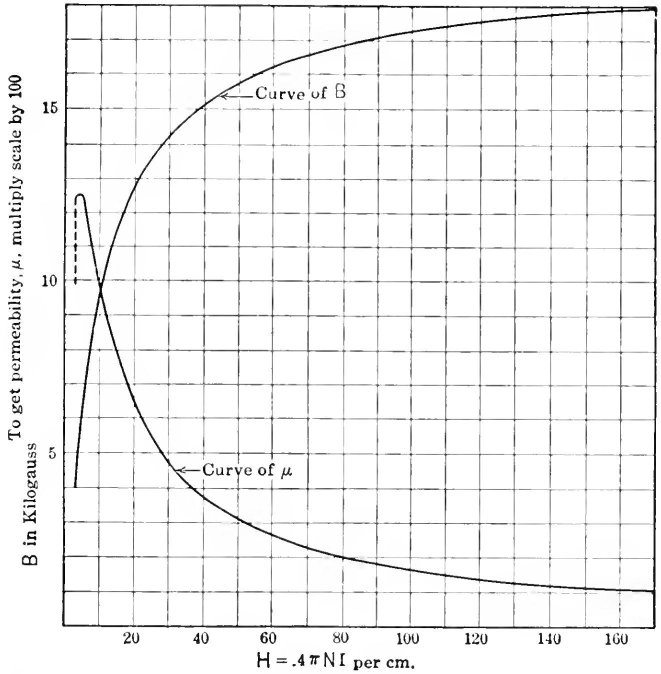


Fig. 48.—Magnetization and permeability curves for cast steel. For extremely weak magnetizing forces the permeability may be as low as fifty, this low value holding for magneto-motive forces less than one ampere-turn per centimeter.

52. B–H Curves.—For the practical solution of problems involving magnetic circuits, use is made of magnetization curves, experimentally determined, giving the relation between m.m.f. per centimeter length and corresponding flux densities for the quality of iron to be used; such curves are shown in Fig. 49.

Since

$$H=\frac{0.4\pi NI}{l},$$

we may express m.m.f. per centimeter, in ampere turns per centimeter, which is the form we find most convenient to use.

Therefore, if in a given magnetic circuit it is desired to set up a given flux, we first determine the flux density $\left(B=\frac{\phi}{A}\right)$ and then, referring to the

B–H curve, determine the required ampere-turns per centimeter length; this figure multiplied by the length gives the necessary total ampere-turns to set up the required flux.

The electrical designer, whose business it is to determine the proper sizes of the various parts of the magnetic circuits of motors and generators, works entirely from *B–H* curves. The testing laboratory furnishes him with an experimentally determined curve, showing the relation between ampere-turns per centimeter and flux densities for the materials used in the magnetic circuit; he then properly proportions the yoke, poles, etc., so that a reasonable number of ampere-turns suffice for magnetizing the field of the machine being designed.

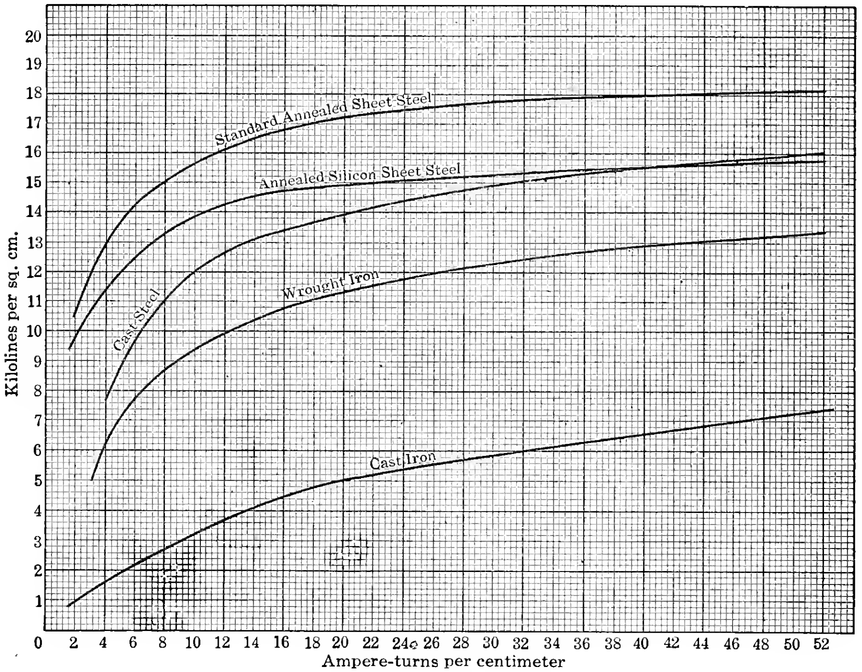


Fig. 49.—Magnetization curves, or *B–H* curves, for various materials used in the construction of the magnetic circuits of electrical apparatus.

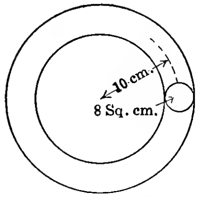


Fig. 50.—The simplest type of magnetic circuit.

53. Applications.—*Example* 1. It is required to set up 40,000 lines in a cast-iron ring with a circular cross-section of 8 sq. cm. (Fig. 50). The ring has a mean radius of 10 cm.

The flux density being 5000 lines per square centimeter, we require, from the curves in Fig. 49, 20 ampere-turns per centimeter length, or a total of $2\pi \cdot 10 \cdot 20 = 1257$ ampere-turns. This can be accomplished by passing 1 ampere through 1257 turns wound on the ring, or 0.2 ampere through 6285 turns, etc.

Example 2. In a ring of the same dimensions, composed half of cast steel and half of wrought iron, it is desired to set up a total flux of 80,000 lines.

Flux density = 10,000 lines per square centimeter
Ampere-turns per centimeter for cast steel = 6.4
Ampere-turns per centimeter for wrought iron = 12.2
Total ampere-turns, cast steel $= \pi \cdot 10 \cdot 6.4 = 201$
Total ampere-turns, wrought iron $= \pi \cdot 10 \cdot 12.2 = 383$

Total ampere-turns 584

If the ring were cut with a hack saw, making an air gap 0.1 cm. long, additional ampere-turns would be required to maintain the same flux density. From the expression

$$lH = 0.4\pi NI,$$

when

$$\mu = 1, \quad B = H,$$

we require for $B = 10{,}000$.

$$\frac{10{,}000}{0.4\pi} = 7962 \text{ ampere-turns per centimeter}$$

or 796 ampere-turns for an air gap of 0.1 cm.

Example 3. A transformer core, made of annealed silicon sheet steel, has the form and dimensions given in Fig. 51. The winding is all in the center leg of the core, and contains 110 turns. How many amperes are required in the winding to produce a flux of 1,400.000 lines in the center leg?

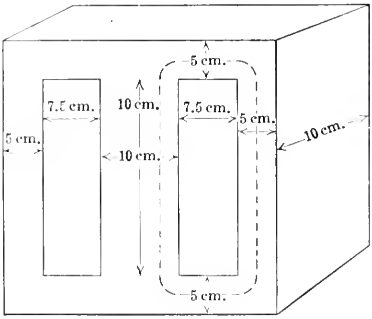


Fig. 51.—A magnetic circuit, in which part of the path is branched, consisting of two parts in parallel.

In this problem, we notice that the flux going through the center leg of the core divides, and half goes each way. The density is therefore the same in each outside leg of the core as in the center one. The average path of the magnetic flux is about as indicated in the figure by the dotted line and has a length of 55 cm. The flux density is equal to $1{,}400{,}000 \div 100 = 14{,}000$ lines per square centimeter. By reference to Fig. 49, it is seen that such a flux density requires 10.8 ampere-turns per centimeter of length. The total ampere-turns required, therefore, is

594 and as there are 110 turns in the coil, the required current must be $594 \div 110 = 5.4$ amperes.

54. Energy Stored in a Magnetic Field.—It was shown on page 41 that when the flux threading a coil of N turns, carrying a current i abamperes, was increased to a final value ϕ, an amount of work was done.

$$w = \phi N i \text{ ergs}, \qquad (29)$$

This flux, it will be noted, is *not* the flux set up in the coil by the current i, in the coil itself, but a flux maintained by some other coil or magnet. There is finally, threading the coil, not only this introduced flux, ϕ, but an additional flux set up by the coil itself, due to current i.

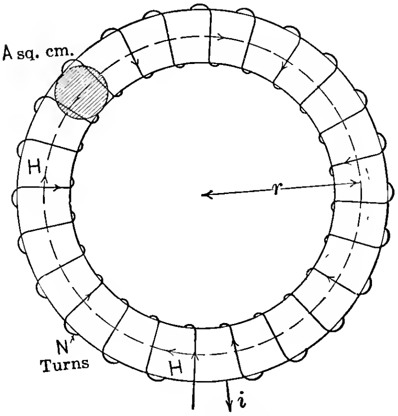


Fig. 52.—Toroidal coil carrying current.

Consider again the toroid of Fig. 46, reproduced as Fig. 52. We have seen that when a steady current, i, is flowing, the value of the flux set up in the coil by the current, is, from Eq. (26),

$$\phi = \frac{4\pi N i \mu A}{l}. \qquad (30)$$

This flux being produced by the current, it is obvious that if the current is allowed to grow at a uniform rate from zero to a value i, in time t seconds, the flux will also increase at the same rate from zero to its final value ϕ. For the t seconds considered, the average value of the current is $i/2$ and similarly the average value of the flux is $\phi/2$. We may apply Eq. (13) by considering that we have increased the flux threading the coil from zero to a value ϕ while the coil was carrying an average current $i/2$, and accordingly the work done is

$$w = \frac{\phi N i}{2} \text{ ergs}, \qquad (31)$$

and this must represent the energy stored in the field at the end of the t seconds.

It will be noticed that this is only one-half as much energy as was represented in Eq. (29). This is due to the fact that in obtaining this equation the field was supposed uniform, and independent of current strength, being set up by the permanent magnets and not by the current in the coil.

If we now substitute in Eq. (31), for ϕ, its value as given by Eq. (30), we have

$$w=\frac{4\pi N^2i^2\mu A}{2l}. \qquad (32)$$

Multiplying numerator and denominator by $4\pi l$ and substituting for $\frac{4\pi Ni}{l}$, its equivalent, H, we have,

$$w=\frac{(4\pi)^2N^2i^2\mu Al}{l^2 4\pi\cdot 2}=\frac{H^2\mu^2Al}{8\pi\mu}. \qquad (33)$$

Since $B=\mu H$,

$$w=\frac{B^2Al}{8\pi\mu}. \qquad (34)$$

The product Al represents the volume of the magnetic field, and the energy stored in the field per unit volume in a medium of permeability μ is therefore

$$w=\frac{B^2}{8\pi\mu}, \qquad (35)$$

and for air,

$$w=\frac{B^2}{8\pi}. \qquad (36)$$

The fact that with the same flux density, the amount of energy stored per unit volume in a medium of permeability μ is less than that stored in air per unit volume, is accounted for by the fact that less ampere-turns are required for the former than for the same flux density in air.

If B is expressed in lines per square centimeter and A and l in centimeters, then w represents the energy in ergs, stored per cubic centimeter.

55. Self-Inductance.—In Eq. (32), for the energy in a magnetic field

$$w=\frac{4\pi N^2i^2\mu A}{2l}=\frac{4\pi N^2\mu A}{l}\left(\frac{i^2}{2}\right),$$

the part $\frac{4\pi N^2\mu A}{l}$ is a constant; it is called the coefficient of self-induction, or merely the self-inductance of the circuit, being generally represented by the symbol L. It obviously depends upon the physical dimensions of the circuit and the value of μ.

Thus, the energy stored in a magnetic circuit may also be represented by

$$w=L\frac{i^2}{2}, \qquad (37)$$

where i is in abamperes. The value of L is ordinarily given in henries, but in the above equation if w is to be in ergs, L is measured in abhenries or centimeters. It may seem strange that such a quantity as self-induction is to be measured in centimeters, but such is actually the case, as may be determined from the dimensional equations for L. (One henry $= 10^9$ abhenries.)

Self-induction will be further discussed in Chapter IV.

56. Pull of Magnets.— Consider a horseshoe magnet as shown in Fig. 53, the keeper of which is separated from the magnet a distance D cm. Let A be the area of separation at right angles to the lines of force (in this case the area of the two pole faces), and let B represent the flux density in the air gap.

The total energy stored in the air gap will be, from Eq. (34),

$$w = \frac{B^2}{8\pi} DA.$$

If the keeper is allowed to move a very small distance, x, nearer the magnet, the change in the amount of energy stored in the air gap will be

$$w' = \frac{B^2}{8\pi} xA,$$

the distance x having been taken so small as not to appreciably change the value of B.

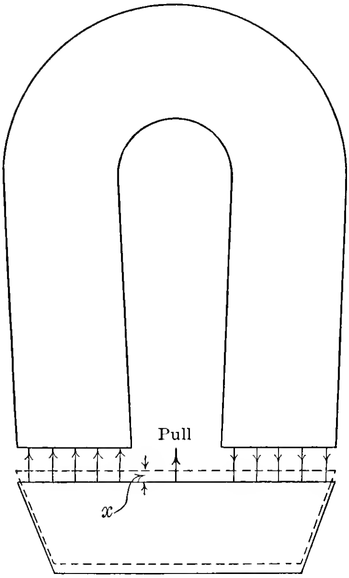


Fig. 53.—The pull of an electromagnet on its armature can be calculated from the change in the energy stored in the air gap, after the armature has been allowed to move an amount so small that the flux density has not changed.

The keeper must have done an amount of work equal to the change in the amount of energy stored in the air gap, and if P represents the pull of the keeper

$$w' = Px = \frac{B^2}{8\pi} xA,$$

from which

$$P = \frac{B^2}{8\pi} A,$$

or, per unit area $P = \dfrac{B^2}{8\pi}$ dynes per square centimeter. . . . (38)

The distance x was chosen very small so that the motion of the keeper produced only minute changes in the reluctance of the circuit. It is evident that if the keeper moves an appreciable distance, the reluctance of the entire magnetic circuit will change, and therefore the flux density may change; in such a case the pull exerted on the keeper would evidently increase as the length of the air gap decreased.

A simple application of the pull of magnets is shown in Fig. 54, where a pivoted keeper is drawn upward and made to strike a blow against a latch or trigger *L*. The shorter the air gap, as controlled by the set screw, the smaller the current through the coil which will pull the keeper up against gravity.

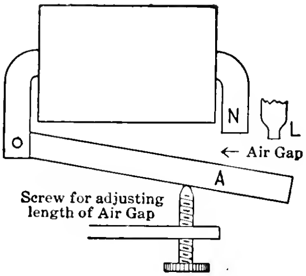


FIG. 54.—The pull of an electromagnet, depending upon the current flowing through its coil, is utilized in all kinds of automatic switches. A non-magnetic stop, *L*, keeps the movable armature, *A*, from touching the pole, *N*, where it would otherwise stick.

In Fig. 55 is shown a commercial electromagnet used for lifting heavy iron masses.

57. Action of Solenoids with Iron Cores.—If a soft iron core is introduced into a solenoid, as in Fig. 56*a*, because of the high permeability of the soft iron, the field set up within the core will be much greater than would be the case without the core and hence the total interlinkages of the system will be greatly increased. If the core were moved partly out of the solenoid, as in Fig. 56*b*, the average flux density within the solenoid, and therefore the total interlinkages of the entire system, would be less. From previous reasoning it follows that in Fig. 56*b*, there must be a force acting between the iron core and the solenoid, tending to pull the core into the solenoid. The core then moves (if free to do so) until the total number of interlinkages of the system is a maximum, as in Fig. 56*a*. The variation of pull on the core will follow some sort of curve, as is shown in Fig. 56*c*.

By completely surrounding the solenoid with iron, as in Fig. 57, a so-called "iron-clad" solenoid is obtained, resulting in a greatly increased flux density within the core. Frequently the space ahead of the core up to the dotted line is also filled with iron, producing a stop. It will be realized that with an appreciable air gap ahead of the core, the air gap will constitute the major portion of the reluctance of the magnetic circuit. As the core moves in and the air gap is shortened, the force on the core will increase gradually until the air gap becomes quite short, when the flux density, and therefore the force, will begin to increase very rapidly toward the end of the stroke. The force on the core thus increasing steadily as the core moves in, the core accelerates and strikes its stop with a considerable blow.

Applications of solenoids with movable cores are very numerous, one of the most important being in the braking of cranes and elevators. On such lifting devices, brakes are applied by powerful springs while the hook or car is not moving. As soon as power is applied to lift or lower, a solenoid pulls a plunger acting to remove spring pressure; at the moment power is removed, the brakes are again applied. Such a mechanism, as applied to an electromagnetic brake, is shown in Fig. 58.

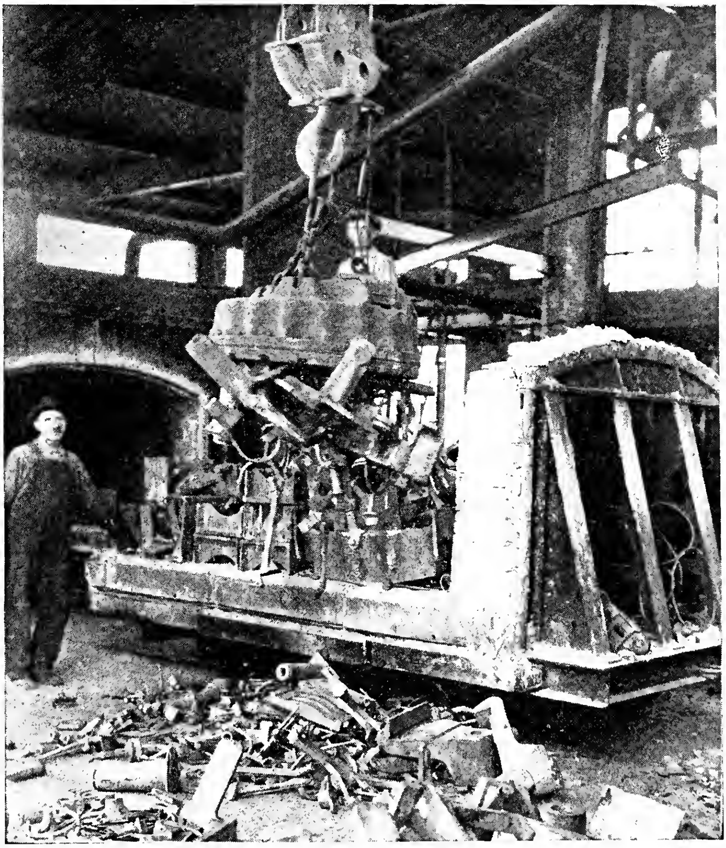


Fig. 55.—This picture shows one of the most useful applications of the electro-magnet. Besides being suitable for lifting tremendous loads of scrap iron, which would be very difficult to handle by other means, it serves in the machine shop for holding material on bed-plates for grinding, and similar operations. A magnet weighing one ton, using about 4 kw. of power for excitation, will lift about 1000 lbs.

58. Circuit-breakers.—A circuit-breaker is a switch used to open an electric circuit automatically when certain conditions arise. In its simplest, or "overload," form, it acts if the current in the circuit in which it is placed exceeds a certain predetermined value. A very simple form is shown in Fig. 59. The current to be opened is passed through a set of flexible copper strips resting on contacts, *CC*, and then around a solenoid The breaker shown is closed against the pressure of a spring coiled behind the plunger *S*, and held shut by the latch *L*. When the current rises to a value sufficient to lift its iron core *P*, the latter, driving a piece of non-magnetic material, *N*, ahead of it, delivers a blow to latch *L*, so that the breaker opens, rupturing the current at the contacts *CC*. The construction of commercial circuit-breakers is more complicated than that shown in Fig. 59, as may be seen in Figs. 60 and 61. In practically all modern types, the circuit is opened over several contacts.

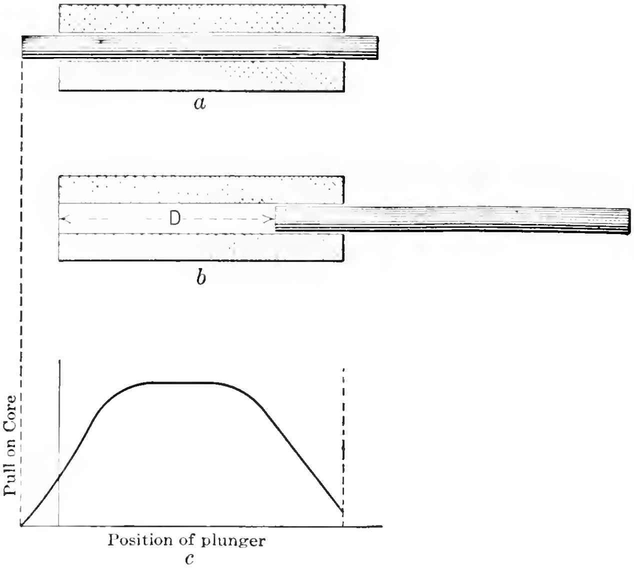


FIG. 56.—For many purposes, a solenoid with a soft iron plunger is utilized to do the pulling.

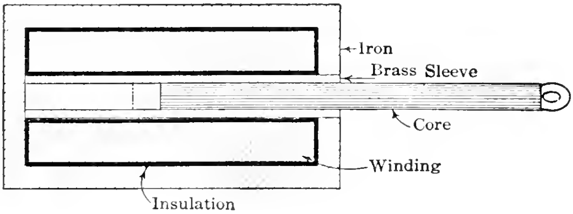


FIG. 57.—The efficacy of the plunger type of magnet is much increased by making the solenoid "iron-clad," thus giving a nearly closed iron path for the magnetic flux.

the main contacts, usually of leaf copper to insure good electrical contacts, opening first, followed by a smaller auxiliary copper contact, and the circuit is finally ruptured across carbon blocks. The idea is to have any burning or arcing take place across carbon blocks, which are readily replaceable. All breakers can be adjusted to open over a fairly wide range of current by suitable adjustment, as in Fig. 59, where the initial position of the core can be adjusted by the screw *A*.

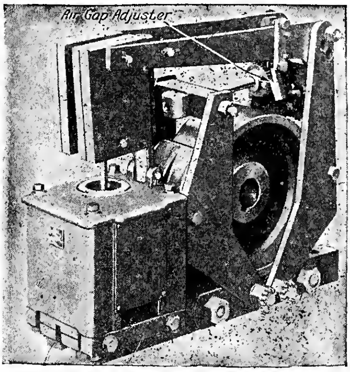


Fig. 58.—An electro-magnet is frequently used in the operation of various kinds of brakes; it is here shown controlling the speed of a hoisting drum.

Circuit-breakers are built to open circuits whenever the load current exceeds a certain value, or flows in the reverse direction, and whenever the line voltage falls below a certain value. In addition, a time limit device may be added, which allows the breaker to open only after these conditions have existed for a definite duration of time, these being styled overload, time-limit relays, etc.

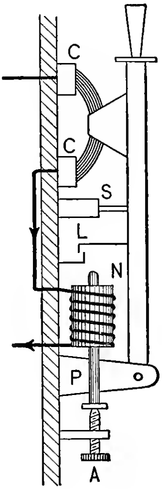


Fig. 59.—A simple type of over-load breaker; when more than a predetermined current flows through the circuit, the plunger, *P*, is lifted by the magnet, then by the trip and spring actions, the switch is opened.

59. Hysteresis.—Consider again the toroid of Fig. 46, wound now on an iron core which has been thoroughly demagnetized. If the exciting current is gradually increased, and corresponding values of flux are measured by some convenient means, and the results plotted as in Fig. 62, the relation between magnetizing force and flux density will be shown by the curve, *OA*. If the exciting current is gradually reduced, it will be found that when the current has reached zero, the value of the flux density is not zero, but some such value as *OC*, which is called the residual magnetism. In order to reduce the flux density to zero, a negative magnetizing force *OD*, called the coercive force, must be applied. Further increase in the exciting current, in the reverse direction, to the same maximum as before, gives the same maximum value of flux as before, but in the reversed direction. If the cycle is completed, the closed curve results.

It can now be shown that the area of the loop represents an amount of energy dissipated as heat during the entire cycle.

The work done in increasing the flux through a coil carrying current by an amount ϕ, we have shown to be

$$w=Ni\phi,$$

and since

$$\phi=BA, \quad \text{and} \quad Ni=\frac{Hl}{4\pi},$$

we have

$$w=\frac{Al}{4\pi}HB. \qquad \qquad (39)$$

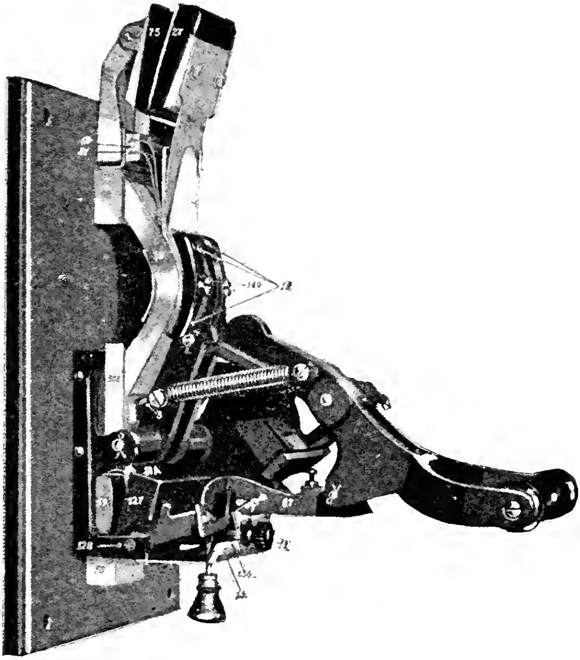


FIG. 60.—A standard type of single pole, over-load, circuit breaker.

Since Al represents the volume of the magnetic circuit, the work per unit volume is

$$\frac{w}{Al}=\frac{1}{4\pi}HB. \qquad \qquad (40)$$

If, in the toroid, we increase the flux density by a very small amount, the work done per unit volume is

$$w=\frac{1}{4\pi}Hb,$$

where b is a small increase in flux.

Referring to the hysteresis loop, as we first increase the excitation, the quantity Hb is represented by the area of the narrow strips shown in Fig. 63*a*, and this area divided by 4π is the work done in increasing the flux density by the small amount b. The total work done in increasing the flux density to its final value is then the area of the total shaded portion in Fig. 63*b*, divided by 4π.

When the excitation decreases to zero, along AC, the work done is negative; that is, energy is restored by the circuit to the amount shown by the shaded area of Fig. 63*c*.

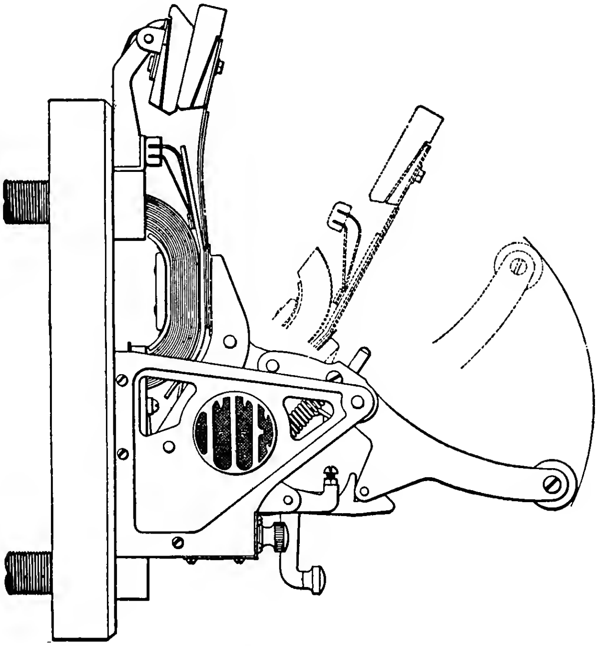


Fig. 61.—Outline sketch of a single pole breaker showing the closed and open positions. Such a breaker suffices for opening a low voltage circuit carrying as much as one thousand amperes.

As the magnetizing force increases in the opposite direction, energy is again put into the circuit, as shown in Fig. 63*d*, and as the excitation decreases a portion of it is again restored as in Fig. 63*e*.

Completion of the cycle requires still more energy, as is shown in Fig. 63*f*.

Figs. 63*b* to 63*f* represent the complete cycle, and it is evident that the area of the loop represents energy dissipated. The relative width of the loop is more or less a property of the material, and it is evident that for circuits in which the flux is reversing many times a second, materials having a narrow hysteresis loop are very desirable.

It is interesting to note that if the permeability had been constant in Fig. 63*b*, the magnetization curve would have been the straight dotted line *OA*. The energy stored in the field would then have been equal to $\frac{1}{4\pi}$ times the area of the triangle, instead of the shaded area, i.e.

$$w=\frac{1}{4\pi}\cdot\frac{BH}{2}=\frac{BH}{8\pi}.$$

Substituting for H its equivalent B/μ (B in air), we have

$$w=\frac{B^2}{8\pi\mu}\quad\text{or, for air,}\quad w=\frac{B^2}{8\pi},$$

which checks Eq. (36).

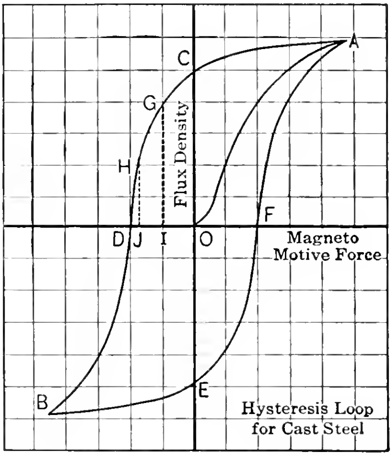


FIG. 62.—A curve showing the relation between flux in an iron core and magnetizing force, as the latter is carried through a complete cycle of positive and negative values; such a closed curve is called a hysteresis loop.

In Fig. 64 are given a number of hysteresis loops for the same material, but for different values of maximum flux densities. The maximum values of B all lie along the magnetization curve *OA*, and the loops indicate that their area increases faster than the maximum value of B. It was found experimentally, by Steinmetz, that the hysteresis loss per cubic centimeter per cycle, for any material, varies approximately as the 1.6th power of the maximum flux density. For the modern silicon-steel alloys, this exponent seems to be considerably different than for the older irons used.

The hysteresis loss for any material may be expressed according to Steinmetz, by

$$W_h=\eta f V B^{1.6}10^{-7}\text{ watts},\quad\ldots\ldots\quad(41)$$

where η = the hysteresis constant, or the loss per unit volume or weight, per cycle;

f = cycles per second;

V = volume or weight, depending on how η is expressed;

B = maximum flux density in lines per square centimeter.

For modern annealed electrical steel, η varies from 0.00095 to 0.004 erg per cm.^{3} per cycle, depending upon kind of steel and flux densities used; for ordinary flux densities in annealed sheets of silicon steel, η varies from 0.0006 to 0.0015 erg per cubic centimeter per cycle.

60. Permanent Magnets.—In many pieces of electrical apparatus, the permanent magnet plays a very important part. Practically all of the better class of continuous-current instruments use movements of the D'Arsonval type, a coil movable in the field of a permanent magnet. All watt-hour meters, for both continuous and alternating current power circuits, use permanent magnets in that part of their mechanism which serves as a speed control. All magnetos used for ignition purposes use permanent magnets for their magnetic fields.

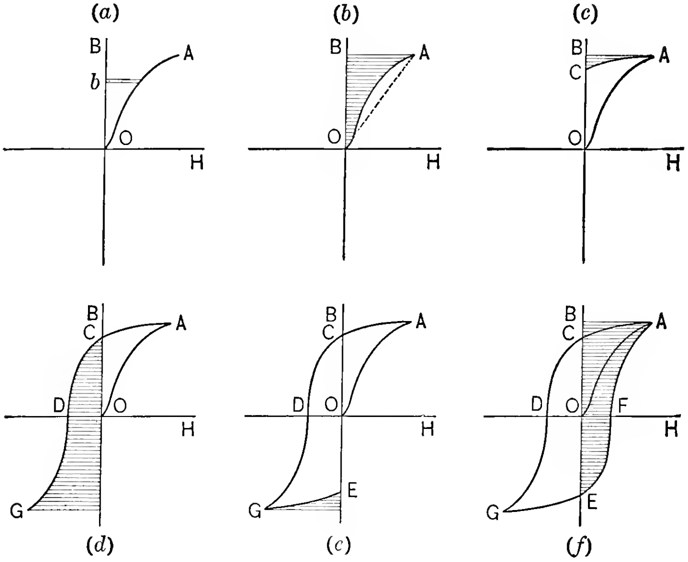


FIG. 63.—This series of figures, being steps of the complete hysteresis loop, shows how it is that the area of the hysteresis loop represents the energy lost in carrying the iron through the cycle.

Of course, no magnet is really permanent, but magnets can be so made that their strength holds sensibly constant (within, say, less than 1 per cent) for long periods of time. A special grade of steel is employed, and it is very carefully heat-treated to give permanence to the magnet. The magnet should not be subjected to vibration, and its armature (iron piece connecting or nearly connecting the poles) should always be left in position. Sometimes armatures are not supplied, but for such magnets the poles must be very close together if they are to retain much magnetic strength.

The general idea involved in the permanence of a magnet may be seen with the help of Fig. 62, which is the hysteresis loop of a piece of hard steel, having a completely closed magnetic circuit. After being properly shaped and tempered, the magnet is magnetized to point *A*; then, if the magnetizing force is taken off, the flux will decrease to point *C* on the loop, leaving a residual magnetization in the steel equal to *OC*. If there is an air gap in the magnet (as there always must be to make it of any use) this amount of flux will not stay in the magnet because of the demagnetizing effect of the air gap. The flux decreases, therefore, along the hysteresis loop, to some point *G*, fixed by the condition that with the flux *IG* through the air gap the m.m.f. required across the gap is equal to *OI*.

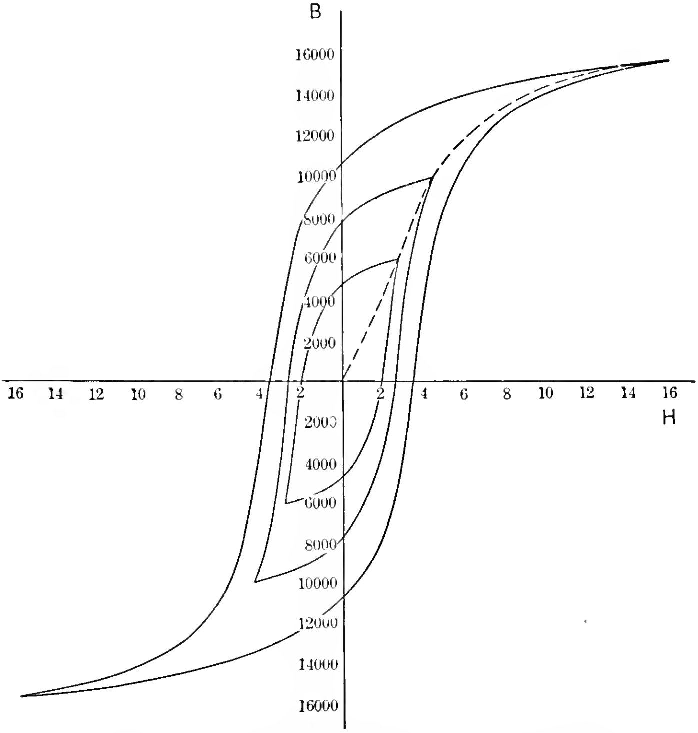


FIG. 64.—The hysteresis loops for cast steel as the sample was carried through cycles of different intensities of magnetizing force. The maximum values of flux density lie on the normal magnetizing curve; the areas of the different loops vary approximately as the 1.6 power of the maximum value of *B*.

If the magnet is jarred, this equilibrium is destroyed, and more flux disappears; jarring the magnet makes it easier for the oriented molecules of the magnet to regain their normal haphazard configuration, thus making the magnet lose some of its strength.

Instead of leaving an amount of flux, *IG*, in the permanent magnet, it is customary to demagnetize it to some point *H*, at which the demagnetizing effect of the air gap, now less than before, because of the decreased flux density through it, is much less than OJ, which is the amount of demagnetizing action required before the magnet can be further weakened. This initial weakening of the magnet by the manufacturer is known as "aging" the magnet. In actual magnets used in meter construction, the normal flux density in the magnet is about 5,000 gausses. This figure may safely be made higher if the ratio of magnet length to air-gap length is made greater; evidently the greater the air-gap length the greater is its demagnetizing action; also, the longer the piece of steel used in making the magnet the greater is its tendency to stay magnetized. The limit of the ratio of magnet length to gap length is generally fixed by economic reasons; magnet steel is expensive, and the commercial forms of magnets do not generally have a ratio greater than about fifty.

PROBLEMS

1. A north pole of a very long, slender magnet, of 40 units strength, is situated 7 cm. from the north pole, and 12 cm. from the south pole, of a slender bar magnet 15 cm. long, the pole strength of the second magnet being 120 units. What force is exerted on the north pole, and what angle does it make with the axis of the bar magnet?

2. What is the flux density at a point 25 cm. away from a supposedly isolated north pole of 65 units strength. With what force would it act on a test pole at this point, if the test pole is of 2 units strength?

3. A slender magnet 12 cm. long, suspended so as to swing in a horizontal plane, has a turning moment in this plane of 30 dyne-centimeters when at right angles to the earth's field at Washington, D. C. What is the strength of its poles?

4. The coil of a tangent galvanometer 1 foot in diameter has 10 turns and is carrying 5.2 amperes. A compass needle at the center of the coil is 3 cm. long and has poles of 10 units strength. What is the turning moment when the needle is in the plane of the coil?

5. On a certain armature there are 60 conductors, each 8 inches long, carrying 15 amperes; average flux density in which the conductors are lying is 20,000 lines per square inch. What is the force, in pounds, tending to move the conductors in a direction perpendicular to the field?

6. A coil of 260 turns is linked with 1,250,000 lines. If the flux dies to zero in 0.15 second, what is the average voltage generated during the decay of the current?

7. A conductor 8 inches long is moving in a direction perpendicular to a magnetic field of density 30,000 lines per square inch, at a rate of 300 feet per minute. What e.m.f. does it generate in abvolts? In volts?

8. A coil having 600 turns is threaded by a flux which is changing at the rate of 100,000 lines per second. The coil has 5.1 ohms resistance and is connected to a storage battery of 2 volts e.m.f. and 0.03 ohm resistance. What current flows in the circuit? (Two answers possible.)

9. A conductor carrying a current of 325 amperes is 30 cm. long and is being moved in a direction perpendicular to a magnetic field of 5,000 gausses intensity, with a velocity of 1 ft. per second. How much work is done, in ergs, and in ft.-lbs., in 0.3 second?

10. At what rate is work being done in a circuit in which 37.3 amperes are flowing under a pressure of 32 volts. How much work is done in 3.2 seconds?

11. Two conductors on an armature are parallel and $\frac{1}{2}$ inch apart. What is the force, in pounds per inch length, exerted by one of the conductors on the other when the current through them is 250 amperes?

12. A toroid, of average diameter 6 inches, has 400 turns which are carrying 5.6 amperes. How much work would be done in carrying once around the magnetic circuit a pole of 150 units strength? (This procedure is, of course, actually impossible.)

13. A cast steel ring has a mean diameter of 45 cm. and a cross-section of 30 sq. cm. The flux is 450 kilolines. How many ampere-turns are necessary?

14. A horse-shoe shaped magnet for lifting iron plates is forged of wrought iron and has a magnetic path 40 cm. long. The oxide on the iron plates is 0.1 mm. thick (note the two gaps) and the path through the plate is 15 cm. long. Density in the air gap is 13,000 lines per square centimeter, and the cross-section of the plate is twice that of the magnet. How many turns are necessary on the magnet, if the current in the winding is to be 2 amperes?

15. The air gap under the pole face of a motor has an area of 18 square inches and is $\frac{1}{4}$ inch long. The flux density is 40,000 lines per square inch. How many ergs of work are stored in the air gap? How many ft.-lbs.?

16. What is the coefficient of self-induction of the magnet described in Problem 14, if the area of each pole face is 25 sq. cm.?

17. Two No. 10 copper wires are 2 feet long, parallel to each other and 2 inches apart. What is the force of attraction between them if they are each carrying 75 amperes of current in the same direction?

18. In order to raise a certain weight the density in a horse-shoe shaped magnet, of cast steel, must be 12,000 gausses. The length of the circuit in the magnet and weight is 50 cm. If there are 1625 turns carrying 2 amperes, how near the weight must the magnet be brought to lift it? If the area of each of the poles is 28 sq. cm., how much is the weight?

19. A core is made of annealed sheet steel; its cross-section is 7.57 inches by 5.22 inches and the average length is 22.3 inches. If there are 800 turns in the coil, and a flux density of 13,500 lines per square centimeter is required, how much current must flow through the coil?

20. A horse-shoe shaped, permanent magnet has pole faces each of $\frac{1}{2}$ square inch cross-section. The flux density in the magnet is 3000 gausses. How many pounds can it lift?

21. What is the hysteresis loss, in watts, in an iron core being magnetized from a 60-cycle alternating-current line, if the core weighs 150 pounds and has a maximum flux density of 11,000 gausses, the iron being high-grade annealed electric steel, weighing 480 pounds per cubic foot?

CHAPTER IV

THE ELECTRIC CIRCUIT

61. Ohm's Law.—It was experimentally determined in 1826, by the physicist, Ohm, that there ordinarily exists a direct proportionality between the e.m.f. in a given circuit and the current which flows through the circuit. For a given circuit, if the e.m.f. is increased to twice its value, the current also increases to twice its value. If the e.m.f. is reduced to one-tenth of its value, the current is decreased in the same ratio. When a direct proportionality exists between two variables, it is always possible to express the relation in the form of an equation, by the use of a proper constant.

If I = current in the circuit,
E = e.m.f. in the circuit;

therefore, the relation discovered by Ohm was $I \propto E$, and we may put $E=kI$, where k is some constant which will generally be different for every circuit. This constant, k, is really what we call *the resistance* of the circuit (designated by R) and so we have the well-known Ohm's law,

$$E=IR, \qquad (42)$$

This expression further states that effect is equal to cause, or that action is equal to reaction. We see from it that if an e.m.f. is impressed on a circuit containing resistance only, the current must increase until the reaction IR becomes equal to the impressed force.

The factor of proportionality might have been chosen so as to write

$$EG=I, \qquad (43)$$

in which case G is called *the conductance* of the circuit; the relation between resistance and conductance is evidently given by

$$R=1/G \quad \text{or} \quad G=1/R. \qquad (44)$$

We may also write Ohm's law in the forms

$$E=IR \qquad\qquad EG=I$$

$$I=\frac{E}{R} \quad (45) \qquad\qquad E=\frac{I}{G} \quad (47)$$

$$R=\frac{E}{I} \quad (46) \qquad\qquad G=\frac{I}{E} \quad (48)$$

62. Unit Resistance. The Ohm.—If, in the first form of Ohm's law, we assume unit current and unit e.m.f., the circuit must have unit resistance, the *abohm* or the *ohm*, depending upon whether absolute or practical units are used for measuring the voltage and current. Practically the ohm is defined as the resistance offered by a column of pure mercury 106.300 cm. long, having a uniform cross-section and weighing 14.4521 grams at 0° C. When an e.m.f. of one volt is impressed across this standard ohm, a current of one ampere results.

Unit conductance is usually taken as the *mho*, so that the conductance of a circuit, measured in mhos, is the reciprocal of its resistance, measured in ohms.

The three practical units have now been defined, the volt in terms of standard cells, the ampere in terms of the amount of silver it deposits in a standard solution, and the ohm in terms of a standard mercury column. By international agreement of many years' standing, the units of current and resistance have been chosen as bases, since they are easiest to obtain for practical purposes. When measurements were crude, the value of the volt as fixed by $\frac{1}{1.434}$ of the voltage of a standard Clark cell at 15° C., apparently satisfied Ohm's law as well as the fact that one volt was generated by a conductor cutting 10^8 lines of force per second. But as measurements became more exact it was found that the value of the volt was more nearly the value now accepted, $\frac{1}{1.4328}$ of the voltage of a standard Clark cell at 15° C., or $\frac{1}{1.01830}$ of the voltage of a standard Weston cell at 20° C.

63. Joule's Law.—It has been shown that when an e.m.f. of E volts produces a current of I amperes in a circuit of resistance of R ohms, the power, or rate of work, is

$$P = EI \text{ watts} = EI \times 10^7 \text{ ergs per second.}$$

Since from Ohm's law, $E = IR$, we have

$$P = I^2R \text{ watts} = I^2R \times 10^7 \text{ ergs per second,}$$

in which form the equation states that the power used as heat in the conductor is dependent directly upon its resistance.

If the current remains constant for t seconds, the heat energy supplied by the circuit is

$$W = Pt = I^2Rt \times 10^7 \text{ ergs,}$$

and since 10^7 ergs have been defined as the equivalent of one joule,

$$W = I^2Rt \text{ joules. . . . . } \quad (49)$$

These equations are the mathematical expression of Joule's law. This law bears the name of the scientist who, in 1843, first clearly formulated the idea that the total heat developed in a resistance in a given time is proportional to the product of the resistance and the square of the current flowing.

Joule, in his experimental work, passed a known current through a known resistance immersed in a volume of water, and determined the amount of heat generated in the resistance by the rise in temperature of the water. Using the gram-calorie, the amount of heat required to raise the temperature of 1 gram or cubic centimeter of water through 1° C., Joule found the relation between heat generated in gram-calories, current, in amperes, and resistance, in ohms, to be

$$Q=0.24EIt=0.24I^2Rt. \quad \ldots \quad (50)$$

From the relation just stated, it is interesting to note that Joule was able to derive his mechanical equivalent of heat; it is evident that as the product of unit voltage and unit current (the watt) is equal to 10^7 ergs per second, from Eq. (50) the gram-calorie must be equivalent to 4.10×10^7 ergs of work. This is known as Joule's equivalent.

Joule's law is one of the most important in electrical engineering, connecting as it does electrical and heat units.

64. Reactions in an Electric Circuit.—If a brick is pushed along slowly as it rests on a board, there must be sufficient force exerted to offset the friction between the two surfaces in contact. The resisting force of friction, which has to be overcome by the force moving the brick, is called the reacting force of friction, or simply frictional reaction. If the brick is pushed up an inclined surface, the impressed force must not only be equal and opposite to the frictional reaction, but must also be large enough to overcome the component of gravitational force which is tending to make the brick slide down. In an analogous manner, if electrons are forced to flow steadily through a conductor, a kind of frictional reaction is set up, which opposes the motion of the electrons; this reaction, called resistance reaction, will be equal in magnitude and opposite in direction to the impressed force. In the electric circuit we may have other forces which the impressed force has to overcome, such as the counter e.m.f. of a storage battery; this corresponds to the force of gravity in the case of the brick pushed up an incline. When the study of alternating currents is taken up, other types of reaction will be considered.

65. Kirchhoff's Laws.—Two simple but fundamental laws, first expounded by Kirchhoff, are of great value in the solution of continuous current problems.

First Law. *The algebraic sum of the potential drops around every closed circuit is always equal to zero.*

This law is practically self-evident; if the sum of the reacting forces were not equal and opposite to the impressed force, some of this would be left over with no opposing force, and so would serve to increase the current. This increase in current flow would continue until the increased resistance reaction resulting therefrom was just sufficient to balance the impressed force.

In the case of a circuit containing the back e.m.f. of a storage battery, it is simple to measure the c.e.m.f. reaction; it exists whether current is flowing or not. Resistance reaction, however, cannot be directly measured; current must be flowing to produce the resistance reaction, and current will not flow unless a force is impressed on the circuit. In the case shown in Fig. 65, the battery, E, forces current to flow through the two resistances in series, in the direction shown; the sum of the two resistance reactions set up must be equal to the impressed voltage, E. Kirchhoff's first law states the fact that if we measure the voltage drop across A–B, and then across B–C, and add these two drops, the resultant must be the same as the impressed voltage, measured across the battery.

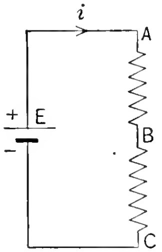


Fig. 65.—The sum of the resistance reactions set up in the two parts of this circuit must be equal in magnitude to the impressed electromotive force.

It is well at this point to distinguish between the "drop," or component of the impressed force used across the part of the circuit in question, and the resistance reaction in the same part of the circuit. They are equal in magnitude, but opposite in direction. The drop across A–B, for example, is evidently from A to B (A being higher in potential than B), and so causes the current to flow in the direction from A to B. The resistance reaction, however, is opposite in direction, from B to A, tending to stop the current. This difference in direction must be carefully considered if errors are to be avoided in solving problems.

Probably the easiest way to solve problems dealing with circuits of the type shown in Fig. 66, is to assume one of the voltages, say E, as the impressed force; then other e.m.fs. in the circuit which tend to make current flow in the same direction as the impressed force are called positive, and those in the opposite direction are negative. We might say then that the total impressed force acting in the circuit is $E+e_2-e_1$, and put this impressed force equal to the sum of the IR drops in the circuit; then,

$$E+e_2-e_1=IR_1+IR_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

or we might reckon the impressed force as $E+e_2$ and place e_1 with the drops, and so get

$$E+e_2=IR_1+IR_2+e_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (52)$$

which will yield the same result as if the e_1 had been put with the impressed force but called negative. Also e_2 might be classed with the drops; but it is to be then noted that, as we go clockwise around the circuit, the direction of potential drop through e_2 is in the opposite direction to that through the resistances and the voltage e_1. We should then write

$$E=IR_1+IR_2+e_1-e_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (53)$$

It will be seen that all three of the Eqs. (51), (52) and (53), are really the same thing and so will give the same solution.

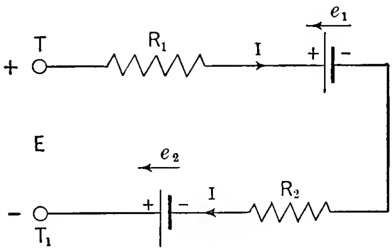


FIG. 66.

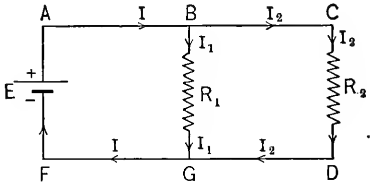


FIG. 67.

FIG. 66.—Kirchhoff's first law states that the sum of the *IR* drops in the circuit must be equal to the algebraic sum of the various e.m.f's. acting in the circuit.

FIG. 67.—In writing the equation to utilize Kirchhoff's first law an *IR* drop may be reckoned as positive in one circuit and negative in another, depending upon which direction has been assumed for the various currents.

It may well be that an incorrect direction has been assumed for the current flow, and that the solution of the problem yields a negative value for I; in this case the current actually flows in the direction opposite to that assumed. The magnitude obtained for the current will be correct however, even though its direction has been incorrectly assumed.

For the circuit shown in Fig. 67, the direction of current flow is obvious, and we write at once the equations required for solution, using Kirchhoff's first law; for circuit *ABGF*

$$E=I_1R_1 \quad \text{or} \quad E-I_1R_1=0,$$

and for circuit *ACDF*

$$E=I_2R_2 \quad \text{or} \quad E-I_2R_2=0.$$

In circuit *BCDG*, there is no voltage acting, and it will be noted that, either way we go around the circuit, one of the *IR* drops must be taken as negative. With clockwise direction of traversing the circuit,

$$0 = I_2R_2 - I_1R_1,$$

and for the opposite direction,

$$0 = I_1R_1 - I_2R_2.$$

Either of the last two equations states the obvious fact that

$$I_1R_1 = I_2R_2.$$

It will be seen that, in applying Kirchhoff's first law, we *assume* a direction for the currents in the various paths of the circuit. We then add up algebraically all the voltages in a closed path, calling positive those voltages that tend to make current flow around the circuit in the direction in which *it is being traversed,* and putting the total voltage equal to the sum of the *IR* drops in the closed path. The *IR* drops due to current in the direction in which the circuit is being traversed are called positive, and those due to oppositely flowing currents are to be called negative. This law is hence summed up for any closed path, by the equation,

$$\Sigma E = \Sigma IR. \qquad (54)$$

66. Second Law.—*The algebraic sum of the currents at any junction of conductors is always zero.* This is the same as saying that the sum of all currents flowing towards a junction is equal to the sum of all currents flowing away from the junction. Were this not so, there would be an accumulation (or the reverse) of electrons at the junction, and the potential of the point would continually change.

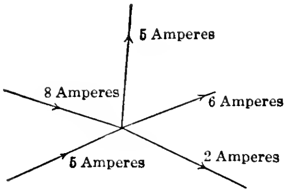


FIG. 68.—Kirchhoff's second law states that the algebraic sum of all the currents flowing towards any point in the circuit must be zero; from the electron viewpoint the law merely expresses the more or less evident fact that electrons cannot accumulate at a point, without the potential of that point continually changing.

The law is illustrated in Fig. 68, the sum of the currents flowing towards the junction being equal to the sum of the currents flowing away.

In the solution of problems involving the second law, we may assume currents flowing in either direction as positive, and those flowing in the opposite direction as negative. Where the direction of a current is uncertain, it may be assumed; if, on solving, the result is negative, the direction is opposite to that assumed.

Example. In the network shown in Fig. 69, the resistance of the cells is neglected. The direction of the various currents will be assumed as indicated in the diagram.

Applying the procedure outlined in the paragraph just preceding Eq. (54) to circuit *ABCDEFA*, we have, starting from *A* in clockwise direction,

$$2+4=2I_3+6I_1 \quad \text{or} \quad I_1=1-\frac{I_3}{3}.$$

For circuit *BCDEB*,

$$2+3=2I_3+4I_2 \quad \text{or} \quad I_2=\frac{5}{4}-\frac{I_3}{2}.$$

Applying the second law to the junction *B*, we have

$$I_1+I_2=I_3 \quad \text{or} \quad I_3-I_2-I_1=0.$$

Substituting the values for I_1 and I_2 already determined and solving, we find

$$I_1=13/22,\ I_2=14/22,\ I_3=27/22 \text{ amperes.}$$

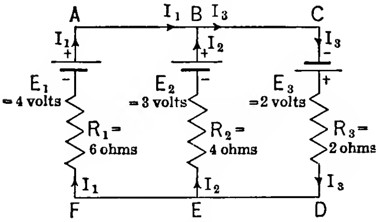


FIG. 69.

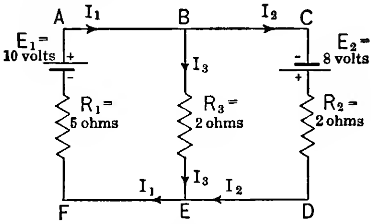


FIG. 70.

FIG. 69.—A simple three-branch network, to illustrate the application of Kirchhoff's two laws.

FIG. 70.—Sometimes the direction of current flow in a branch of the network is not at all evident, as is the case for the current I_3 in this circuit. A direction is assumed for the current in such a case, and if the solution yields a negative answer it signifies that the direction assumed for the current was incorrect.

Had we used circuit *ABEF*, traversing it clockwise, we should have considered the *IR* drop in branch 2 ($4I_2$), as negative, since the current is flowing in a direction opposite to that with which we are passing around the circuit. We should then have

$$4-3=-4I_2+6I_1,$$

and substituting the values obtained above for I_2 and I_1, this extra equation evidently serves as a check;

$$1+56/22-78/22=0.$$

which is an identity.

Example. In the network shown in Fig. 70, the direction of the current, I_3, is not obvious. Let us assume it as flowing toward the junction *E*.

For circuit $ACDF$ we have, starting from A,

$$8+10=2I_2+5I_1 \quad \text{or} \quad I_2=9-\frac{5I_1}{2}.$$

For circuit $ABEF$ we have,

$$10=2I_3+5I_1 \quad \text{or} \quad I_3=5-\frac{5I_1}{2}.$$

We also have

$$I_1=I_2+I_3,$$

Solving, we find

$$I_1=14/6, \quad I_2=19/6, \quad \text{and} \quad I_3=-5/6 \text{ amperes},$$

showing that the current is flowing in the direction opposite to that assumed and is supplied by the cell marked E_2.

Example. Wheatstone Bridge.—A special network much used for comparing resistances, originated by Wheatstone, is shown in Fig. 71. Four resistances, A, B, R and X, are connected in a mesh or square, a battery being connected across two corners, b-b', and a galvanometer or other sensitive detector of current across the other two corners, a-a'. A and B are usually fixed resistances of appropriate values, R is a resistance which can be varied through a wide range, and X is some unknown resistance.

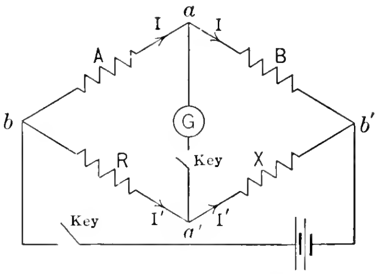


Fig. 71.—The ordinary Wheatstone bridge is a good illustration of the application of Kirchhoff's laws.

But by varying the resistance R, the desired result, that of no current flowing through the galvanometer G, is achieved. Under these conditions, calling I the current flowing through resistances A and B, and I' the current through R and X, and applying Kirchhoff's first law to the mesh, we have passing around in clockwise direction,

$$IR_A+IR_B-I'R_X-I'R_R=0,$$

or

$$IR_A+IR_B=I'R_X+I'R_R.$$

As no current is flowing through the galvanometer G, the points a and a' are at the same potential, and hence

$$IR_A=I'R_R,$$

and

$$IR_B=I'R_X.$$

Dividing one equation by the other, we have

$$\frac{R_A}{R_B}=\frac{R_R}{R_X} \quad \text{and} \quad R_X=\frac{R_B}{R_A}\times R_R. \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

Commercially, all the above members, except the unknown resistance, are suitably arranged in a convenient case with terminals for connecting in the unknown resistance. The variable resistance can be varied in 1-ohm steps from 1 to perhaps 9999 ohms, and the fixed resistances are generally of such values that their ratio may be changed at will, in multiples of 10, from 1/1000 to 1000, permitting of wide ranges in the measurement of resistance.

67. The Series Circuit.—When a number of resistances, r_1, r_2, r_3, etc., are put in series, as in Fig. 72, it is evident that the same current must flow through all of them when an e.m.f. is impressed on the entire circuit. Applying Ohm's law and Kirchhoff's first law, we have

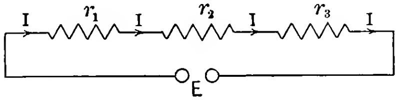


FIG. 72.—The resistance of a circuit consisting of several resistances in series is obtained by adding the various resistances making up the circuit.

$$E=IR=Ir_1+Ir_2+Ir_3,$$

where E = the impressed e.m.f.,
and R = the total resistance of the circuit.

Dividing by the current, I,

$$R=r_1+r_2+r_3, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

which states that resistances in series are to be added to obtain the total resistance.

68. The Parallel Circuit.—When a number of resistances, r_1, r_2, r_3, are put in parallel as in Fig. 73, the e.m.f. across each separate resistance must be the same, and the total current supplied must, from Kirchhoff's second law, be equal to the sum of all the branch currents.

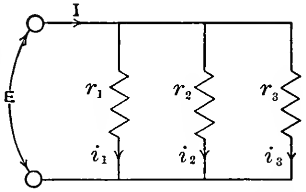


FIG. 73.—When the resistances are connected in parallel the solution of the problem shows that the circuit resistance is the reciprocal of the sum of the reciprocals of the resistances in the various branches.

$$I=i_1+i_2+i_3,$$

$$E=IR=i_1r_1=i_2r_2=i_3r_3,$$

and since

$$I=\frac{E}{R}, \quad i_1=\frac{E}{r_1}, \quad i_2=\frac{E}{r_2}, \quad i_3=\frac{E}{r_3},$$

we have

$$\frac{E}{R}=\frac{E}{r_1}+\frac{E}{r_2}+\frac{E}{r_3}.$$

Hence

$$\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}. \qquad (57)$$

The advantage of using conductances instead of resistances in parallel circuits is easily evident. Since

$$\text{conductance}=\frac{1}{\text{resistance}}.$$

Eq. (57) may be written

$$G=g_1+g_2+g_3. \qquad (58)$$

69. Resistance.—It is found experimentally that the resistance of a homogeneous conductor, of length l, and cross-section a, varies directly as its length and inversely as its cross-section or

$$r=\rho\frac{l}{a}, \qquad (59)$$

where the factor of proportionality ρ, is called the *resistivity* or *specific resistance* of the material. The resistivity of a material is dependent upon temperature, as will be discussed later. The explanation of the variation of the resistance of a conductor with length and cross-section, according to the electron theory, has been given in Section 19.

Resistivity being, from Eq. (59), the resistance of unit volume, in calculations involving conductors other than round wires, it is usually expressed in microhms (ohms$\times10^{-6}$) per centimeter cube or inch cube. Values of resistivity for the metals and alloys in common use are given in Table I.

In calculations involving round wires, the length of which, in America, is usually taken in feet, it is more convenient to express the area in circular measure. A convenient unit for this purpose is the *circular mil,* which is the area of a circle one mil, or one one-thousandth of an inch in diameter. Thus, to obtain the area of a circle in circular mils, we need only square its diameter in mils.

A square mil is the area of a square each side of which is one mil. Accordingly, the area in square mils, of a circle one mil in diameter is $\frac{\pi}{4}\times1^2$. Hence, area in square mils is equal to area in circular mils times $\frac{\pi}{4}$ or 0.7854, or

$$\text{Square mils}=\text{circular mils}\times0.7854;$$

or

$$\text{Circular mils}=\text{square mils}\times1.2732.$$

This relation is indicated in Fig. 74.

TABLE I *

RESISTIVITY AND TEMPERATURE COEFFICIENT

Material	Resistivity		Temperature Coefficient Referred to 0° C.
	Microhms per Cm. Cube at 0° C.	Ohms per Mil Foot at 0° C.	
Aluminum	2.62	15.76	0.00423
Copper, standard	1.589	9.56	0.00427
" hard-drawn	1.60	9.62	0.00408
Gold	2.20	13.23	0.00368
Iron, cast, soft	74.4	447.5	
" cast, hard	97.8	588.3	
Lead	19.8	119.1	0.00411
Mercury	94.07	565.9	0.00086
Nickel, commercial wire	9.9	59.55	0.0039
Platinum, drawn	11.0	66.17	0.00367
Silver	1.47	8.84	0.00400
Steel, soft	11.8	70.98	0.00423
" hard	45.6	274.3	0.00161
Tungsten, hard-drawn	5.42	32.60	0.0051
" annealed	4.37	26.29	0.0051
Zinc	5.38	32.36	0.00402
Calido	100.00	601.5	0.00034
Climax	87.1	523.9	0.00055
Ia Ia, soft	47.1	283.3	0.000005
" hard	50.2	301.9	−0.000011
Ideal	49.0	294.7	0.0000±
Manganin	41.4 to 73.8	249 to 443.0	0.000011 to 0.000039

1 microhm per centimeter cube = 6.0153 ohms per mil foot.

* Values taken from Pender's Handbook for Electrical Engineers, 1922.

When length is in feet and area in circular mils, resistivity is expressed in ohms per mil-foot, or the resistance of a round wire 1 foot long and 1 mil in diameter (Fig. 75). Values for resistivity in ohms per mil-foot are also given in Table I.

The circular inch is also occasionally used, it being the area of a circle 1 inch, or 1000 mils, in diameter. One circular inch thus equals 1,000,000 circular mils.

Up to 1912, Matthiessen's standard for the resistivity of copper, 1.594 microhms per centimeter cube at 0° C., was universally used. This standard was the result of measurements made by Matthiessen in 1862, on supposedly pure copper. Because of the insufficiency of his data, his results could not be uniformly reproduced and led to later determinations. The present standard for the resistivity of copper, given in Table I, of 1.589 microhms per centimeter cube at 0° C. or 1.7241 microhms per centimeter cube at 20° C., is known as the "International Annealed Standard" and is the result of many measurements upon commercial copper (density 8.89) made by the Bureau of Standards. It was accepted as standard by the International Electrotechnical Commission in 1913, by the A.I.E.E. (American Institute of Electrical Engineers) in 1914 and since then by several national engineering societies in Europe.

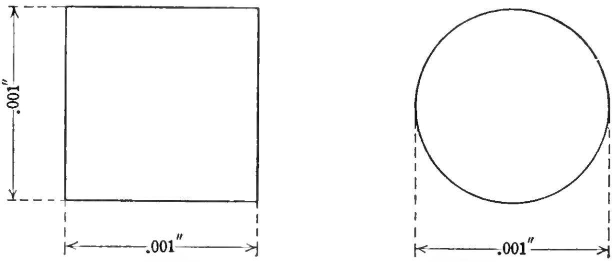


FIG. 74.—A circular mil is the area of a circle having a diameter of one thousandth of an inch; the square mil is the area of a square one thousandth of an inch on a side, or one millionth of a square inch.

70. Conductors.—It will be seen from Table I that, with the exception of silver, copper has the lowest resistivity; because of its high cost, silver is used only to a very limited degree. Copper, having high conductivity and tensile strength, is most frequently used as a conductor, because of its reasonable cost and excellent properties, and because it can easily be drawn and soldered.

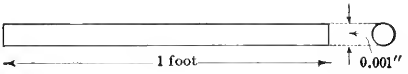


FIG. 75.—The mil-foot is a common term in connection with conductors; it is a wire one foot long, circular in cross-section, with a diameter of one thousandth of an inch.

Aluminum has about 60 per cent of the conductivity and only 30 per cent the weight of copper. For wires of the same resistance per unit length, the area of cross-section of aluminum wires must be 63 per cent greater than that of copper, and round aluminum wires are therefore about 27 per cent greater in diameter. However, for the same conductance, the weight of aluminum is only about one-half that of copper and in price aluminum is usually about 10 per cent less. One difficulty with aluminum is that it cannot readily be soldered.

Because of its greater diameter for a conductance equal to that of copper, its use in insulated wires is prohibited at usual prices of both metals, by the extra cost of insulation. Its use in machine construction is evidently out of the question because of the resulting increase in size, and, therefore cost, of the machine. Where it can be used bare, as in high-tension transmission systems and low-tension bus-bars, it is employed to some extent. Although the cost of the wire itself is often greatly reduced when aluminum is used in transmission lines this is nearly offset because, owing to its lower tensile strength, either the number of poles per mile, or their height, must be increased. Although few data on comparative costs are available, it would appear that the total cost of a transmission system, with the price of aluminum per pound about twice that of copper (which is the general proportion), is about the same for either aluminum or copper wires.

About the only commercial use of iron and steel as conductor is in resistance units and for third rails of electric railways.

Copper-clad steel wire (steel wire coated with copper) is somewhat used for long spans where higher tensile strength is desired. Its conductivity naturally depends upon the relative cross-sections of copper and steel. As now found on the market, the conductivity of copper-clad steel wire varies from 30 to 40 per cent of that of copper wire of the same total cross-section.

71. Effect of Temperature on Resistance.—*Metals.* It is shown by experiment that the resistance of conductors varies with temperature, and this phenomenon has been explained, according to the electron theory, in section 19. The variation of resistance with temperature is found to be practically linear within ordinary temperature ranges, and the relation may be expressed as follows:

$$R_{t_1}=R_t[1+\alpha(t_1-t)], \quad . \; . \; . \; . \; . \; . \; . \; . \quad (60)$$

where R_{t_1} is the resistance at any temperature t_1;

R_t is the resistance at some standard temperature t;

and α is the temperature coefficient at the standard temperature t.

If the standard temperature is taken as 0° C., the expression reduces to

$$R_{t_1}=R_0(1+\alpha_0 t_1), \quad . \; . \; . \; . \; . \; . \; . \; . \quad (61)$$

where α_0 is the temperature coefficient at 0° C.

Copper being used more than any other metal for carrying current, it is important to know the change of resistance with temperature of copper windings in machines and other apparatus.

If the values for resistance of copper at ordinary temperatures are plotted against temperature, as said before, the results lie on a practically straight line, which line, if extended, reaches zero resistance at −234.5° C. (Fig. 76). This assumes that the temperature coefficient at ordinary temperatures applies continuously down to very low temperatures. The same result may be obtained by substituting the temperature coefficient

TABLE II

TEMPERATURE COEFFICIENT OF COPPER AT DIFFERENT TEMPERATURES

$\left(\text{From equation } \alpha_t = \frac{1}{234.5+t}\right)$.

Initial Temperature	Increase in Resistance per 1° C.	Initial Temperature	Increase in Resistance per 1° C.
0	0.00427	30	0.00378
5	0.00418	35	0.00371
10	0.00409	40	0.00364
15	0.00401	45	0.00358
20	0.00393	50	0.00351
25	0.00385		

at 0° C., and the value for resistivity from Table I, in Eq. (61), equating it to zero and solving for t_1.

$$0 = 1.589(1 + 0.00427t_1),$$

from which $t_1 = -234.5°$ C.

Similar solutions for other conductors indicate that their resistance also becomes zero at about the same temperature, and there is experimental evidence available that the resistance of metals is practically nothing in the region of absolute zero.[1]

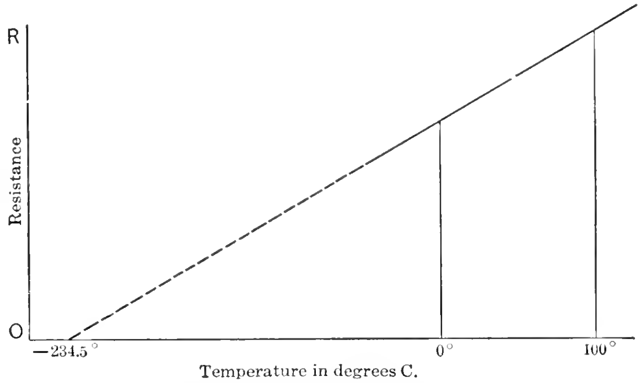


FIG. 76.—In the ordinary range of temperatures the curve plotted with resistance and temperature as co-ordinates is practically a straight line, which if continued backwards cuts the line of zero resistance at −234.5 degrees Centigrade.

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[1] That the resistance of metals is practically zero in the region of absolute zero was shown by an ingenious experiment, in which current was induced in a lead ring held at nearly absolute zero temperature by suitable cooling apparatus. The induced current was maintained practically constant for several hours after the induced voltage, which started the current, had ceased.

The temperature coefficient at 20° C. from Table II, is 0.00393, which means that the resistance of copper increases or decreases by 0.393 of 1 per cent of what it was at 20°, for each degree Centigrade above or below 20° respectively (at constant mass). Hence, for the resistance to become zero, the temperature may fall $\frac{100}{0.393}=254.5°$ below 20° or $-234.5°$. Or we may write

$$234.5+20=\frac{1}{0.00393}=\frac{1}{\alpha_{20}},$$

and

$$\alpha_{20}=\frac{1}{234.5+20}.$$

Hence, the temperature coefficient at constant mass may be determined for any temperature from the expression

$$\alpha_t=\frac{1}{234.5+t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

where t is in degrees C.

It often happens that we know the value of a resistance at one temperature and desire to know it at some other temperature. The required result may be obtained by first calculating the value of the resistance at some standard temperature, say 0° C. and from this the value at the desired temperature.

We frequently desire to determine the rise in temperature from two resistance measurements. A very simple formula can be deduced from Eq. (61) to fit this case.

Let $R_{\text{cold}}=$ resistance at any initial temperature, t_c.
$R_{\text{hot}}=$ resistance at some higher temperature, t_h.

Then

$$R_{\text{cold}}=R_0(1+\alpha_0 t_c)$$
$$R_{\text{hot}}=R_0(1+\alpha_0 t_h)$$

From Eq. (62).

$$\alpha_0=\frac{1}{234.5}.$$

Dividing one equation by the other, we have

$$\frac{R_c}{R_h}=\frac{1+\alpha_0 t_c}{1+\alpha_0 t_h},$$

from which

$$\frac{R_c}{R_h-R_c}=\frac{1+\alpha_0 t_c}{1+\alpha_0 t_h-1-\alpha_0 t_c}=\frac{1+\alpha_0 t_c}{\alpha_0(t_h-t_c)},$$

So

$$t_h-t_c=\frac{R_h-R_c}{R_c}\times\frac{1+\alpha_0 t_c}{\alpha_0},$$

and substituting the value of α_0

$$t_h-t_c=\frac{R_h-R_c}{R_c}(234°.5+t_c) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

It will be noticed in Table I that several of the alloys listed have temperature coefficients which are nearly zero. Such alloys are particularly valuable for use as standard resistances and in meters where a variation of resistance with temperature change is likely to cause considerable error.

Non-metals. The variation of resistance with temperature of most non-metals which are conductors is found to be different from that in metals, in that their resistance decreases with increase of temperature; they therefore have a negative coefficient.

This is true for carbon, gases, solutions of acids and salts, and many materials like glass which are conductors only at high temperatures. The variation in resistance of the latter class of materials is, however, irregular and is best expressed by curves. The change of resistance with temperature of materials which are much used for insulation is of great interest. In Fig. 77 is shown the variation of resistance with temperature of certain impregnated insulating materials much used for slot and cable insulation. It will be seen that their insulating properties decrease very rapidly as the temperature increases; it is because of this effect of heat on such insulating materials that a definite temperature is specified, above which it is not safe to operate an electric machine.

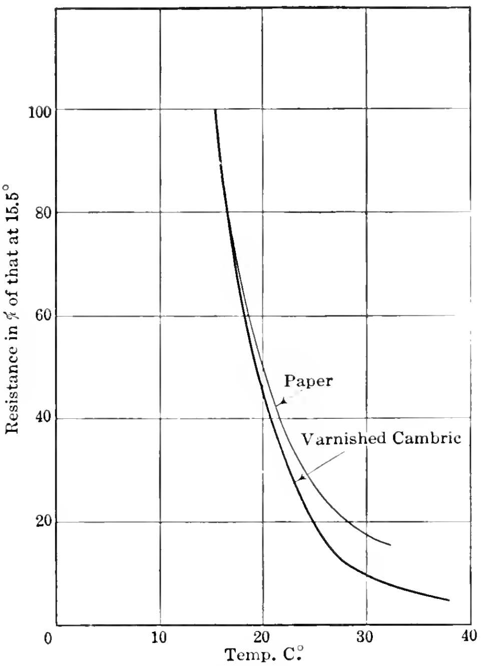

Fig. 77.—The resistance of most insulating substances decreases rapidly with temperature increase, a fact which must be considered in machine design; the above curves illustrate this idea for two of the more commonly used insulating materials. These materials are used in armature windings and in the construction of electric cables.

72. Wire Table.—The manufacture of copper wire has been standardized by gauge numbers, according to a table, originally known as the Brown and Sharpe, but now properly known as the American Wire Gage. The wire numbers are retrogressive, a larger number denoting a smaller size. The wire sizes are related according to simple mathematical law, which is most simply expressed by saying that the ratio of any diameter to the next smaller is a constant number, 1.123. The sixth power of this ratio is approximately 2, which means that the diameter doubles every six sizes, as may be seen from the wire table on page 83.

The resistance, mass, and cross-section of a wire vary as the square of the diameter; hence the ratio of the cross-section of any size to the next smaller is the square of 1.123, or 1.261. The cube of 1.261 is so very nearly 2 that it follows that the cross-section is doubled or halved every three numbers, and the resistance and mass accordingly changed.

Inspection of the wire table shows that No. 10 wire has practically,

a resistance of 1 ohm per 1000 feet at 20° C.,
a diameter of 0.1 inch
and a cross-section of 10,000 circular mils.

With these characteristics in mind, the application of the "three number" rule enables one to determine quickly the approximate resistance and area of any size wire.

If the resistance and cross-section of, say, No. 3 wire is wanted, we know at once that the resistance of No. 7 is one-half, and that of No. 4 is one-fourth, that of No. 10. Hence the resistance of No. 3 is $\frac{1}{4\times1.26}=\frac{1}{5}=$ 0.2 ohm per 1000 feet.

Similarly, its area is $10{,}000\times4\times1.26$ or 50,000 cir. mils. These figures are sufficiently close for all ordinary problems, being within 2 per cent at 20° C.

73. Electrical Factors which Determine the Size of Wire in Practice.—In selecting the size of wire for transmitting and distributing electric current, the electrical factors to be considered are: (*a*) The allowable voltage variation at the load or the voltage lost in the line as IR drop. (*b*) The allowable current-carrying capacity of the wire. (*c*) The power lost in the lines, or I^2R loss.

The weight of conductor necessary to transmit a given amount of power a certain distance, with the same loss, varies inversely as the square of the voltage.

Let V_1 and V_2 be two voltages at which an amount of power, P, is to be transmitted. The currents will then be

$$I_1=\frac{P}{V_1} \quad \text{and} \quad I_2=\frac{P}{V_2},$$

and the power loss

$$P_1=I_1^2R_1 \quad \text{and} \quad P_2=I_2^2R_2,$$

where R_1 and R_2 are the respective resistances of conductor used.

If the losses in the two cases are to be the same, we have

$$I_1^2R_1=I_2^2R_2,$$

so that

$$\frac{R_1}{R_2}=\frac{I_2^2}{I_1^2}=\frac{(P/V_2)^2}{(P/V_1)^2}=\frac{V_1^2}{V_2^2}. \quad \quad (64)$$

TABLE 111 *

SOLID COPPER WIRE—AMERICAN WIRE GAGE

100 Per Cent Conductivity; Density 8.89 at 20° C.

Gage No.	Diameter in mils	Cross-section		Resistance at 20° C. or 68° F.†		Weight in Pounds		Feet per Pound
		Circular Mi's	Square Inch	Ohms per 1000 Feet	Ohms Per Mile	Per 1000 Feet	Per Mile	
0000	460.0	211,600	0.1662	0.04901	0.259	640.5	3380	1.561
000	409.6	167,800	0.1318	0.06180	0.326	507.9	2680	1.968
00	364.8	133,100	0.1045	0.07793	0.411	402.8	2130	2.482
0	324.9	105,500	0.08289	0.09827	0.519	319.5	1680	3.130
1	289.3	83,690	0.06573	0.1239	0.654	253.3	1340	3.947
2	257.6	66,370	0.05213	0.1563	0.825	200.9	1060	4.977
3	229.4	52,640	0.04134	0.1970	1.04	159.3	841	6.276
4	204.3	41,740	0.03278	0.2485	1.31	126.4	667	7.944
5	181.9	33,100	0.02600	0.3133	1.65	100.2	529	9.980
6	162.0	26,250	0.02062	0.3951	2.09	79.46	420	12.58
7	144.3	20,820	0.01635	0.4982	2.63	63.02	333	15.87
8	128.5	16,510	0.01297	0.6282	3.32	49.98	264	20.01
10	101.9	10,380	0.008155	0.9989	5.28	31.43	166	31.82
12	80.81	6,530	0.005129	1.588	8.38	19.77	104	50.59
14	64.08	4,107	0.003225	2.525	13.3	12.43	63.3	80.44
15	57.07	3,257	0.002558	3.184	16.8	9.858	52.0	101.4
16	50.82	2,583	0.002028	4.015	21.2	7.818	41.3	127.9
17	45.26	2,048	0.001609	5.061	26.7	6.200	32.7	161.3
18	40.30	1,624	0.001276	6.385	33.7	4.917	26.0	203.4
19	35.89	1,288	0.001012	8.051	42.5	3.899	20.6	256.5
20	31.96	1,022	0.0008023	10.15	53.6	3.092	16.3	323.4
21	28.46	810.1	0.0006363	12.80	67.6	2.452	12.9	407.8
22	25.35	642.4	0.0005046	16.14	85.2	1.945	10.3	514.2
23	22.57	509.5	0.0004002	20.36	108	1.542	8.14	648.4
24	20.10	404.0	0.0003173	25.67	135	1.223	6.46	817.7
25	17.90	320.4	0.0002517	32.37	171	0.9699	5.12	1,031
26	15.94	254.1	0.0001996	40.82	216	0.7692	4.06	1,300
27	14.20	201.5	0.0001583	51.46	272	0.6100	3.22	1,639
28	12.64	159.8	0.0001255	64.90	343	0.4837	2.55	2,067
29	11.26	126.7	0.00009953	81.84	432	0.3836	2.03	2,607
30	10.03	100.5	0.00007894	103.2	545	0.3042	1.61	3,287
31	8.928	79.70	0.00006260	130.1	687	0.2413	1.27	4,145
32	7.950	63.21	0.00004964	164.1	866	0.1913	1.01	5,227
33	7.080	50.13	0.00003937	206.9	1090	0.1517	0.814	6,591
34	6.305	39.75	0.00003122	260.9	1380	0.1203	0.635	8,310
35	5.615	31.52	0.00002476	329.0	1740	0.09542	0.504	10,480
36	5.000	25.00	0.00001964	414.8	2190	0.07568	0.400	13,210
38	3.965	15.72	0.00001235	659.6	3480	0.04759	0.251	21,010
40	3.145	9.888	0.000007766	1049	5540	0.02993	0.158	33,410

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* Reproduced from Pender's Handbook for Electrical Engineers, 1922.

† Let C = per cent conductivity, R_{20} = resistance of 100 per cent conductivity wire at 20° C. (from table), R_t = resistance of wire of conductivity C at any temperature t° C., then

$$R_t = R_{20}\left[\frac{100}{C} + 0.00393(t-20)\right].$$

Weight of conductor for the same length varies inversely as resistance hence, if weight of conductor is designated by W,

$$\frac{W_1}{W_2}=\frac{V_2^2}{V_1^2}. \qquad (65)$$

Further, since resistance of conductor of constant cross-section varies directly as length, the relative distances, L, for equal power loss, are given by

$$\frac{L_1}{L_2}=\frac{V_1^2}{V_2^2}. \qquad (66)$$

It appears from these relations that the higher the voltage, the less is the weight of copper necessary to transmit a given amount of power, or the greater is the distance a certain amount of power can be transmitted, with a given loss.

When alternating current is used to transmit power, voltages as high as 250,000 volts are feasible, being made so by the transformer, a stationary piece of apparatus, which makes it possible to economically transform from one voltage to another. Thus, we might generate power at 15,000 volts, step it up through transformers, in one or two steps, to 250,000 volts, transmit it many miles and then again step it down through other transformers, in several steps, to 110 volts or any voltage at which the power is to be used.

In the case of continuous currents,[1] to transform from one voltage to another it is necessary to use rotating machinery, usually a motor-generator set. If we had c.c. power at 500 volts available, and required it at 110 volts, we would probably use a 500-volt motor to drive a 110-volt generator. Such transformation is relatively inefficient, and we are further limited by the fact that the maximum voltage for which large c.c. power generators can successfully be built is about 1500 to 2000 volts.

Whereas voltages of from 500 to 3600 volts are used for c.c. trolley service (obtained sometimes from two generators in series), the voltage for incandescent lamps and general household service is practically limited to 110 volts. This is done partly, because shocks from high voltages are dangerous, but more particularly because it is difficult to make economical lamps for higher voltages without using long and extremely thin filaments in order to obtain the required resistance. Filaments of such small cross-section are more difficult to make and are likely to be unduly fragile.

Transmission by c.c., being thus limited to low voltages, is hence confined to comparatively short distances.

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[1] In speaking of circuits where the current is uni-directional and non-varying, the words *direct-current* (d.c.) and *continuous-current* (c.c.) are both used; either is correct and they are generally used synonymously—we shall use the term continuous-current as it seems to describe the nature of the current better than the term direct-current.

In Europe a c.c. series transmission system at fairly high voltages, called the Thury system, is used to some extent. Special, constant-speed, series generators, generating up to about 5000 volts per machine, are used, enough generators being put in series to obtain transmission voltages as high as 90,000 volts. The load consists of similar series motors, which in turn drive generators for local distribution. One advantage of the system is that all of the generators or load motors need not be located together, it being possible to run several generating and load stations in series scattered anywhere on the line.

Continuous-current series systems are now obsolete in the United States.

74. Voltage Regulation.—If constant voltage is maintained at the generator end of a transmission system, the voltage at the receiving end will vary with the load, on account of the IR drop in the line. At no load, the voltage at both ends will be the same, $E_g = E_r$; but at full load, the voltage at the receiving end will be less, according to the relation

$$E_r = E_g - IR, \quad \dots \dots \dots \quad (67)$$

where E_r = voltage at receiving end;
E_g = voltage at generator end;
I = current;
R = resistance of outgoing and return wires.

The regulation of a line is an important matter, as it affects the saleability of the energy. It is found that the light given out by modern tungsten or mazda lamps varies directly as the 3.6 power of the ratio of the applied voltage to the rated voltage, and their life varies inversely about as the 13th power of the ratio of the applied voltage to the rated voltage. Further, if the load over the line varies rapidly through wide limits, unpleasant flickering of the lamps results. Voltage change also affects the speed of continuous-current motors.

Circuits designed to transmit relatively large amounts of power, with few or no branches, are generally called transmission lines, while circuits used for delivering power in small amounts, to numerous points, by many branches, are termed distribution circuits. A transmission line usually has a distribution system at its receiving end. Because of the low voltages employed, continuous-current systems generally serve only as distribution circuits.

The usual loss in voltage in c.c. systems supplying light and power varies from 2 to 5 per cent of the delivered voltage. In some cases where provision is made for raising the voltage at the supply end, these values may be exceeded, but in any case the voltage variation at the lamps should never exceed 5 per cent, i.e., never vary more than 2.5 per cent above or below normal. For systems supplying power only, the allowable voltage variation is usually greater, because the voltage variation is here not so important, and by allowing a greater voltage variation smaller and correspondingly cheaper wires may be used to deliver the power.

75. Current-carrying Capacity of Conductors.—Inasmuch as the passage of current through a conductor generates heat, and as the temperature of a conductor must rise until the rate of dissipation of heat is equal to the rate of generation, it becomes necessary to limit the current so as not to allow the temperature of the conductor to reach such values as might cause deterioration of the insulation. The current-carrying capacity of a conductor is therefore dependent upon the type of insulation used.

The principal materials used for insulating electric conductors intended for transmission, distribution, house wiring, etc., are compounds containing rubber, varnished cambric, paper, and impregnated cotton braid. The last is much used in connection with the others, to serve as protection against weather and mechanical injury. It is generally known as "weatherproof" insulation.

To insulate wires used in machinery, the materials most used are those of a fibrous character, such as cotton and silk, which may or may not be completely impregnated with insulating compounds, mica in special form known as micanite, and asbestos. As the conditions under which wire is used in machinery are different from those found in transmission and general wiring, only the insulation of power cables, distributing wires, etc., will be considered at this time.

Rubber insulation, so-called, is a compound of rubber with various other substances. It has been found that from 60 to 70 per cent adulterant may be added to rubber gum without destroying its insulating and mechanical qualities after vulcanization, a heat process which increases its chemical stability and its mechanical and electrical strength. Accordingly, 30 per cent pure rubber is taken as the general lowest standard proportion. Rubber insulation is found to deteriorate slightly even at ordinary temperatures, but when exposed to temperatures above 50° C. deterioration is rapid, the insulation becoming brittle and losing strength. As the temperature is increased above 100° C., the insulation melts and finally burns. Subsequent cooling does not restore any of the properties lost in heating. As generally used, rubber compound is pressed around the bare or stranded wire, and one or two coverings of weatherproofed braid added. The cost of rubber-covered wire is governed by the rubber market, as well as by the cost of manufacture; in smaller sizes of wire, say up to about No. 8, it is the cheapest for interior wiring and is generally so used for voltages up to 600. Its competitor is wire with varnished cambric insulation.

Varnished cambric is a prepared cotton fabric, coated with insulating varnish. For use on wire, the material is made up in the form of tape and wound spirally over the copper. Overlapping joints are staggered in successive layers, and between layers a film of a non-drying viscous adhesive filler is used. Every two or three layers, the wrap is reversed. For protection, weatherproof braid, a lead sheath or galvanized iron tape is added.

Varnished cambric does not absorb moisture as paper does, and is generally more flexible; contact with lubricating or transformer oil does not injure it, as is the case with rubber. It is not, however, as good a heat conductor as rubber. In price, varnished cambric cable is generally cheaper than rubber-covered cable in sizes larger than No. 8. However, since varnished cambric can be permitted to attain a maximum temperature of 80° C., the current-carrying capacity of a cable of given size is greater with varnished cambric insulation than with rubber, even though rubber is a better heat conductor. It is thus often possible to use a smaller varnished cambric cable and thereby cause a reduction in the cost of conduit, hangers, etc.

The paper used for cable insulation is a good grade of manila, and is wound spirally, in a sufficient number of layers, to the desired thickness. The cable is then thoroughly impregnated with some oily insulating compound, and the whole enclosed in a lead sheath, the only thoroughly waterproof covering yet devised for cables. Paper-insulated cable is cheaper than varnished cambric cable or good rubber-covered cable, and is much used for voltages above 5000. The maximum operating temperature for paper insulation is 80° C.

When several layers of cotton braid, thoroughly impregnated with a black, weatherproofing compound, are put upon bare hard-drawn copper wire, so-called "weatherproof wire" results. This wire is much used for outdoor overhead work, where great insulating strength is not required. It is a very cheap form of insulation.

In American practice overhead lines carrying power at voltages greater than 2300 are not insulated at all; for the higher voltages insulation which has been exposed to the weather is not reliable as a safeguard against shock, and so gives apparent security where there really is none. Hence the wires of overhead power lines for voltages above 2300 are always bare.

The limiting current capacity for cable and wires is dependent upon many factors. For interior wiring in buildings, the limits are set by the National Board of Fire Underwriters. For other uses, current limitations are recommended by the various wire manufacturers. In order that some comparison may be made, Table IV is given.

In Fig. 78 are shown cross-sections of representative cable for c.c. distribution. In Fig. 78(*a*) is shown the construction of a single cable

TABLE IV *

CARRYING CAPACITIES, IN AMPERES, ALLOWED BY THE REGULATIONS OF THE NATIONAL BOARD OF FIRE UNDERWRITERS FOR INTERIOR COPPER CONDUCTORS

(For Aluminum 84 Per cent of These Currents is Allowed)

Single Conductor Cables or Each Conductor of Multiple Conductor Cable

A. W. G.	Area in Circular Mils	Table A. Rubber Insulation	Table B. Varnished Cloth	Table C. Other Insulation
18	1,624	3	....	5
16	2,583	6	....	10
14	4,107	15	(18)	20
12	6,530	20	(25)	25
10	10,380	25	(30)	30
8	16,510	35	(40)	50
6	26,250	50	60	70
5	33,100	55	65	80
4	41,740	70	85	90
3	52,630	80	95	100
2	66,370	90	110	125
1	83,690	100	120	150
0	105,500	125	150	200
00	133,100	150	180	225
000	167,800	175	210	275
	200,000	200	240	300
0000	211,600	225	270	325
	250,000	250	300	350
	300,000	275	330	400
	400,000	325	390	500
	500,000	400	480	600
	600,000	450	540	680
	700,000	500	600	760
	800,000	550	660	840
	900,000	600	720	920
	1,000,000	650	780	1000
	1,100,000	690	830	1080
	1,200,000	730	880	1150
	1,300,000	770	920	1220
	1,400,000	810	970	1290
	1,500,000	850	1020	1360
	1,600,000	890	1070	1430
	1,700,000	930	1120	1490
	1,800,000	970	1160	1550
	1,900,000	1010	1210	1610
	2,000,000	1050	1260	1670

Varnished cloth smaller than No. 6 may be used by special permission only.

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* Reproduced from Pender's Handbook for Electrical Engineers, 1922.

conductor, such as is used for the underground distribution on the low-voltage continuous-current systems in the larger cities. In Fig. 78(*b*) is shown the construction of a two-conductor concentric cable; the cross-section of copper in the inner and outer cables is about the same. Woven in with the outer part of the cables are extra, insulated wires called pressure wires; they are connected across the distribution system at some point where it is desired to read the voltage, on station instruments. At the station, these pressure wires connect to a switchboard voltmeter; by its indications the voltage at the distribution point may be suitably adjusted.

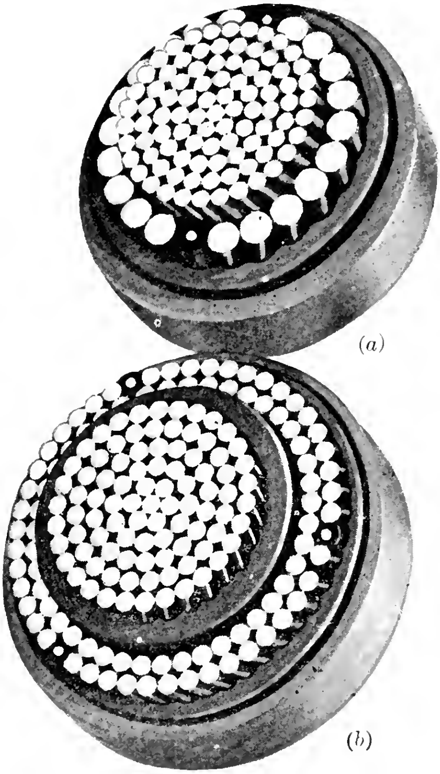


FIG. 78.—The cables used for continuous-current distribution systems have comparatively thin insulation, as they are always used on low-voltage circuits. They are built in various forms of which the two shown here are common ones; these cables are frequently an inch in diameter, having copper of one million circular mils cross-section.

76. Power Loss in Line.—The energy converted into heat, or I^2Rt loss, besides heating the conductors, also represents an actual money loss. The amount of permissible power loss is governed by such factors as the cost and sale price of the power, cost of erection, wire, supports, etc., taxes, depreciation, etc. The student is referred to the various electrical handbooks for further information on this subject.

77. Calculation of Simple Distribution Systems.—There are various types of circuits used in distribution systems, of which we shall consider only the more common forms.

(*a*) *Series System.*—Find the size of wire necessary to supply 100 series incandescent lamps over a 5-mile series circuit. Each lamp requires 6.6 amperes at 30 volts. Allowable loss in voltage 3 per cent. (Fig. 79.)

Let R = resistance of wire per 1000 feet.

Total voltage consumed = 3000.

Length of wire in thousands of feet = $5 \times 5.28 = 26.4$.

Allowable voltage loss $= 3000 \times 0.03 = 90$.
Allowable IR drop $= 6.6 \times 26.4 \times R = 90$.

$$R = \frac{90}{6.6 \times 26.4} = 0.517 \text{ ohm}$$

From wire table, the nearest size of wire is No. 7 wire.

(*b*) *Multiple System.*—Two loads, each of 10 amperes, are 600 and 1000 feet, respectively, from the distribution center. Find the size of wire necessary to keep the drop within 5 volts, using the same size wire throughout. (Fig. 80).

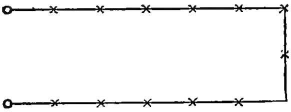


FIG. 79.

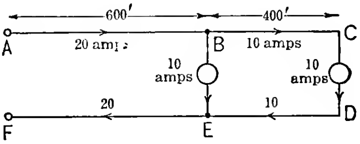


FIG. 80.

FIG. 79.—In the series distribution system, used principally in street lighting systems, but one wire is used, all the lamps being in series; the voltage to ground of such a system varies progressively around the circuit, being zero at two points and very high (possibly several thousand volts) at others.

FIG. 80.—In the more commonly used parallel distribution system the voltage to ground is practically the same at all points on one side of the circuit, but the current in the circuit varies widely at different points; in the series circuit of Fig. 79 the current is the same throughout the entire circuit.

Let R = resistance of wire per 1000 feet, in ohms.

Then
$$5 = I_{AB}R_{AB} + I_{BC}R_{BC} + I_{DE}R_{DE} + I_{EF}R_{EF};$$
$$5 = 2 \times 20 \times 0.6R + 2 \times 10 \times 0.4R;$$
$$5 = 24R + 8R;$$
$$R = \tfrac{5}{32} = 0.156. \quad \text{Choose No. 2 wire.}$$

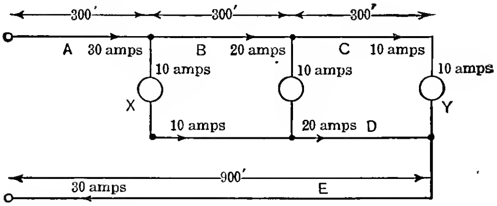


FIG. 81.—In the so-called "anti-parallel" system of distribution there is less variation of voltage throughout the circuit than if the straight parallel circuit of Fig. 80 is used; more wire is generally required for the anti-parallel scheme than for the straight parallel connection.

(*c*) *Anti-parallel System.*—With connections of three 10-ampere loads, as in Fig. 81, determine size of wire necessary when allowing a maximum voltage drop of 5 volts at middle load.

Let R = resistance of wire per 1000 feet, in ohms.

Then $5 = I_A R_A + I_B R_B + I_D R_D + I_E R_E$;
$5 = 30 \times 0.3R + 20 \times 0.3R + 20 \times 0.3R + 30 \times 0.9R$;
$5 = 9R + 6R + 6R + 27R = 48R$;
$R = \frac{5}{48} = 0.104$. Choose No. 0 wire.

Voltage drop to X

$= 30 \times 0.3R + 10 \times 0.3R + 20 \times 0.3R + 30 \times 0.9R$;
$= 45R = 4.68$ volts.

Voltage drop to Y is the same.

(*d*) *Tapered Conductor.*—Given three loads, as in Fig. 82. This figure shows the distribution by means of a one-line diagram, but it will be understood that actually two wires are required wherever one is shown; thus the length of wire causing IR drop between A and B is actually 400 feet. The allowed drop to each load is 5 volts. Find sizes of wires.

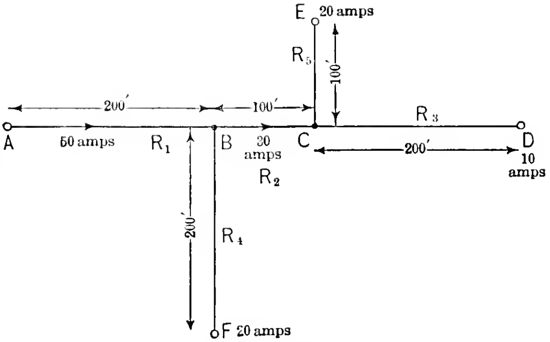


FIG. 82.—In parallel networks of any appreciable magnitude conductors of decreasing sizes are used at points more distant from the power supply; this sketch illustrates the ordinary scheme of showing circuit layouts, one wire only being shown where really a pair is actually installed.

Assume drop to most distant load to be proportional to distance; this assumption, which gives uniform current density in the copper from the generator to the most distant load, results in a minimum amount of copper for a specified IR drop to this load. On this assumption, the drop in AB is equal to two volts, in BC it is one, and in CD, two. Then if the total drop to loads F and E is to be 5 volts, that in $BF = 3$ and in $CE = 2$.

Let R_1 = resistance of wire per 1000 feet between A and B;
R_2 = resistance of wire per 1000 feet between B and C, etc.

Then $2 = 50 \times 2 \times 0.2R_1$; $R_1 = 0.1$. Choose No. 0 wire.
$1 = 30 \times 2 \times 0.1R_2$; $R_2 = 0.167$. Choose No. 2 wire.
$2 = 10 \times 2 \times 0.2R_3$; $R_3 = 0.500$. Choose No. 7 wire.
$3 = 20 \times 2 \times 0.2R_4$; $R_4 = 0.375$. Choose No. 6 wire.
$2 = 20 \times 2 \times 0.1R_5$; $R_5 = 0.500$. Choose No. 7 wire.

(*e*) *Feeder and Main.*—Given five loads, each of 10 amperes, as in Fig. 83, the maximum drop to be 10 volts. Find size of feeder and mains.

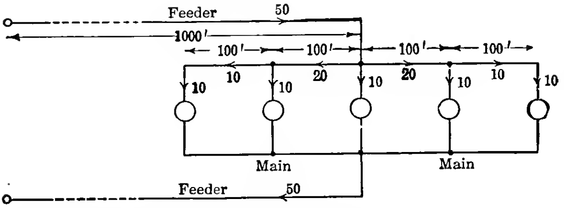


FIG. 83.—A simple parallel distribution for calculation of feeder and mains.

Assume drop over feeder as 8 volts, leaving two volts drop for the mains. This is about the proportion used in practice.

Let R_1 = resistance of feeder per 1000 feet;
R_2 = resistance of main per 1000 feet.

Then $8 = 50 \times 2R_1$; $R_1 = 0.080$. Choose No. 00.
$2 = 20 \times 2 \times 0.1R_2 + 10 \times 2 \times 0.1R$; $R_2 = 0.333$. Choose No. 5.

78. The Three-wire System.—In order to take advantage of the saving in copper by the use of higher voltages, and yet use low-voltage incandescent lamps, Edison devised the three-wire system of distribution. Originally, two 110-volt generators were put in series at the station and three wires led out, one from each outside terminal of the two generators and one from their common connection, as shown in Fig. 84.

By this arrangement, we have 110 volts between the middle wire, or *neutral*, and either outside wire, and 220 volts across the two outer conductors.

If there were no neutral, to transmit a given amount of power at 220 volts would require one-fourth the copper necessary for transmission at 110 volts. If the cross-section of the neutral wire be taken the same as that of the outside wires, then we should require 37.5 per cent of the copper required by a 110-volt system. In some three-wire distribution systems, the cross-section of the neutral is less than that of the outer wires, so that the saving in copper is even greater.

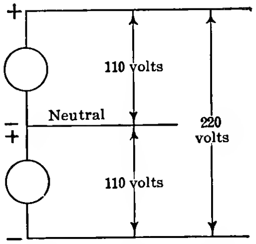


FIG. 84.—Nearly all power for lighting loads is delivered at the customer's premises by the three-wire system; the voltage is twice as great between the two outside wires as between either outside wire and the central, or neutral, wire. This neutral wire is generally grounded as a protection against shock.

The scheme of operation of the three-wire system is simple. If the load is exactly balanced, the neutral carries no current, as shown in Fig. 85. If the load on the lower leg be reduced to 2 lamps, the system is said to be unbalanced, as in Fig. 86, and the neutral will carry current.

The neutral wire is maintained by the generators at a difference of potential of 110 volts from each of the outer wires, being thus negative with respect to the upper, and positive to the lower wire. Since the voltage across the lamps is thus maintained at 110 volts (neglecting the effect of the IR drop in the wires), each lamp must still continue to take 5 amperes. Hence, the upper bank of lamps will require 20 amperes and the lower bank, 10 amperes. The difference must be taken care of by the neutral, the current of which will therefore flow toward the generators.

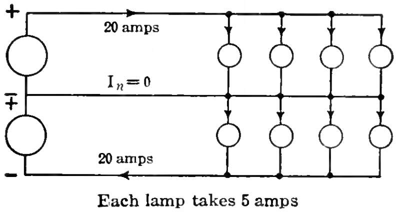


FIG. 85.

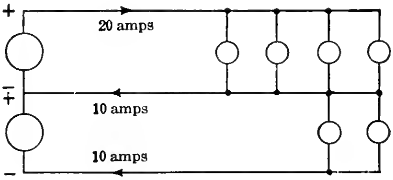


FIG. 86.

FIG. 85.—In a well maintained three-wire system the neutral wire carries practically no current, as the two loads are kept balanced by shifting various loads from one side to the other as required.

FIG. 86.—In case of unbalanced loads, as illustrated here, the neutral may carry a large current; in this case the current in the neutral is towards the power supply.

If the unbalancing is reversed, as in Fig. 87, the current distribution must be as shown. The neutral again carries the difference between the two outer currents, but as the lower leg requires more current than the upper, its current is now out from the generators. The current in the neutral may thus flow in either direction, and is always the difference between those in the two outer conductors. Every effort is made to keep the amount of unbalancing as small as possible, by proper connection of the load. In house wiring, complete unbalancing is apt to occur, and for this reason the neutral is usually of full size. In distribution systems, however, by careful connection of the customers to the system, the amount of unbalance is very small.

Modern methods for maintaining a constant voltage in c.c. three-wire systems will be discussed later.

The accidental opening of the neutral of a three-wire system, with balanced loads as in Fig. 85, would not be felt, but with unbalanced loads trouble is apt to occur. If the system in Fig. 87 is considered as having 110 volts across each side, and the feeders as having no voltage drops, the resistance of each of the two lamps on the upper side taking 10 amperes would be $110/10=11$ ohms; the resistance of the four lamps on the lower side would be $110/20=5.5$ ohms.

If the neutral were opened somewhere between the supply and the loads, as in Fig. 88, 220 volts would be impressed on $5.5+11.0=16.5$ ohms, and a current of $220/16.5=13.33$ amperes would flow through the two loads in series. The drop across the upper lamps would be $13.33\times11=146.7$ volts and across the lower ones, $13.33\times5.5=73.33$.

This calculation assumed that the resistance of the lamps would remain constant with other than their rated voltages, which is, of course, not the case. It is evident, however, that the upper bank of lamps would have impressed on it a higher voltage than normal, resulting in increased candle power and shorter life, the reverse being true for the lower bank. The amount of voltage unbalance in such a case is dependent upon the extent to which the load is unbalanced.

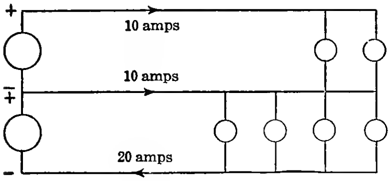


FIG. 87.

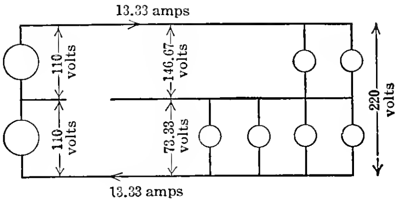


FIG. 88.

FIG. 87.—Here the unbalance is the opposite of that shown in Fig. 86 and the direction of the neutral current is also reversed.

FIG. 88.—In case the neutral wire opens on an unbalanced system a big difference in voltage on the two sides of the system will result, probably burning out the lamps on the lightly loaded side.

To obviate the results of an open neutral, the neutral wire is not fused, and it is the custom to ground this wire at both the distribution centers and on each customer's premises. This practice also limits the voltage of a shock to which a person might be subjected in simultaneously touching any live parts of the wiring system and a ground connection, such as water pipes, radiators, bath-tub, etc.

79. Calculation of Voltage Drop in Three-wire System.—With loads connected as in Fig. 89, we shall calculate the voltage at each load. Outside wires are No. 4 ($R=0.25$ ohm per 1000 feet) and neutral is No. 7 ($R=0.5$ ohm per 1000 feet).

By inspection of the distribution of the various loads, it will be evident that the various lines must carry currents as indicated in the diagram.

Using Kirchhoff's first law, the section drops are determined as follows:

$$\begin{aligned} A &= 25\times0.2\times0.25=1.25; & F &= 5\times0.1\times0.50=0.25;\\ B &= 20\times0.2\times0.25=1.00; & G &= 10\times0.2\times0.50=1.00;\\ C &= 10\times0.1\times0.25=0.25; & H &= 5\times0.2\times0.25=0.25;\\ D &= 5\times0.1\times0.50=0.25; & J &= 15\times0.3\times0.25=1.125.\\ E &= 15\times0.1\times0.50=0.75. \end{aligned}$$

$$\begin{aligned}
\text{Voltage at load } K &= 120 - I_A R_A - I_G R_G \\
&= 120 - 1.25 - 1.00 = 117.75 \\
\text{Voltage at load } L &= 117.75 - I_B R_B - I_E R_E - I_F I_F \\
&= 117.75 - 1.00 - 0.75 - 0.25 = 115.75. \\
\text{Voltage at load } M &= 115.75 - I_C R_C - I_D R_D \\
&= 115.75 - 0.25 - 0.25 = 115.25. \\
\text{Voltage at load } P &= 120 + I_G R_G + I_F R_F - I_J R_J \\
&= 120 + 1.00 + 0.25 - 1.125 = 120.125. \\
\text{Voltage at load } N &= 120.125 + I_E R_E + I_D R_D - I_H R_H \\
&= 120.125 + 0.75 + 0.25 - 0.25 = 120.875
\end{aligned}$$

The problem indicates one of the disadvantages of the three-wire system; the voltage on the loaded side falls, while that on the unloaded side may even rise. This is accounted for by the fact that there must be a drop in voltage along the neutral from right to left with the condition given in the problem, to cause the unbalanced current to flow, and this drop tends to offset any drop occurring in the lower wire (*J–H*); in case the drop in *D, E, F, G,* is greater than the drop in *J, H,* the voltage at load *N* will be greater than the generator voltage. This occurs, however, only with very badly unbalanced loads.

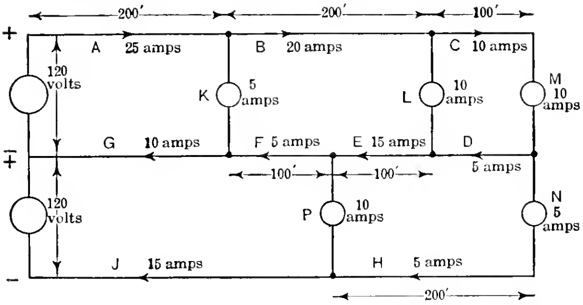


FIG. 89.—A problem circuit, for the calculation of voltages at various points in the three-wire system.

As a check on the figures obtained in the above problem, we can find the voltage across *M* and *N* in series

$$\begin{aligned}
E_{MN} &= 240 - I_A R_A - I_B R_B - I_C R_C - I_H R_H - I_J R_J \\
&= 240 - 1.25 - 1.00 - 0.25 - 0.25 - 1.125 = 236.125.
\end{aligned}$$

From our first calculations

$$E_{MN} = E_M + E_N = 115.25 + 120.875 = 236.125$$

Calculation of Size of Wire.—Given three loads, as shown in Fig. 90, assume maximum drop to be 5 volts; neutral wire of same size as others.

Let R = resistance of wire per 1000 feet.

Then $5 = 15 \times 0.3R + 10 \times 0.4R + 10 \times 0.7R.$
$5 = 4.5R + 4R + 7R = 15.5R.$
$R = 5/15.5 = 0.323.$ Choose No. 5 wire.

80. Distribution Networks.—The use of continuous current for light and power (except for railway motive power), is practically confined to the largest cities, the systems being the outgrowth of the old Edison installations. The areas so supplied are heavily congested, with large office buildings, theaters, apartment houses, etc. To supply the tremendous demand for current at 120 and 240 volts, the three-wire system is used entirely, the three mains being laid underground, generally on both sides of each street, and electrically interconnected at street crossings, forming a closed network. An idea of this is given by Fig. 91.

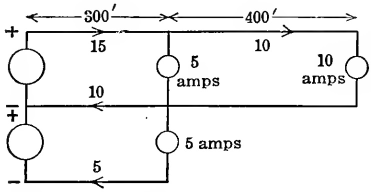


FIG. 90.—A problem circuit, for calculating the proper sizes of conductors, the allowable drop to the various loads being specified.

Scattered throughout the distribution area at appropriate places, are sub-stations from which the power is fed into the distribution network through feeders. These sub-stations contain, for the most part, synchronous converters, which convert alternating-current power from the main generating power plants, into continuous-current power. Some of the sub-stations may contain continuous-current generators and, as far as this discussion is concerned, all sub-stations may be considered as consisting of such machines.

From the sub-station, feeders run underground to appropriate places and are tapped into the network, as is roughly shown in Fig. 92, the distribution mains being shown in single lines. The number of feeders from a given sub-station depends entirely upon the capacity of the station, and the location of the tapping points upon the distribution of the load.

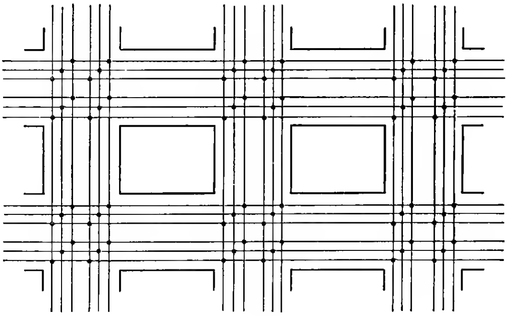


FIG. 91.—In the Edison distribution systems the mains are interconnected wherever possible, as this tends to maintain an even voltage over the whole system, and also increases the reliability of the power supply.

In a closed network, it is evident that if the voltage at the station ends of the feeders is the same, the shorter feeders will carry more current than the longer ones. For the same reason, points in the network nearest the sub-stations will be several volts higher in potential than those farther away. It is necessary to keep the entire network at as nearly constant potential as practically possible, and to do this, the voltage at the station ends of the feeders is adjusted.

In a station there are three sets of bus-bars, maintained by separate machines at, say 250, 260 and 270 volts and known as the low, intermediate, and high buses. Any feeder may be connected to any of the three buses and its load and the voltage of the network thus controlled. A feeder can at any time be opened without bad effects, since the other feeders will take up its load. In some stations only a high and a low bus are used.

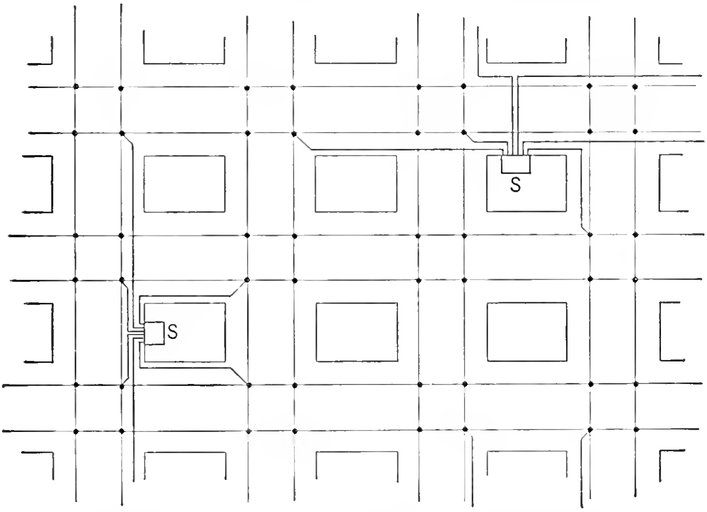


Fig. 92.—The sub-stations, from which the power is supplied to the mains, are connected to the network of Fig. 91 at various points, as suggested in this figure; a great many feeders are sent out from each sub-station.

81. Protection of Distribution Networks.—The entire system being so closely tied together, the question of protection naturally assumes importance. In case of trouble due to grounds and short circuits, it is customary to allow them to burn themselves clear. This may momentarily reduce the voltage in the region where the short circuit occurs, but once the short circuit is burnt off, the system is still operative. While the short circuit is burning off, the nearest feeder may be badly overloaded, but it is only necessary to disconnect it to balance the overload current on the other feeders. The greatest danger has been found, by experience, to lie in the feeders themselves, a short circuit being likely to spread from one feeder to another in the neighborhood of the sub-stations. The latest practice, by one of the largest companies, has been to equip their feeders with automatic circuit-breakers at the feeding end and even at the station ends. In case of trouble on a feeder, it is automatically disconnected.

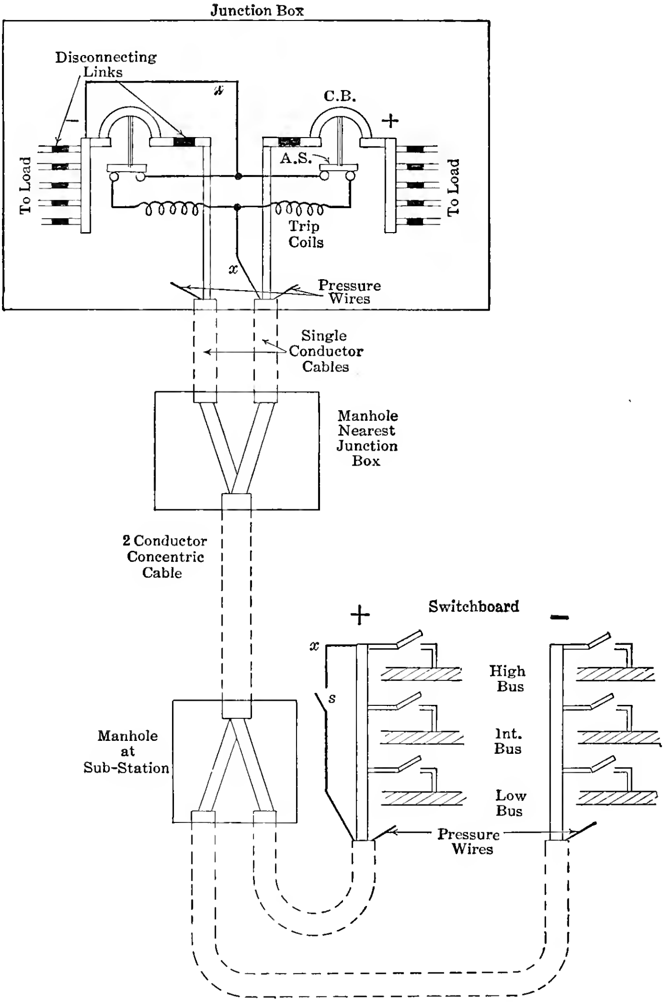


FIG. 93.—A modern scheme for protecting a feeder in case of short circuit; when the voltage across the system, at any selected point, falls below a predetermined value, the circuit breakers blow, disconnecting the feeder from the network.

The scheme, as used in connection with a 1,000,000 circular mil concentric feeder, is shown in outline in Fig. 93. Between the mains and the positive and negative ends of the feeder, circuit-breakers, held closed against spring pressure by a latch, are connected. One of the pressure wires, *x*, built into the cable, is connected to the positive end of the feeder at the station. At the load end of the feeder, the two-trip coils of the breakers are connected in parallel between the pressure wire and the negative side of the feeder. Normally, the trip coils have full voltage impressed upon them, so that they hold up a plunger through their cores. If the voltage impressed upon the trip coils falls to about 175, they drop their plungers, which in turn knock out the latches of the circuit breakers and cause the latter to open by spring pressure. If a breakdown occurs in the cable, causing a short circuit between positive and negative conductors, the voltage across the trip coils will become very low, and if either the positive or the negative conductor become grounded, the voltage will fall to 120, thus opening these breakers. The operator at the station can, at any time he desires, disconnect a feeder from the network by opening switch, *s*.

It is to be noted that the object of these automatic breakers is to disconnect the faulty feeder from the distribution network, thus preventing power from the network from flowing into the short circuit. The operator at the local sub-station, noting the excessive current taken by the shorted feeder, may disconnect it by opening the proper switch, or this may be done by proper overload breakers or fuses at the sub-station.

It should also be noted that these automatic circuit breakers cannot be classed as normal overload breakers, as the drop in voltage required to operate them is so high that it could not be produced by ordinary overloads.

In Fig. 93 only the positive and negative feeders were considered; it is, of course, necessary to have feeders to the neutral as well. In the case of one of the largest companies, the mains, positive, neutral, and negative are all of the same cross-section. The load is kept so well balanced between the two sides of the system, however, that it has been found satisfactory to have the total area of the neutral feeders installed, only 12.8 per cent of the area of the total positive (or negative) feeders; this is accomplished by using comparatively few neutral feeders.

PROBLEMS

1. A tungsten lamp has resistance of 440 ohms when hot; how much current does it draw from a 110-volt line, and how much power is used? What is the conductance of the lamp?

2. Neglecting losses due to radiation and convection, how long will it take to heat 5 gallons of water from 60° F. to boiling, if the power being supplied to heat the water is 4.75 kw.? How many joules are supplied? How many calories are supplied?

3. If, in the above problem, losses due to radiation and convection are equal to the energy used in raising the water temperature, how much does the boiling of the water cost, if power is worth 12 cents per kilowatt-hour?

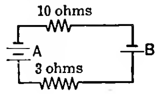
4. An electric automobile weighing 2600 pounds is moving up a 10 per cent grade at a speed of 20 miles per hour. The force required to overcome friction due to tires, gears, etc., at this speed, is 120 pounds. What power is being developed by the motor? At what rate is energy being wasted in heat, and at what rate stored?

5. A charging current of 15 amperes is being sent through a storage battery which has a resistance of 0.16 ohm and 12 volts c.e.m.f. At what rate is energy being wasted, and at what rate stored?

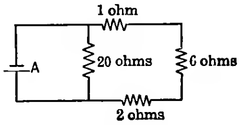
6. If 70 per cent of the ampere-hours used in charging the above battery is available for discharge, how long must the charging current be kept on if the battery is to furnish 8 amperes for twelve hours? How many calories of heat are wasted during discharge?

7. A storage battery, being charged at the rate of 12 amperes, requires an impressed voltage of 13.6 volts. When drawing 12 amperes discharge current, the terminal voltage is 12.85. What is the internal resistance of the battery?

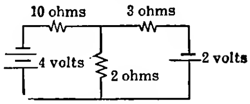
8. E.m.f. of $B=2.1$ volts. Resistance of $B=0.16$ ohm. E.m.f. of $A=6.2$ volts. Resistance of $A=0.22$ ohm. How much heat is generated in B, in one hour and twenty minutes? What is the voltage across B?



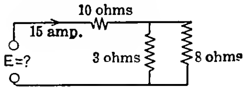
9. E.m.f. of $A=60$ volts. Resistance of $A=0.82$ ohm. How much current flows through it and what is the voltage across its terminals?



10. What is the current in the 2-ohm branch, and in what direction does it flow?



11. How much voltage is required to force 15 amperes through this circuit?



12. The resistance of a certain tungsten lamp, when lighted, is 220 ohms, the temperature being 2200° C. What is the resistance of the lamp at 20° C.?

13. Twenty-five feet of Calido wire is used in making a toaster which draws 7.3 amperes from a 110-volt line. What is the diameter of the wire if its temperature is 1000° F.?

14. What must be the diameter of an aluminum wire which has a resistance of 0.24 ohm per mile at 0° C.?

15. A hard-drawn copper wire, 0.125 inch diameter, has a resistance of 2.26 ohms at 30° C. What is its length?

16. How much No. 20 Calido wire must be used to make the heating element of a flatiron which is to consume 550 watts from a 110-volt line, the operating temperature of the heating element being 400° F.? How much power does it draw from the line when first connected?

17. The resistance of the field coils of a generator is 22.3 ohms at 19.6° C. After the machine has operated for several hours, the resistance of the coils is found to be 29.2 ohms. What is the temperature of the coils?

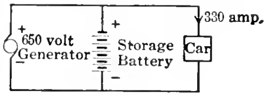
18. If an armature has a resistance of 0.0162 ohm at 18.5° C., what will its resistance be at 80° C.?

19. How many circular mils in a bus-bar 1 inch by $\frac{3}{8}$ inch? In a copper bar $\frac{1}{4}$ inch square?

20. Fifteen 500-ohm lamps and twelve 350-ohm lamps are in parallel. Resistance of circuit between lamps and generator is 0.36 ohm; voltage at the generator is 118. Find the voltage across the lamps.

21. A resistance of 1.7 ohms is connected across two batteries, *A* and *B*, which are themselves connected in parallel. The e.m.f. of *A* is 2.1 volts, and its resistance is 0.08 ohm; the e.m.f. of *B* is 1.9 volts, and its resistance is 0.106 ohm. How much current is supplied by each battery?

22. Distance from generator to battery is 7 miles, and from battery to car is 3 miles. Resistance of battery is 0.63 ohm, and its e.m.f. is 600 volts. Resistance of feeders, including rail return, is 0.12 ohm per mile. How much current is the storage battery delivering, and what is the voltage at the car?



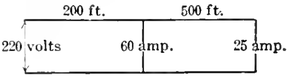
23. In Problem 22, to what value would the voltage of the generator have to be raised in order that the battery supply 100 amperes, the car still taking 330 amperes? To what value would the voltage of the generator have to be raised in order that the battery supply no current to the car? What would be the value of the voltage at the car in each case?

24. One hundred kilowatts of power is to be transmitted 10 miles, the voltage at the generator being 240 volts, and 10 per cent line drop being allowed. What size wire must be used, and how many watts of power are used in the line? Would the line be feasible?

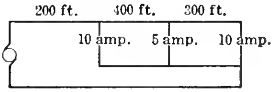
25. How many kilowatts of power can be transmitted over an aluminum cable, 750,000 circular mils area, 60 miles long, if the voltage at the station is 45,000 volts and 10 per cent line drop is allowed? How much could be transmitted if copper cable of the same cross-section were used?

26. The insulation resistance of a certain machine using varnished cambric insulation is 55 megohms at 15.5° C. How much will it probably be at 35° C.?

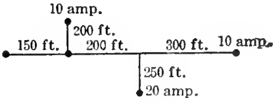
27. What size wire must be used, if the voltage at the 25-ampere load is to be 200 volts?



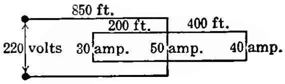
28. Drop to the middle load is 8 volts. What size wire is used?



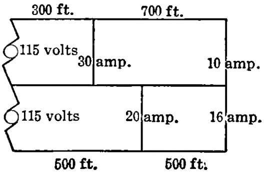
29. The allowed drop to each load is 12 volts. Find sizes of wire, using proper taper.



30. Maximum drop allowed is 10 per cent, of which 7 per cent is in the feeder. Find proper size of feeder and main, and calculate voltage at each load.



31. Outside wires No. 1 and neutral No. 4. Find voltage at each load.



CHAPTER V

SELF AND MUTUAL INDUCTION

82. Coil Moving in a Magnetic Field.—In section 41, the case of a coil moving in a magnetic field was considered. The conclusion reached from the analysis given was that when the flux threading a coil is changing, e.m.f. is induced (in amount proportional to the rate of change of interlinkages between flux and turns), causing current to flow, and that the action of this resulting induced current is always such that it tends to prevent any change in the number of interlinkages. The system therefore develops reactions, which, lasting as long as the flux interlinkages are changing, always act so as to oppose the change in the number of flux interlinkages.

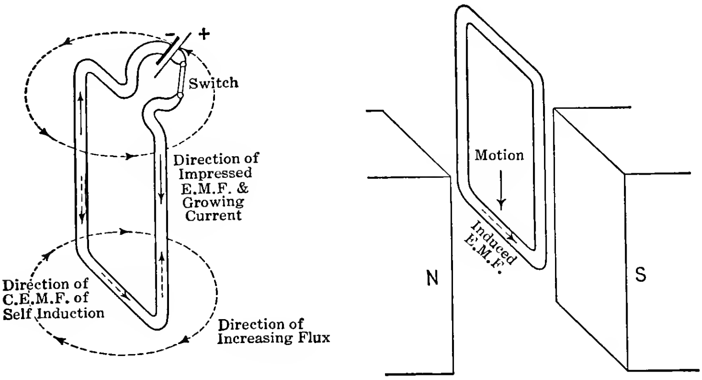
The act of " filling " a coil with flux by moving it into a field, as in Fig. 37, establishes a certain number of flux interlinkages; conversely, the act of " emptying " the coil of flux by moving it out of a field, as in Fig. 39, destroys a certain number of flux interlinkages. In either case we found that an e.m.f. was induced in such a direction that the current set up by it opposed the change taking place in the number of interlinkages.

Now it is evidently logical to reason that if a coil is " filled " with, or " emptied " of, a certain number of flux lines, (in other words, the same number of flux interlinkages are established or destroyed) in the same time, *by any method other than that of moving the coil into or out of a field, the same value of voltage must be induced.*

83. Self-induction.—When the current flowing in an electric circuit varies with time, any magnetic field set up by the current will correspondingly vary with time. Consider a turn of wire; if no current flows through it, no magnetic field is set up. However, if current is passed through the turn, as in Fig. 94*a*, being made to increase uniformly to a final value, I, by the application of a voltage, E, a uniformly increasing magnetic field, of final value, ϕ, will be set up. The current has been so chosen as to set up a field through the turn in a direction from left to right, as indicated by dotted lines. The increasing current thus " fills " the turn with flux or establishes a definite number of flux interlinkages, just as motion of the turn does in Fig. 94*b*, or in Fig. 37.

From Fig. 94*b*, we see that a voltage must be induced, which, by application of the rule given in section 40, is seen to be toward us in the lower turn-side; since the turn in Fig. 94*a* is being " filled " with flux in the same direction, a voltage must be induced, which, in its lower turn-side, must be in the same direction as that shown in Fig. 94*b*. This induced voltage, *e*, is called the *counter e.m.f. of self-induction* and is obviously in a direction opposite to that of the impressed voltage, *E*, thus resulting in a decrease in the net voltage of the circuit. The current at any instant is proportional to the net voltage acting in the circuit, and it is therefore evident that the c.e.m.f tends to limit the increase of current, to the rate of change of which it is due.

The action is the same as the force of inertia developed by an accelerating body; this force always acts in such a way as to limit the acceleration, or rate of change of velocity, of the body. The force of inertia comes into play only when the body starts to accelerate, and as soon as the velocity becomes uniform it disappears.

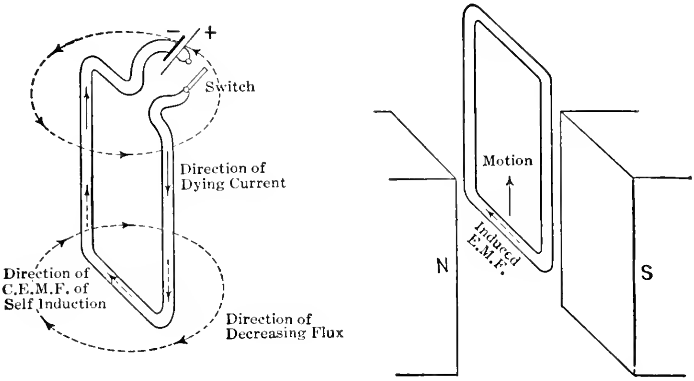


(*a*) (*b*)

FIG. 94.—As the current in a coil increases from zero, flux is set up in the circuit, establishing interlinkages; the voltage produced thereby is the same as though the interlinkages had been established by moving the coil into the field of a permanent magnet.

The whole effect in the electrical circuit is in accord with the idea expressed by Lenz's law; the original increase in current strength which produces the increase in flux, immediately brings into existence an opposing or counter e.m.f., *which tends to retard the increase in current, thereby opposing the increase in flux interlinkages.*

Conversely, when the current dies down within the turn, the number of flux interlinkages decreases, just as if the turn were moved out of the field. From Fig. 95*a*, it will be seen that the decrease in current strength, which causes a reduction in interlinkages, at once induces a voltage, which, being in the same direction as the impressed voltage, tends to retard the decrease in current, thereby again opposing the change taking place in the flux interlinkages.



(*a*) (*b*)

Fig. 95.—When the current in a coil decreases, the e.m.f. of self-induction acts to aid the impressed e.m.f., thus tending to prevent the current decrease, to which the e.m f of self-induction is due.

When the turn of Fig. 94*b* is moved into the field, its lower turn-side cuts the flux, or, in other words, flux interlinkages are being established only along the lower turn-side; voltage, therefore, is being induced only in the lower turn-side. However, when an increasing current is passed around the turn, as in Fig. 94*a*, the flux interlinkages set up are due to the current, and as the strength of the current at any instant is uniform over the entire turn, it follows that the flux interlinkages are being set up uniformly about the entire length of the turn (Fig. 96). Hence, the generation of the c.e.m.f. of self-induction must be uniform along the entire length of the turn, or, in other words, each element of the turn is assisting in the generation of the c.e.m.f. of self-induction. If the turn is not circular the above statement is not quite true; the e.m.f. generated per centimeter length is greatest where the curvature of the turn is the greatest.

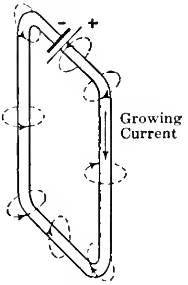


Fig. 96.—The e.m.f. of self-induction is set up nearly uniformly throughout the length of the turn; each centimeter of length develops nearly the same amount of voltage of self-induction.

Any circuit capable of setting up a magnetic field is said to be *inductive*, or to possess inductance. Whenever the current in an inductive circuit changes, a c.e.m.f. of self-induction will be generated, which lasts as long as the current (and flux) varies, and always has such direction as to prevent the change in current (and flux) taking place. This property of a circuit, by virtue of which it tends to prevent a change of current (or flux) *in itself,* is called its self-induction.

84. E.M.F. of Self-induction.—Consider again the toroid of Fig. 46, of N turns wound on an iron core of permeability μ, of mean radius r cm., and area of cross-section A sq. cm. When a steady current of I amperes flows through the toroid we have, as the value of the flux set up, from Eq. (30).

$$\phi=\frac{0.4\pi NI\mu A}{l}.$$

If the current changes uniformly at a rate of I/t amperes per second, and the permeability of the iron is, for the time considered, constant, the flux will correspondingly change at a uniform rate of ϕ/t lines per second. Since the flux is changing at this rate through N turns in series, the average value of the c.e.m.f. of self-induction will be

$$\text{Average } e=\frac{\phi N}{t10^8} \text{ volts.} \quad \quad (68)$$

Substituting for ϕ its value above, we have,

$$\text{Average } e=\frac{0.4\pi N^2\mu A}{l\,10^8}\cdot\frac{I}{t} \text{ volts.} \quad \quad (69)$$

It may seem strange at first that the back e.m.f. of self-induction varies as the *square* of the number of turns in the coil, instead of the first power of their number. It is to be remembered, however, that the flux set up by a given current depends directly upon the number of turns and so the voltage induced *per turn* (for a given rate of change of current) is directly proportional to the number of turns in the coil. There being N turns connected in series, and thus adding their respective e.m.fs., the total e.m.f. of self-induction will vary with the square of the number of turns.

As noted in section 55, the part $\frac{0.4\pi N^2\mu A}{l\,10^8}$ is a constant and is called the *coefficient of self-induction* or merely the *self-inductance* of the circuit, and is generally represented by the symbol L. Thus:

$$L=\frac{0.4\pi N^2\mu A}{l\,10^8}. \quad \quad (70)$$

It obviously depends upon the physical dimensions of the circuit and the value of μ. If no iron is used in the construction of the coil, the flux set up per ampere is constant; hence the self-inductance of the coil is also constant. Where iron is used, the flux per ampere changes as the iron becomes saturated, and the value of L decreases.

By reference to the magnetization and permeability curves of Fig. 48, it is evident that for the very low values of current the value of L increases with increase of current. In such low values of flux density the electrical engineer in general is not interested. It is, however, important to the telephone engineer who frequently uses iron core coils operated at these low densities.

We may now write, in place of Eq. (69), the shorter form,

$$\text{Average } e = L\frac{I}{t}, \quad \ldots \ldots \ldots \ldots \quad (71)$$

$\frac{I}{t}$ being the average rate of change of current.

85. Unit of Self-inductance.—The practical unit of self-inductance is called the *henry;* it is obtained by making all the terms in Eq. (71) equal to unity. The henry is thus the self-inductance of a circuit of such construction that a c.e.m.f. of one volt is generated by a rate of change of current of one ampere per second. This is the legal definition.

Another definition, which perhaps gives a better concept of self-inductance, is derived through the use of Eqs. (68) and (71). Since

$$e = L\cdot\frac{I}{t} \quad \text{or} \quad L = \frac{et}{I},$$

substituting for e, its value from Eq. (68),

$$L = \frac{\phi N \ 10^8}{I}. \quad \ldots \ldots \ldots \ldots \quad (72)$$

From this equation we may also define the henry as the amount of self-inductance possessed by a circuit in which one ampere is capable of setting up 10^8 interlinkages. If the current changes at a rate of one ampere per second, the flux interlinkages change at a rate of 10^8 per second, and so generate a c.e.m.f. of one volt.

It is to be noted that if the rate of charge of current is not uniform, an equation of the form given in Eq. (71) is not applicable; in such a case we use the same idea but confine our attention to such a short interval, or increment, of time, Δt, that the change of current, Δi, taking place during this small time interval, *does* occur at a uniform rate. An equation connecting an *increment of current* with the *corresponding increment in time* is much more general than is Eq. (71); even if the current increases on a curved line we can still speak of the rate of change of the current, by considering a short enough interval of time, Δt. Thus, in place of Eq. (71), we write the more general form

$$e = L\frac{\Delta i}{\Delta t}. \quad \ldots \ldots \ldots \ldots \quad (73)$$

86. Growth of Current in an Inductive Circuit.—If a voltage is impressed upon a circuit possessing no inductance, the current rises immediately to a value as determined by Ohm's law. In Fig. 97 is shown the rise in current when 20 volts is impressed on a circuit of 4 ohms resistance, by closing of the switch S; the current rises at once to a value of 5 amperes.

When voltage is impressed on a circuit possessing inductance, and the current starts to increase, the change of flux interlinkages induces a voltage of self-induction which opposes the increase of the current.

Consider a circuit possessing inductance and resistance as in Fig. 98*a*, in which closing of the switch S impresses a voltage E on the circuit. Just before the closing of the switch, the current is necessarily zero and is not changing, and no c.e.m.f. of self-induction exists; but as soon as the switch is closed the current immediately starts to rise. Now the rate with which the current will increase is obviously fixed by the amount of voltage available for the purpose; at the instant of closing the switch there is no appreciable iR drop in the circuit, and so the entire impressed voltage is available for producing current increase; the current, therefore, begins rising at a high average rate, as shown by the line *oa* in Fig. 98*b*.

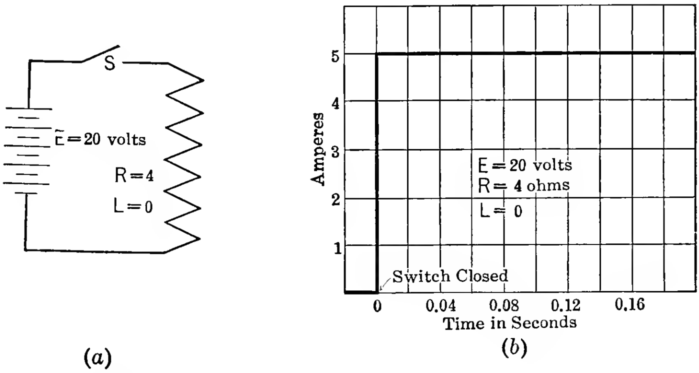


FIG. 97.—In a circuit containing no self-induction the current rises to its final value the instant the switch is closed.

During the first short interval of time considered (Δt_1), the resistance drop, although actually present, may be neglected, because the current is so small. Hence the c.e.m.f. of self-induction must balance the impressed voltage E, and so we have, using Eq. (73)

$$E = L\frac{(\Delta i)_1}{(\Delta t)_1},$$

and from this we get,

$$\frac{(\Delta i)_1}{(\Delta t)_1} = \frac{E}{L}, \qquad (74)$$

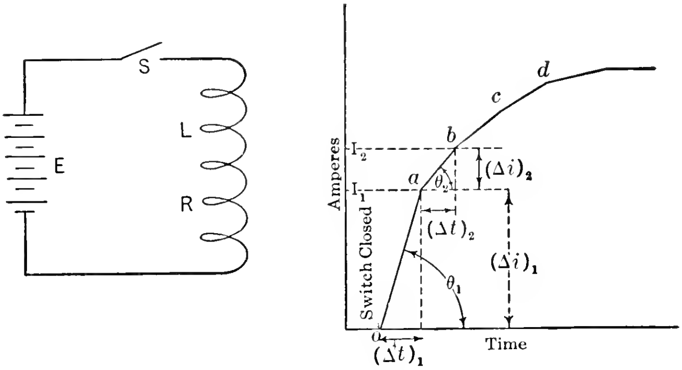
It therefore appears that the rate of increase in the current, $\frac{(\Delta i)_1}{(\Delta t)_1}$, when the switch is first closed, is given by the value of E/L. In Fig. 98*b* this rate of current increase is given by tan θ_1, so we have tan $\theta_1 = E/L$.

After the current has risen to an appreciable value (say at the end of time interval $(\Delta t)_1$), the resistance drop can no longer be neglected; it is equal to I_1R. At this instant of time we may write,

$$E = I_1R + L\frac{(\Delta i)_2}{(\Delta t)_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

in which $\frac{(\Delta i)_2}{(\Delta t)_2}$ represents the rate of increase in current at the beginning of interval of time $(\Delta t)_2$. This relation may evidently be written,

$$\frac{(\Delta i)_2}{(\Delta t)_2} = \frac{E - I_1R}{L}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$



(*a*) (*b*)

FIG. 98.—When a circuit contains appreciable self-induction the current cannot at once assume its final value; it starts to increase, from zero value, at a rate determined by the impressed voltage and the self induction of the circuit.

which shows that the rate of increase in the current is decreased by the effect of the circuit resistance; this decreased rate of current increase is shown in Fig. 98*b*, by the lower value of tan θ_2, as compared to tan θ_1.

At the end of the second increment of time $(\Delta t)_2$ the current has risen to the value I_2 and there is, therefore, a greater iR drop. For a third increment of time $(\Delta t)_3$ the rate of current increase would be given by

$$\frac{(\Delta i)_3}{(\Delta t)_3} = \frac{E - I_2R}{L}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

which is evidently less than the value given in Eq. (76). So, with increasing time, the increase in current becomes correspondingly less, until finally (actually in the ordinary circuit after a small fraction of a second) the iR drop is essentially equal to E; for this condition the rate of current increase is zero.

The form of current growth in an inductive circuit, considered from the standpoint of current increments, is shown by the broken line *oabcd* of Fig. 98*b*; it will be evident that this solution is approximate only, because, to make our analysis accurate, the increments of time and current should be taken extremely small. The result of so taking the time and current increments is given in the next section.

This case of current rise in an inductive circuit is analogous to that of the motion of a body being accelerated in a liquid. When the body is at rest and a constant force is applied to it, the body accelerates rapidly, the entire force applied being available to overcome its inertia. However, as the body starts to move, the friction reaction comes into play and some of the impressed force is used to overcome it, and only the remainder of the impressed force is available to overcome inertia for further acceleration. As more and more of the impressed force is required to overcome the increasing friction reaction, and less and less is available for acceleration, the velocity of the body increases more and more slowly, until finally the entire impressed force is required to overcome friction alone, and the velocity becomes constant.

87. Exact Form of Current Growth in an Inductive Circuit.—The equation for the growth of current in an inductive circuit of constant inductance, L, can be derived by the use of higher mathematics, and is found to be as follows:

$$i=\frac{E}{R}\left(1-\epsilon^{-\frac{Rt}{L}}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (78)$$

where i is the current at any elapsed time, t, after closing switch;

E is the constant impressed voltage;

R is the resistance of the circuit;

L is the inductance;

ϵ is the Naperian logarithmic base = 2.71828.

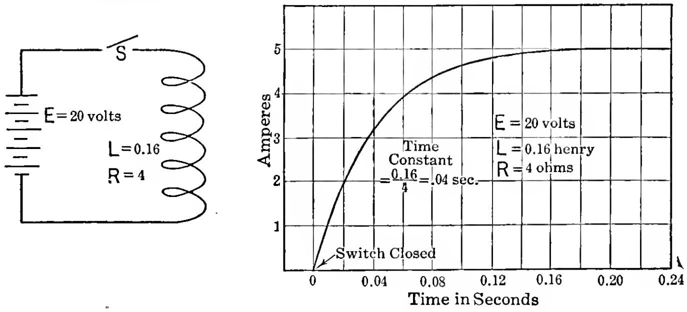


Fig. 99.—Exact analysis of the growth of current in an inductive circuit shows it to be an exponential curve, as given here.

In Fig. 99 is shown the rise of current in a circuit possessing 0.16 henry inductance and 4 ohms resistance, when 20 volts is impressed, the final value of the current being 5 amperes. Figure 100 is an oscillograph record of the growth of current in an air-core inductance, the permeability and inductance, L, of which remains constant. Fig. 101 is a similar record for a coil wound on an iron core; the permeability and L of this coil varied

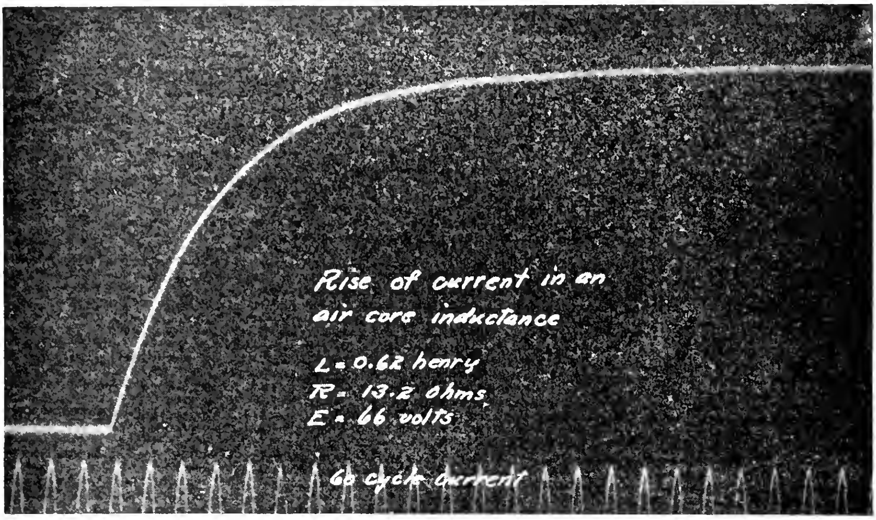


Fig. 100.—An oscillogram of the rise of current in an inductive circuit; it is seen to be of exactly the same form as the theoretical curve of Fig. 99.

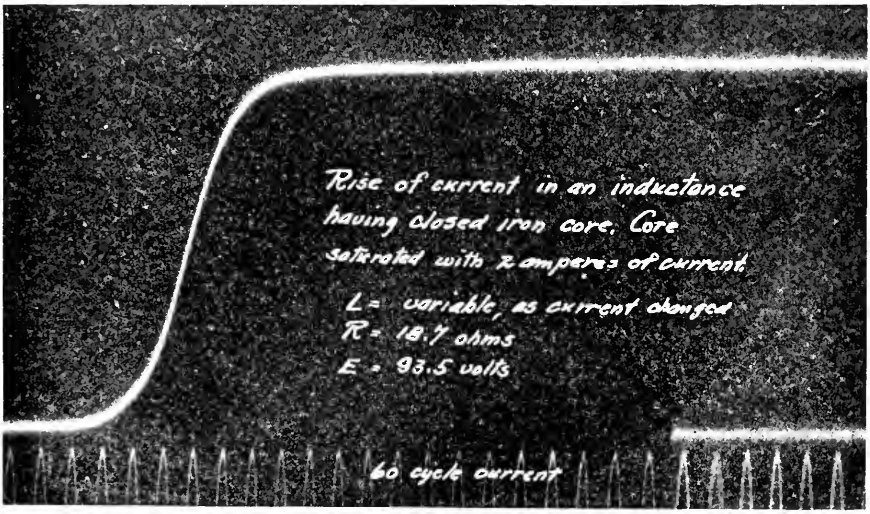


Fig. 101 If the inductance has a closed iron core, the permeability is likely to change as the current rises, thus giving a variable L in Eq. 78. The form of current rise is therefore considerably different from that of Fig. 100.

with the current, accounting for the difference in the form of this curve as compared with that of the air-core coil.

88. Decay of Current in an Inductive Circuit.—If an inductive circuit is short-circuited, as in Fig. 102*a*, the current in the coil does not die down immediately, as it would in a non-inductive circuit, but continues to flow for an appreciable time after the short-circuit is applied; this is due again to the c.e.m.f. of self-induction.

As soon as the short circuit is applied and the current starts to decrease, the c.e.m.f. of self-induction makes its appearance and acts to maintain the current. As there is now no impressed e.m.f. on the circuit, this c.e.m.f. of self-induction is the only agent causing current to flow, and it must therefore be overcoming the iR drop, which exists so long as current flows.

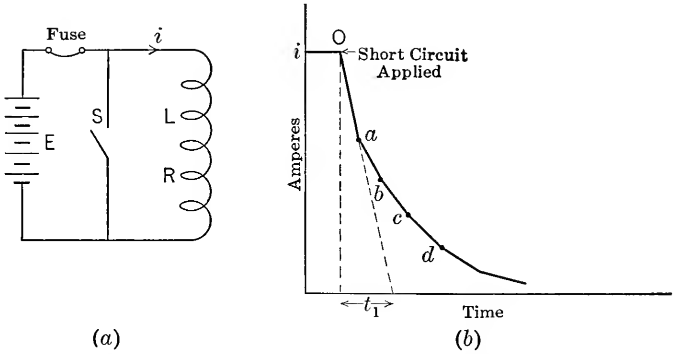


FIG. 102.—When an inductive circuit is shorted, as will be done by closing switch S of the above circuit, the form of current decay will resemble, in general form, the broken line shown in diagram (*b*).

If we assume the current to decrease to zero along the line *oa* in Fig. 102*b*, the time required to reach zero current would be t_1, and the rate of change of current would evidently be $\frac{i}{t_1}$. We may then write,

$$0=iR+L\frac{i}{t_1},$$

or

$$-L\frac{i}{t_1}=iR \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (79)$$

The last equation may be transposed to give,

$$\frac{i}{t_1}=-\frac{iR}{L}, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (80)$$

which indicates that the greater the iR drop the greater is the slope of the line oa, in Fig. 102b.

But as iR, at the instant the short circuit is established, is equal to E, the voltage which has been impressed on the circuit, we may write $\frac{i}{t_1} = -\frac{E}{L}$, which expresses the idea that the current starts to decrease at the same rate it started to rise when the voltage was impressed on the circuit. This is seen by reference to Eq. 74.

With the high rate of decrease of the current, as shown by the line oa, the value of the c.e.m.f. is large, as is necessary, inasmuch as the iR drop is still considerable. However, as the current falls, the iR drop decreases in proportion and the rate of change of the current need not be as large to generate the necessary c.e.m.f. to balance the iR drop. From the standpoint of time increments, the current, therefore, decreases its rate of decrease, first along the line ab for a while, and then along bc and so on.

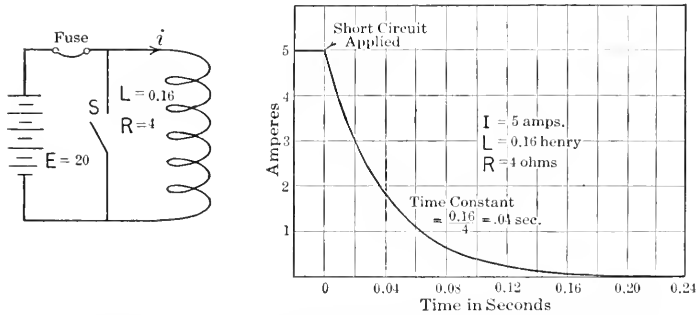


FIG. 103.—The exact form of current decay in an inductive circuit is exponential, as shown in this curve, which was calculated from Eq. 81.

The exact equation for the decay of current in a circuit having constant inductance, when derived by the use of higher mathematics, is found to be

$$i = I\epsilon^{-\frac{Rt}{L}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

where I is the value of the steady current through the inductive circuit, an instant before it is short-circuited; the other terms have the same significance as in Eq. (78).

Figures 103, 104 and 105 show the decrease in current in inductive circuits, Fig. 103 showing it for the circuit possessing 0.16 henry inductance and 4 ohms resistance, Fig. 104 being an oscillograph record of current decay in an air-core coil and Fig. 105 a similar record for an iron-core coil. Changing permeability accounts for the shape of the curve in the last figure.

89. Time Constant.—Study of Eq. (78) will indicate that the value of the fraction R/L is the factor which actually fixes the rate of increase of the current. In circuits of equal resistance, the greater the inductance, the longer it will take for a given impressed e.m.f. to set up the same final value of current in the circuit.

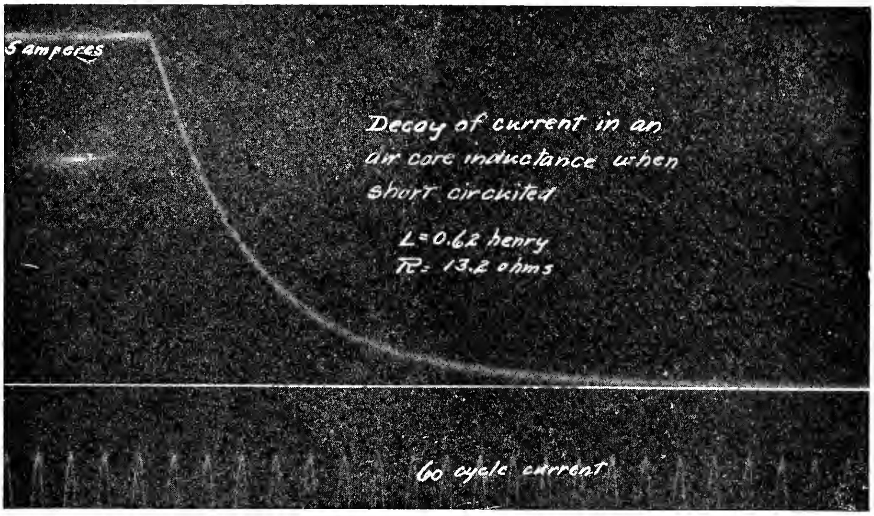


Fig. 104.—Oscillogram of current decay in an inductive circuit which has been short-circuited.

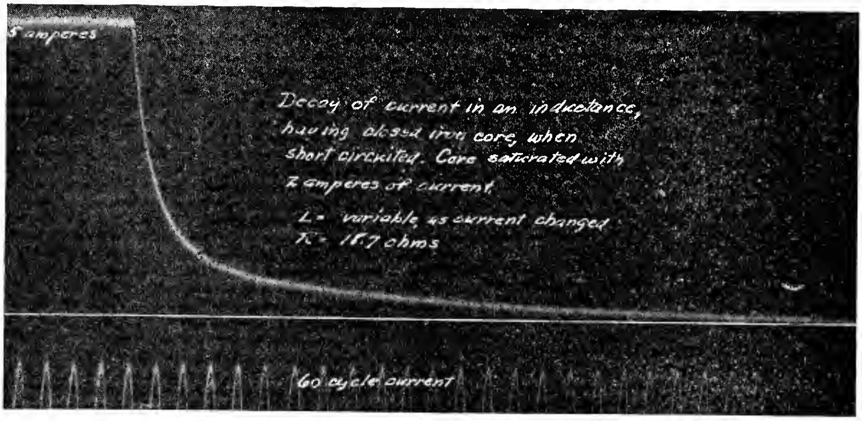


Fig. 105.—In case a closed iron core is used, the decay of current may be very rapid at first, and then the decrease takes place more slowly as the self-induction of the circuit increases.

In the case of a body being accelerated in a liquid, bodies of the same size, shape and surface will encounter the same frictional force at the same velocity; but a mass having high inertia, as lead, will accelerate much more slowly than a mass of low inertia, as aluminum, when the same force is applied. In the end, however, each will have the same velocity.

The greater the resistance of a circuit, the lower the final value of the current becomes with a given impressed e.m.f. The initial rate of increase of the current being fixed by the inductance, it follows that in circuits of equal inductance, the current will reach it's final value very much sooner in a circuit of high resistance than in one of low resistance; that is, the effect of inductance is less evident in a high resistance circuit than in one of low resistance.

The ratio of the inductance to the resistance of a circuit, (L/R), is called the time constant of the circuit. If the numerical value of L/R, in seconds, is substituted for t in Eq. (78), it will be found that it is the time it takes for the current to reach 63.2 per cent of its final value; if substituted in Eq. (81), it represents the time it takes the current to fall 63.2 per cent of its initial steady value.

The time constant is therefore useful in determining the rapidity with which current rises or falls in one inductive circuit in comparison with others. The time constant of the circuit used in Figs. 99 and 103 is $\frac{0.16}{4}=0.04$ second.

In order to get a large time constant, it is evidently necessary to have a high ratio of $\frac{L}{R}$; this requires the use of a large amount of copper or iron. This is illustrated by the fact that an air-core coil using even 100 pounds of copper wire will have a time constant of less than one-tenth of a second; the field circuits of large generators, having iron magnetic circuits of large dimensions, with many pounds of wire in the field coils, may have a time constant of several seconds.

90. Energy Stored in a Magnetic Field.—It was shown in section (54) that the energy stored in a magnetic field was,

$$w=\frac{4\pi N^2i^2\mu A}{2l}=L\frac{i^2}{2}\text{ ergs,}$$

where i is in abamperes and L in abhenries.

If the current is expressed in amperes and the coefficient of self-induction in henries, the above equation becomes,

$$W=\frac{0.4\pi N^2I^2\mu A}{2l10^8}=L\frac{I^2}{2}\text{ joules} \quad \dots\dots \quad (82)$$

When a magnetic field is established by a current, this amount of energy must be stored in the field as the current increases from zero to its final value, I.

Let us consider the first part of the current rise in an inductive circuit during the time this curve is essentially a straight line, as, for example, the first hundredth of a second in Fig. 99, which is shown enlarged in Fig. 106. During this time the current is so small compared with its final value that the resistance drop is negligible, so that Eq. (71) may be used, that is

$$E=L\frac{i_1}{t_1}.$$

The average current during this time is evidently $\frac{i_1}{2}$, so that the energy supplied to the circuit during this time is,

$$E\cdot\frac{i_1}{2}\cdot t_1=L\frac{i_1^2}{2t_1}\cdot t_1=L\frac{i_1^2}{2}. \qquad (83)$$

If we had taken into account the heat lost in the resistance of the circuit, the energy supplied to the circuit would have been found, by an accurate analysis, equal to $L\frac{i_1^2}{2}$ plus the amount of heat generated in the resistance. The amount of this heat cannot be obtained by ordinary arithmetic because the current is not truly a straight line during the time considered.

In any case, the amount of energy stored in the magnetic field, for any value of current I is found to be just equal to

$$W=\frac{LI^2}{2}. \qquad (84)$$

This is true no matter what the form of the current curve, as it rises from zero to I. This expression for the energy stored in a magnetic field checks with that derived in section (55) by an entirely different method.

91. Danger of Opening Circuits Possessing Much Inductance.—In discussing the decay of current in inductive circuits, it was considered that the coil was short-circuited, and the current was found to decrease to zero; all the energy stored in the magnetic field must therefore have been dissipated in the form of heat generated within the coil.

If the switch of a circuit possessing inductance, as in Fig. 99, is opened, it will be noticed that an appreciable arc occurs across the switch contacts, much greater in magnitude than would be the case if the circuit contained no inductance, as in Fig. 97. This arc is due to the voltage of self-induction.

The value of the voltage of self-induction is always given by the expression $L\frac{\Delta i}{\Delta t}$, where $\Delta i/\Delta t$ is the rate of change of the current. If the switch is opened very rapidly, so that the current is quickly ruptured, the rate of change of the current is high, as is also the value of the voltage of self-induction, the action of which is to maintain the flow of current even across the open switch contacts, in the form of an arc.

When an inductive circuit is opened, the energy stored in the magnetic field must be dissipated in some way by the time the current reaches zero, and this energy generally appears as heat in the arc at the switch contacts.

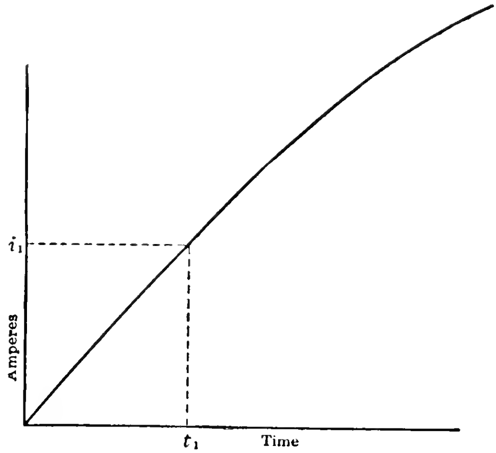


FIG. 106.—During the first short time interval after the switch of an inductive circuit is closed, the current rise is essentially a straight line, the slope being equal to E/L.

If the energy stored in the field is $L\frac{I^2}{2}$, and opening the switch reduces the current to zero in time t, then the average power must be $\frac{LI^2}{2t}$, which, if t is very small, will be large.

(It is to be pointed out that the circuit is not actually opened as quickly as the switch is opened, because the arc at the switch contacts maintains the circuit closed as long as it lasts, even though the switch itself is actually open.)

The average power must in turn be equal to the product of the average voltage induced and the average current. The current can, however, only decrease, making its average value small, so that the voltage induced must be large.

Suppose that the field circuit of a large generator has 5 henries inductance, and that its current of 10 amperes is opened completely in 0.05 second. The average induced voltage of self-induction will then be $5\times\frac{10}{0.05}=1000$ volts. The energy originally stored in the field is, from Eq. (84), $\frac{5\times10^2}{2}=250$ joules, which, if dissipated in 0.05 second, represents power to the amount of $\frac{250}{0.05}=5000$ watts. The average current is $10/2=5$ amperes and the product of 1000 volts and 5 amperes is again 5000 watts.

The case is analogous to the stopping of moving bodies; a rifle bullet, striking a steel wall, develops great force and great power for a fraction of a second.

The opening of circuits containing much inductance, as the fields of generators, motors, etc., is likely to cause severe burning at the switch contacts, but the real danger lies in the possibility that the voltage of self-induction may break down the insulation between the coil and the frame of the machine. If the opening of the switch induces a high voltage across the ends of a field coil, as diagrammatically represented in Fig. 107, this high voltage is also impressed on the path *WXYZ*, that is, across the insulation at *WX*, through the steel pole of the machine, *XY*, and the insulation at *YZ*. By puncturing the insulation at both *WX* and *YZ* and thus "grounding" the coil at two places, current may flow over the ground circuit, and the energy stored in the field may be thus dissipated. Such grounding of the field circuit may be dangerous for the operator, resulting in severe shocks, or may even make the machine inoperative in that it will not excite itself (as will appear later in the text).

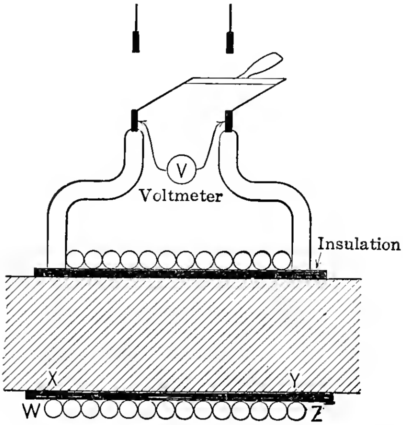


FIG. 107.—If a field circuit is opened quickly the high voltage of self-induction of the winding is likely to cause rupture of the insulation of the field winding; a breakdown is likely to occur through the insulation to the frame of the machine at two points, such as across *W–X* and *Z–Y*.

Incidentally, if a voltmeter is connected across the field side of the switch, to measure the voltage originally impressed on the coil, say 110 volts, and the switch is opened, the voltage of self-induction will be impressed on the voltmeter. If this voltage reaches high values, even only momentarily, the meter will be injured and may even be burnt out.

92. Field Discharge Switches.—In order to guard the fields of machines against puncture of their insulation when the field circuit is opened, special switches, called discharge switches, are used. The action of these switches is diagrammatically indicated in Fig. 108; in the position shown, current flows only through the field *F*, but before the blades of the switch, *bb*, part company with the main jaws, *aa*, one blade makes contact with an auxiliary jaw, *c*. This action places a discharge resistance, *R*, in parallel with the field, and, in this position of the switch, current from the supply continues to flow through the field, besides flowing through the resistance. Further motion of the switch opens the contacts between the blades and the main jaws, opening the line but leaving the discharge resistance in parallel with the field. The energy stored in the field is thereby dissipated slowly and without the induction of high voltages. A common form of field discharge switch is illustrated in Fig. 109.

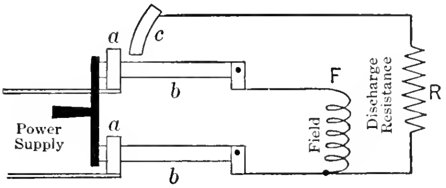


FIG. 108.

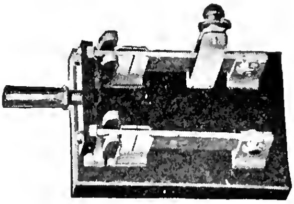


FIG. 109.

FIG. 108.—A common method of lessening the chances of the rupture of the field-winding insulation is to open the field circuit by a specially designed switch, such as that indicated here; before the field is disconnected from the power supply a resistance *R* is connected across the field circuit, and this serves to keep the rate of decay of the current to reasonable values.

FIG. 109.—A common type of the switch indicated in Fig. 108; the extra clip, for connecting the field discharge resistance across the field winding is seen on the upper blade.

93. Mutual Induction.—Consider two turns of wire in close proximity, as in Fig. 110. If an increasing current is made to flow in turn 1, some of the flux set up will thread turn 2 and establish flux interlinkages with it. Voltage must therefore be induced in turn 2, as long as its flux interlinkages are increasing, and this voltage, from previous reasoning, will have a direction as shown, or opposite to the direction of the impressed e.m.f. in turn 1. The induced voltage in turn 2 will set up current in the same direction as itself, or in a direction opposite to that of the current in

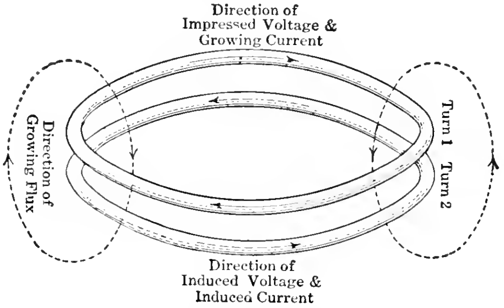


FIG. 110.—If current rises in one circuit in proximity to another, a voltage is induced in the second, by the change of flux linking the second, this flux being set up by the first circuit. If the second circuit is closed a current will flow in it, due to this induced voltage.

turn 1. Hence, the induced current in turn 2 sets up a m.m.f. in the opposite direction to that due to the current in turn 1; but inasmuch as it owes its existence to the growing field due to the current in turn 1, it cannot actually overcome the flux set up by turn 1 and set up a flux of its own in the opposite direction; the action of this m.m.f. in turn 2 is merely to decrease the amount of flux from turn 1, which links turn 2. In other words, the effect of the current in the second circuit is to limit the rapidity of the increase of flux through itself, the flux being due to the current in circuit 1.

If the current in turn 1 is allowed to die down, the decreasing flux linking turn 2 induces a voltage in turn 2, the direction of which is obviously as shown in Fig. 111.

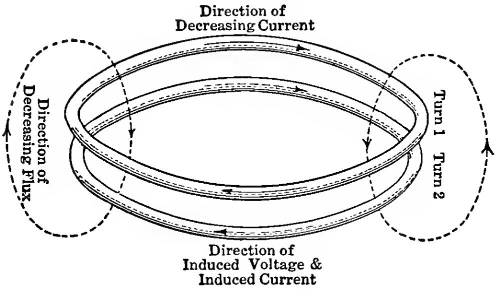


FIG. 111.—If the current in the first circuit decreases, the voltage induced in the second will be in the opposite direction to that it had when the current in the first was rising.

The induced current in turn 2 will now tend to set up a field in the same direction as the decreasing field of turn 1, thereby again opposing the change taking place in the latter.

Therefore, if two circuits are so placed with respect to each other that the magnetic field due to current in one circuit links wholly or partly with the other, any change in the strength of the current in the first circuit will induce in the second circuit a *voltage of mutual induction.* If the second circuit is closed, this voltage of mutual induction sets up a current in such direction in the second circuit as to oppose the change of the flux linking this circuit. This, again, is in accord with Lenz's law. If the second circuit is open, it offers no reaction to the establishment of a field by the first circuit, because no current can flow in this open circuit even though a voltage is induced in the second circuit; it is only by current flow that a back m.m.f. is set up.

The magnitude of the e.m.f. of mutual induction depends only upon the rate of change of the flux interlinkages of the second circuit; i.e.,

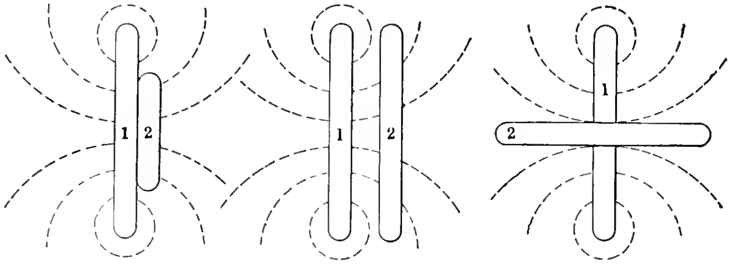
$$e_2 = \frac{\phi_m N_2}{10^8 t} \quad \text{. . . . .} \quad (85)$$

where e_2 is the voltage of mutual induction in the second circuit;
ϕ_m is the *mutual* flux that is set up, linking both circuits;

N_2 is the number of turns in the second circuit;

and t is time in seconds, taken for the flux ϕ_m to be set up.

The amount of flux from the first circuit actually linking with the second circuit will depend upon the relative shapes and relative positions of the two circuits. If the first circuit is larger than the second (Fig. 112*a*), or at a distance from the second (Fig. 112*b*), only a small portion of the flux set up by the first circuit will thread the second.



(*a*) (*b*) (*c*)

FIG. 112.—The value of the mutual induction between two circuits depends upon their relative shapes and positions, being about the same for cases *a* and *b*, and being zero for case *c* shown above.

If the two circuits have their axes at right angles (Fig. 112*c*), no flux from either will thread the other; hence there is no mutual induction between them.

94. Unit of Mutual Inductance.—Even though the two turns considered before are brought as close together as possible, only a fraction of the total flux set up by the first will link the second; some of the primary flux will leak past the second turn. We may write then, if ϕ_1 is the total flux set up in the first circuit,

$$\phi_m = K\phi_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (86)$$

where K is a constant for any particular circuit and position of the two turns; it is called the *coefficient of coupling* of the two circuits.

Substituting the above value for ϕ_m in Eq. (85), we have

$$e_2 = \frac{K\phi_1 N_2}{10^8 t}.$$

Since K and N_2 are constants and ϕ_1 is proportional to the current i_1, in the first circuit, we may further write,

$$e_2 = \frac{K\phi_1 N_2}{10^8\ i_1} \cdot \frac{i_1}{t} = M\frac{i_1}{t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

where

$$M=\frac{K\phi_1 N_2}{10^8 i_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

and is called the coefficient of mutual induction, or mutual inductance between the two coils.

In this analysis also, if the change in i_1 or ϕ_1, is not uniform, the equations must be expressed in the form of increments; thus, instead of Eq. (87), we should have

$$e_2=M\frac{\Delta i_1}{\Delta t_1}, \text{ etc.}$$

The henry is also the unit of mutual inductance; it may be defined from either of the last two equations. From Eq. (87), we find that two circuits have mutual inductance of one henry, if a rate of change of one ampere per second of the current in the first, or inducing, circuit, generates one volt in the second circuit; from Eq. (88) we may define the henry as the amount of mutual inductance possessed by two circuits when one ampere in the first circuit sets up 10^8 flux interlinkages in the second.

It is to be noted that M has the same value whichever coil is treated as primary; that is

$$M=e_2\frac{\Delta t}{\Delta i_1}=e_1\frac{\Delta t}{\Delta i_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (89)$$

in which e_2 is the voltage induced in circuit two by a rate of change of current $\frac{\Delta i_1}{\Delta t}$ in circuit one and correspondingly e_1 is the voltage induced in circuit one by a rate of change of current $\frac{\Delta i_2}{\Delta t}$ in circuit two.

Also

$$M=\frac{\phi'_2 N_2}{10^8 i_1}=\frac{\phi'_1 N_1}{10^8 i_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (90)$$

in which ϕ'_2 is the flux set up through circuit two by a current i_1 in circuit one and correspondingly ϕ'_1 is the flux set up through circuit one by a current of i_2 in circuit two.

95. Currents Due to Mutual Induction.—The exact form of currents set up in circuits due to mutual induction with other circuits in which the current is varying, is difficult to deduce without the use of the calculus, but a typical case is shown in Fig. 113, which is an oscillogram showing the currents in two circuits coupled by mutual induction. A continuous voltage was impressed on the first circuit, while the second was short-circuited. It is to be seen that when the current in the primary circuit rises, a negative current is set up in the secondary, and when the current in the primary decreases, a positive current is set up in the secondary.

The peculiar form of decay of current in the primary, accompanied by a corresponding peculiarity in the form of the secondary current, was due to arcing at the switch blades when the primary circuit was opened, this arcing being caused by the self and mutual inductances of the two circuits.

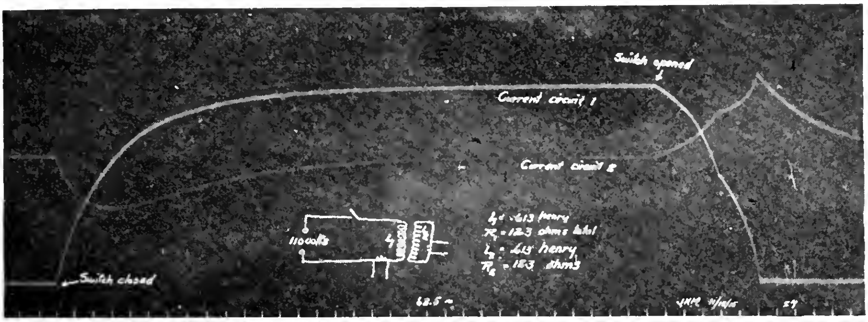


FIG. 113.—The form of the current due to induced electromotive forces is not easy to deduce with elementary mathematics; for a typical case, the current in the second circuit due to rise and fall of the current in the first was as shown by the above oscillogram. The form of the primary, or inducing current, is also shown in the film.

PROBLEMS

1. A conductor 10 inches long is moved, in a direction perpendicular to its length, at a velocity of 1200 feet per minute, through a magnetic field of 8000 gausses. How many abvolts does it generate, and how many volts?

2. A loop of one turn is linked with 10^6 lines of flux; the flux is reduced to zero in 0.0123 second. What is the average induced voltage?

3. The inductance of a certain generator field is 40 henries. The current in the field is changed from 1.25 amperes to 0.87 ampere in 0.0136 second. What is the induced voltage?

4. With a current of 7.24 amperes through 860 turns, the reluctance of the magnetic circuit is 0.130 oersted. When the current increases 3 per cent, the reluctance increases 1 per cent. The current increase takes place at the rate of 18,000 amperes per second. What is the average induced voltage, and what is the inductance in abhenries and henries for each of the current values?

5. A coil of 1000 turns has a magnetic circuit of 18 sq. cm. cross-section. With 9 amperes, the density is 14,500 gausses; and with 4 amperes, it is 10,000 gausses. What is the inductance for each current?

6. In the above problem, the current change takes place in 0.00146 second. What is the average induced voltage?

7. From the equation, $e = L\frac{\Delta i}{\Delta t}$, what is the average inductance of the circuit of Problem 6, which opposes the change of current? What is the average value of L of this circuit, from the data as given in Problem 5? Explain the difference.

8. A circuit of 0.216 henry and a resistance of 4.63 ohms are connected to a battery of 102 volts and 1.63 ohms. At what rate does the current start to rise immediately after the switch is closed?

9. What is the time constant of the above circuit? What is the current this number of seconds after closing the switch?

10. A certain 110-volt generator field circuit has an inductance of 12 henries, and 88 ohms resistance. How long does it take for the current to reach 95 per cent of its final value? To reach 99 per cent?

11. The current in the above circuit is reduced to zero, from full value, in 0.06 second. What is the average voltage of self-induction?

12. How much energy is stored in the circuit of Problem 3 when the current is 1.25 amperes, in ergs, joules, and ft.-lbs.? If the current is reduced to zero (by opening the switch) in $\frac{1}{60}$ second, what is the average power expended in the arc?

13. Two circuits are magnetically coupled by a mutual inductance of 3.26 henries. If the current in one circuit changes from 2.63 amperes to 0.97 ampere in 0.0142 second, what voltage is induced in the second circuit?

14. A coil of 200 turns of wire is threaded by 100,000 lines of force. The exciting current is halved in 0.01 second. What is the average induced e.m.f.? If the original exciting current is 5 amperes, what is the inductance of the circuit?

15. The core of a certain transformer is 15 cm. by 20 cm. in cross-section; the winding has 500 turns. The density with a current of 0.2 ampere, is 5000 gausses. What is the coefficient of self-induction, and what is the average induced e.m.f., if the current is reduced to zero in 0.001 second?

CHAPTER VI

CONTINUOUS-CURRENT METERS

96. Classes of Meters.—The quantities generally measured by continuous-current meters are current, voltage, quantity of electricity, and electrical energy. The instruments for measuring these quantities are, respectively, ammeters, voltmeters, ampere-hour meters and watt-hour meters. The first two are generally indicating instruments, and the discussion of them will be taken up first.

In addition to the above-named instruments, the electrical engineer uses several other types, such as the oscillograph, megger, etc.; these will be taken up later.

97. Requirements for Indicating Meters.—There must first be an actuating force which is proportional to the quantity to be measured. In order that the needle or pointer may come to rest at some definite point on the scale, some counter or restoring force, the intensity of which will increase in proportion to the displacement of the pointer, must be provided. Since the needle or pointer must move over a scale, bearings of some sort are necessary. That readings may easily be taken, some sort of device is required to rapidly damp out oscillations of the needle; the instrument must be " dead beat." A uniform scale is generally desirable, but is not always possible, as this feature depends upon the variation of the actuating force with the quantity to be measured. In some types of meters, the actuating force varies as the square of the current or voltage, as the case may be.

98. Actuating Force.—Continuous-current indicating instruments obtain their actuating forces in one of three ways, the most important being that employed in the type known as the *D'Arsonval*, in which a movable coil, carrying a current proportional to the quantity to be measured, is placed in the field of a permanent magnet. In the *movable-core* type of c.c. instrument, one or more pieces of soft iron are acted upon by the electromagnetic field of a coil carrying current proportional to the quantity to be measured. In the third type, known as the *electro-dynamometer*, the force between two or more conductors carrying current is utilized. Instruments are generally classified according to their actuating forces, and these three types will be described in greater detail later.

99. Restoring Force.—As restoring or counterbalancing forces are used: (1) the resisting force of a spring; (2) the attraction of gravity; (3) the torsion of some filament. The first-named is most frequently used.

100. Damping Devices.—The means employed for this purpose may be either the attraction of currents, induced in properly placed conductors by motion of the moving element, or the air friction of a suitable fan or vane.

101. D'Arsonval Meters.—In Fig. 114 are shown the general features of this type of meter. A rectangular aluminum frame, on which is wound a coil of very fine insulated copper wire, is mounted between the poles of a permanent horse-shoe magnet, these poles being so shaped, by the addition of soft-iron pole pieces, that the clearance between the coil and the pole piece faces is constant. Within the coil and supported by a brass plate attached to the pole pieces, is a soft iron cylinder, which helps to complete the magnetic circuit. The air gap between the cylinder and the pole pieces, within which the coil moves, is thus of constant length, and the coil can move for a considerable arc in a radial field of uniform flux density. Two spiral springs, one at the upper, the other at the lower end of the shaft, furnish both the restoring force and a means of current connection between the coil and the external circuit. The springs are so arranged that as the coil is deflected, one is coiled up and the other uncoiled, compensating for errors due to elongation brought about by temperature changes. The moving element is supported at both top and bottom by hardened steel pivots turning in cup-shaped jewels, which

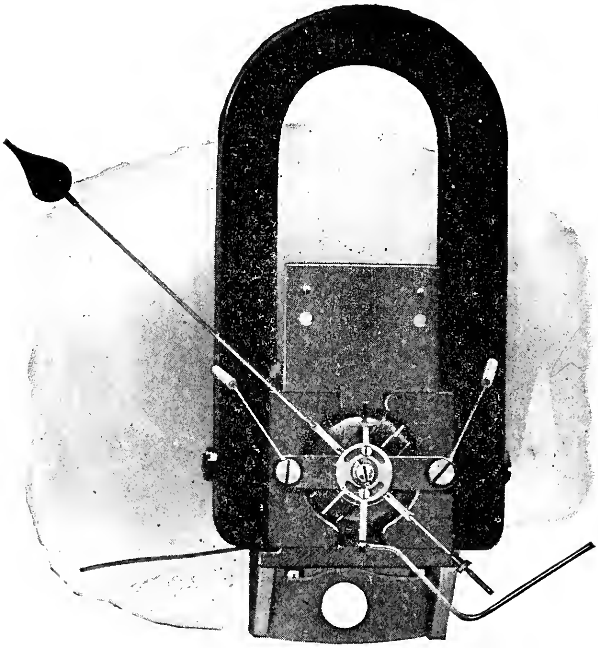


FIG. 114.—This cut shows the construction of the moving parts of a modern D'Arsonval type of meter.

are generally white sapphires. The pointer is made of very thin aluminum tubing and is balanced by small counterweights.

Since the turns of the coil are in a uniform magnetic field, when current flows through it a force results which is directly proportional to the current. By the use of spiral springs, the restoring force is made directly proportional to the distortion, and a uniform scale results.

The rectangular aluminum frame on which the coil is wound serves also as the damping device. When the current in the coil changes, the equality between the actuating and restoring forces is destroyed, and the coil must move until this equality is again restored. If the moving element is very light, the coil moves very quickly; in any case, without a suitable damping device, it would overshoot the new point of equilibrium, drop back, and continue to oscillate until air and bearing friction brought it to rest. Within the aluminum frame (on which the coil is wound) eddy currents are induced, while the coil is moving. The direction of these currents is always such that, reacting with the magnetic field, a force is produced which opposes the motion. This form of damping device is very effective, the needle coming to rest with only one or two

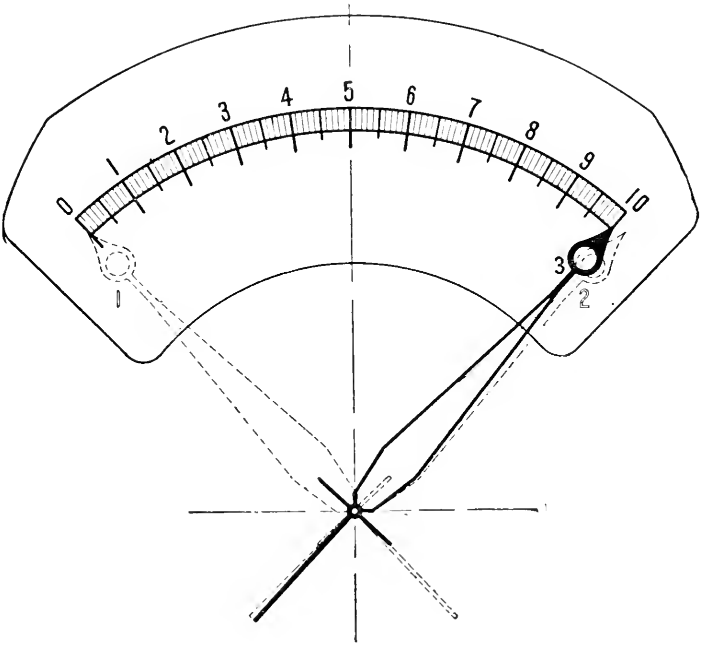


FIG. 115.—The moving system of a meter should be damped sufficiently so that it overshoots its proper position only a small amount, coming to rest with not more than one or two small oscillations; a reasonable amount of overshooting is shown in the above figure.

oscillations, as indicated in Fig. 115. Overdamping is never desirable, as it is apt to cause sluggishness; slight underdamping gives opportunity to observe that the movement of the needle is perfectly free.

The moving element of good commercial meters of the D'Arsonval type is made to deflect to full scale value with a current of about 0.015 to 0.020 ampere through the coil. A permanent magnet being used, the coil will move in a desired direction only when the current flows in the right direction. If the current is reversed, the direction of deflection will be in the opposite direction, or backwards. Such a meter, therefore, has polarity and cannot be used on alternating current circuits. It is customary to indicate the polarity of the terminals of c.c. meters by a plus sign on the proper post.

102. Ammeters and Shunts.—In order that the instrument described above may be used as an ammeter to measure currents greater than, say, 0.020 ampere, it is placed in parallel with a resistance, known as a *shunt*, as in Fig. 116. In order that the meter may be deflected to full scale when a line current, I, flows through the combination of meter and shunt, a required value of current, I_m, must be passed through the meter. This leaves a current, I_{sh} (equal to $I-I_m$), to flow through the shunt. The drop in potential across the shunt is

$$E_{sh}=I_{sh}R_{sh},$$

and this is also the voltage impressed upon the meter. Hence, the resistance of the meter must be

$$R_m=\frac{E_{sh}}{I_m}. \qquad (91)$$

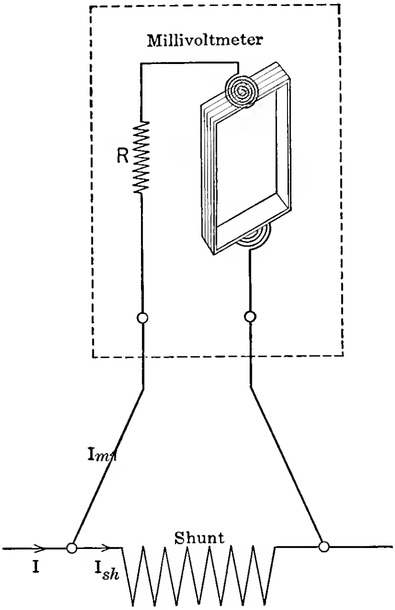


FIG. 116.—When a D'Arsonval type of meter is used as an ammeter, connections are so made that only a small fraction of the current to be measured goes through the moving coil; the moving coil is connected in parallel with a low-resistance shunt which generally carries more than 99 per cent of the line current.

In the best types of D'Arsonval meters, the drop across the shunt, when it is carrying its rated current, is made from 0.100 to 0.200 volt or from 100 to 200 millivolts. Assuming that the current required to cause the needle to go to full-scale deflection is 0.020 ampere, when 100 millivolts are impressed across the instrument, the resistance of the meter together with the leads must then be 0.1/0.02 or 5 ohms. In cheaper makes of meters the drop across the shunt is about 50 to 60 millivolts. The moving coil, although wound of fine wire, has a resistance usually lower than 5 ohms, but in order to bring the resistance of coil and leads to exactly 5 ohms, or whatever value is necessary, sufficient resistance is put in series with the coil, as is shown in Fig. 116.

If the meter just described is intended to be used as a 10-ampere ammeter, the current through the required shunt would be $10-0.02=9.98$ amperes, and the resistance of the shunt, to give a drop of 100 millivolts, would be $0.100/9.98=0.01$ ohms, very nearly.

The same meter, when used as a 200-ampere ammeter, would require a shunt with a resistance of 0.100/199.98 or 0.0005 ohm, approximately.

The meter in operation really reads the drop across the shunt. If the line current is one-half the value for which the shunt is rated, and its rated current gives a drop of say 100 millivolts, the drop across the shunt is 50 millivolts, and only 0.01 ampere passes through the meter, causing the needle to stop at half-scale. This would indicate a current of 5 amperes if the shunt used is a 10-ampere one, or 100 amperes for the shunt having a rating of 200 amperes.

The meter alone, together with its leads, is called a *millivoltmeter*, and it is thus possible to use a millivoltmeter as an ammeter of any range, provided it is used with the proper shunt. In commercial meters, in smaller ranges, it is customary to place the shunt inside the meter case and to calibrate the scale of the meter in amperes direct. Such meters are called " self-contained " and are made for as high as 500 amperes by some manufacturers. To make the supply of meters as flexible as possible, it is frequently the practice in laboratories to have all shunts separate from the meter. Wherever external shunts are used, however, the leads must have a certain resistance to make the meter read accurately.

Shunts are usually made of a metal the resistance of which does not change with temperature to any appreciable degree, such as manganin; for large values of current these shunts become very large and bulky (Fig. 117).

One reason for using 100 millivolts drop or higher across the shunt for the best millivoltmeters is that the change in resistance (due to temperature variation) of the copper wire used on the moving coil can be compensated. The adjusting resistance is made of a metal with negative temperature coefficient; enough metal must be used for proper compensation, and the increased resistance requires the use of high drop across the shunt. In cheaper instruments, this compensation is not added to the meter.

103. Voltmeters and Multipliers.—To use the D'Arsonval type of meter as a voltmeter, it is only necessary to place in series with the moving element a high resistance, as in Fig. 118. The moving coil in voltmeters is usually wound with more turns, and is of finer wire, than that used in millivoltmeters, and the needle will therefore go to full-scale deflection with smaller currents; a value of about 0.01 ampere is ordinarily used. The resistance of the coil in a certain reliable make of meters is about 150 ohms.

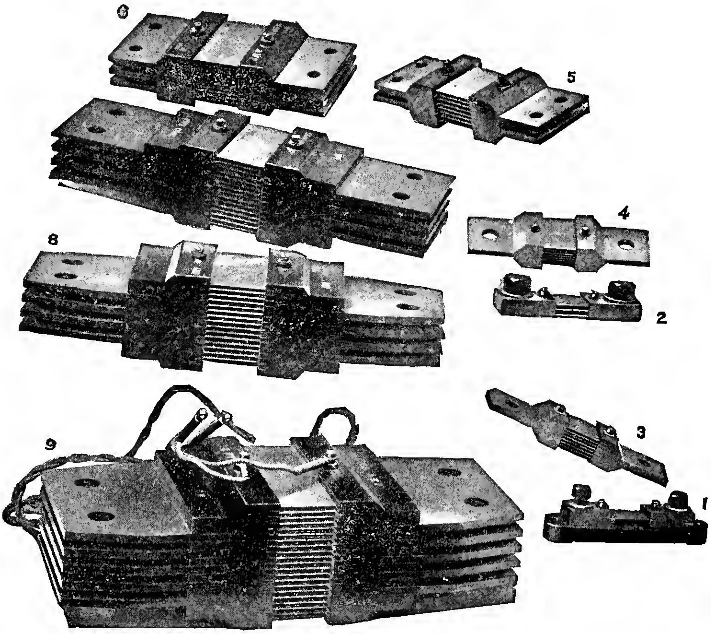


FIG. 117.—Shunts intended to carry large currents must be suitably designed; to prevent undue heating, with resultant error in the meter reading, much metal must be used for the large currents and a liberal amount of heat radiating surface allowed.

To draw a current of 0.01 ampere from a 150-volt source of potential, a total resistance of 15,000 ohms is required. If the resistance of the moving coil is taken as 150 ohms, then 14,850 ohms of additional resistance must be added in series, as shown in Fig. 118*a*. This series resistance is ordinarily of a metal having a zero resistance-temperature coefficient. The scale of the instrument is naturally calibrated in volts, the current taken (and therefore the deflection) being proportional to the voltage impressed.

It is a very simple matter to provide a number of different ranges on the same voltmeter. If a 75-volt range is desired, it is only necessary to bring out a tap from the total resistance, so that 7350 ohms are added to the coil. With a coil resistance of 150 ohms, using the coil alone, a 1.5-volt range results. (Fig. 118*b*.)

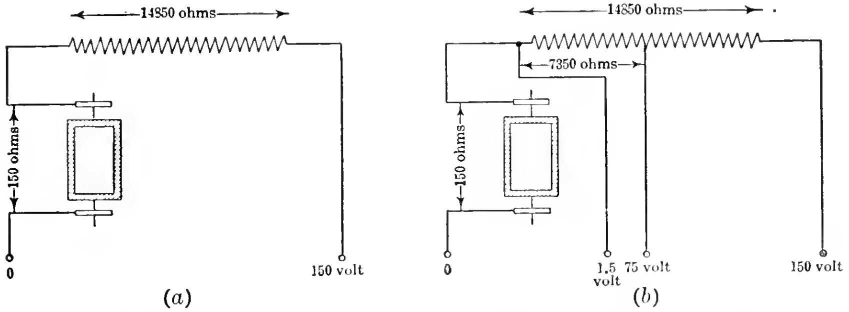


FIG. 118.—When a D'Arsonval type of meter is used for a voltmeter, a suitable resistance must be used in series with the moving coil; in case several voltage ranges are to be obtained with the same meter, suitable taps from this added resistance are brought out to binding posts.

If a voltmeter is to be used for higher ranges of voltage than the instrument was built for, proper external resistances, called multipliers, can be used in series with the meter (Fig. 119). For the case considered, if an external resistance of 15,000 ohms is connected in series with the meter, the combination of meter and multiplier is good for 300 volts, the reading of the meter being multiplied by 2 to get the voltage of the circuit.

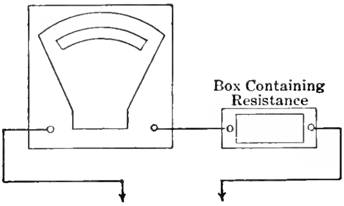


FIG. 119.—In case a single-range meter is to be used for voltages higher than its rated range, a suitable external resistance may be connected in series with it; this resistance must be able to safely carry whatever current is required to give full scale reading of the meter.

104. Movable-core Type Meters.—To this type of meters belong all of those in which the actuating force is obtained by the action of a coil carrying current, on a piece of soft iron. One of the earliest types is shown in Fig. 120; it is readily seen that the current magnetizes the soft iron core and draws it into the coil. The restoring force is gravity. The force drawing in the core is proportional to the square of the current until the core is well up in the coil, when it increases with the current less rapidly; doubling the current doubles the flux through the iron (provided it does not become saturated) and quadruples the force. This results in a scale of uneven divisions, which is objectionable, the divisions for low values of current being generally very small. Another objection to the type illustrated is that the moving parts are heavy, resulting in objectionable friction in the supports, and also making an instrument which it is difficult to damp. When the instrument is used as an ammeter, the entire current to be measured is passed through the coil; shunts are never used in connection with it. The actuating force is independent of the direction of the current, so that these instruments do not possess polarity.

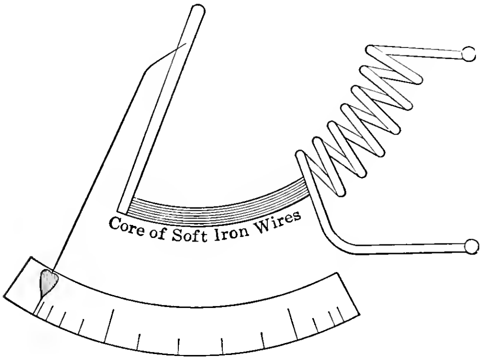


FIG. 120.—An old-style iron-vane meter, useful only for installation on a switchboard; such an instrument cannot easily be made portable.

Modern meters employing this principle show great improvement, and are entirely satisfactory except for their uneven scales. Although the same principle is used, the method of application is such that light moving parts and excellent damping are possible. However, for c.c. purposes, meters on the D'Arsonval principle are so nearly perfect and, for the same accuracy and sensibility, so cheap, that at present this is practically the only type employed.

Movable-core type instruments are extensively employed in alternating-current circuits, simply because there are no other types that can be made at moderate cost. The forms so employed will be discussed in the section on alternating-current meters.

105. Electro-dynamometer Meters.—As noted before, the principle of operation is the mutual attraction and repulsion between circuits carrying currents, or the interaction of the magnetic fields produced by the currents. As no iron is employed, the action is called electrodynamic, instead of electromagnetic.

The principle of operation, as applied in a voltmeter, is illustrated in Fig. 121, which gives a sketch, from the top, of a meter of this type. Two stationary coils and one movable coil, all of many turns of fine wire, are connected in series, the movable coil being pivoted to rotate between the two stationary coils. Current flowing through the three coils causes the movable coil to rotate against the tension of springs, its extreme displacement being about as shown in broken lines.

Evidently, since the force between the coils varies as the current squared, such a meter will also have an uneven scale. To set up forces without the use of iron requires comparatively high values of current, resulting in a meter which, compared to those utilizing the D'Arsonval principle, requires considerably greater power.

It will readily be appreciated that it is almost impossible to pass any but very small currents through the moving coil; the connections to the coil can only be of the lightest, and the coil itself must also be very light. The use of this principle for portable ammeters is thus impracticable except where small currents are to be measured. For measuring large currents with meters of this type, suitable shunts are sometimes used, so that only a small fraction of the total current flows through the movable coil.

For laboratory purposes, it can be used, and formerly was much employed in a form known as the Siemens dynamometer, particularly to measure alternating currents. Its form for such purposes consisted of a large rectangular, or circular, stationary coil of several turns of large wire, and a similar but slightly smaller movable coil, moving within the former. The movable coil was suspended by a helical spring, and contact to the coil was made by having its ends dip into mercury cups.

For portable instruments, the electro-dynamometer principle is used at present only in a.c. voltmeters, being generally inferior to the D'Arsonval. However, a type of electro-dynamometer known as the Kelvin balance is very convenient for laboratory work, being used to compare alternating currents with continuous currents, that is, as a transfer instrument.

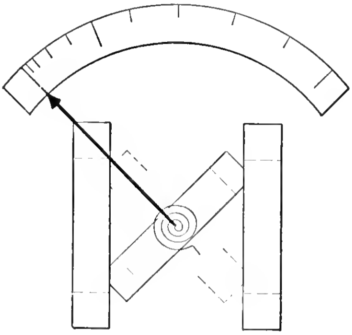


FIG. 121.—Top view of a dynamometer type of meter; the moving coil is pivoted so as to be rotatable inside of the two fixed coils. The same current flows through both fixed and movable coils as they are connected in series.

106. The Watt-hour Meter.—In connection with the supply and sale of electric energy, a meter to record the total amount of energy used is necessary. It is standard commercial practice to measure electrical energy in kilowatt-hours, the kilowatt-hour being defined as the total amount of energy supplied in one hour to a circuit in which the average rate at which the energy is expended is 1000 watts.

A watt-hour meter consists essentially of (*a*) a small electric motor as actuating force, the torque of which is proportional to the power taken, (*b*) a brake system, so constructed that the retarding force is proportional to the speed of the motor armature spindle to which it is attached, and (*c*) a system of gears with graduated dials for registering the number of revolutions of the spindle.

When the speed of the rotating spindle has reached a constant value, the actuating torque must be exactly equal to the retarding torque. If the driving torque of the motor is made proportional to the power taken, and the retarding torque of the brake system is proportional to the speed of rotation of the spindle, then the speed of the spindle is proportional to the actuating torque and also to the power.

If speed of the spindle is proportional to power, each revolution represents a certain number of watt-hours passed through the meter, whether the power is variable or steady, so that the total number of revolutions of the shaft during a given interval is a measure of the total energy during the interval.

The two common types of c.c. watt-hour meters are the Thomson or "commutator," and the mercury flotation, the former being more generally used.

107. Brake System.—As the brake system of all modern watt-hour meters is virtually the same, it will be taken up first. An aluminum disc is mounted on the motor armature spindle so as to rotate between the poles of one or more permanent magnets. The rotating disc generates eddy currents within itself which, reacting with the magnets, produce a drag on the disc. The strength of the magnets and the resistance of the eddy current paths being constant, the strength of the eddy currents, and hence the force between them and the field of the magnets, will be proportional to the speed. Therefore, the drag on the disc is directly proportional to the speed.

108. Thomson, or Commutator, Watt-hour Meter.—In this type, a motor having fields and an armature with a tiny commutator is employed. The field coils, of low resistance, are placed in series with the line supplying the current; no iron being used in the construction of the motor, their flux is directly proportional to the current.

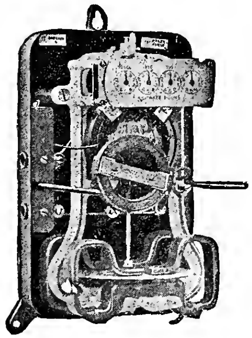


Fig. 122.—General view of the most common type of watt-hour meter for continuous current circuits.

The armature is placed directly across the line and its current (which is reduced to a very low value by the use of a high series resistance) is then directly proportional to the e.m.f. of the line. The torque of a motor being proportional to the product of armature current and field strength, it follows that the actuating torque of the meter is directly proportional to the product of line voltage and line current, or to the power supplied to the load.

In Fig. 122 is shown a commercial form of a Thomson or commutator type of meter, with the cover removed; in Fig. 123, the circuits of such a meter are shown diagrammatically. A vertical spindle, S, supported at the top by a guide bearing, and at the bottom by a sapphire or diamond bearing, carries the armature, A, made in spherical form to fit closely to the fields, and the aluminum disc D.

The main field coils, FF, are in series and carry the entire current passing through the meter. The armature, or potential, circuit, shown in dotted lines, starts at one side of the line, and passes successively through the compensating field C, the armature, and a fixed high resistance, R, ending on the other side of the line. In some meters all the necessary resistance in series with the armature is contained in the compensating field and no extra resistance, R, is used.

The function of the compensating field is to compensate for the friction of the main bearings, the brushes on the commutator, and the gear train. The friction is reduced to a minimum by using an extremely small commutator (about $\frac{1}{10}$ inch in diameter), jewel bearings and a very light gear train. The material used in the commutator is generally pure silver, to maintain a smooth surface and prevent oxidation, and the brushes are generally silver-tipped for the same reasons.

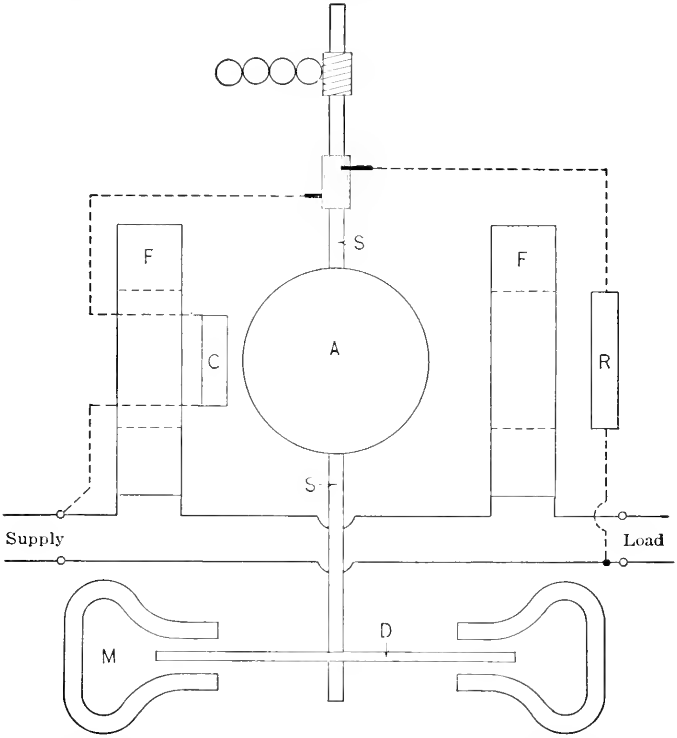


FIG. 123.—Schematic diagram of the Thomson watt-hour meter, showing the field coils, F–F, connected in series with the load, and the armature circuit, consisting of the armature, A, compensating winding, C, and extra resistance, R, connected across the line.

The strength of the compensating field is not adjustable, but its effect on the armature may be changed by varying its position with respect to the armature; enough of its flux is made to pass through the armature to create sufficient torque to just balance friction, so that the meter is just on the point of starting at no load. If the compensating coil is too close to the armature, more torque than necessary results, and the meter tends to "creep," or rotate slowly, without any load current through its fields, and register "fast," or too high, at light loads.

As the load on the meter increases, the main fields become stronger and stronger, and the effect of the compensating field comparatively less and less. In order to control the speed of the disc for loads above 10 per cent, the position of the permanent magnets is changed. If they are moved out from the center of the disc, the speed of the disc will decrease for a given load, for the same rate of cutting of the flux from the permanent magnets can now be maintained with the lower rotational speed of the disc. On moving the magnets in toward the center of the disc, the latter will speed up. At very light load, the effect of the permanent magnets is insignificant because of the low speed.

The resistance of the armature circuit is about 2500 ohms for 110-volt meters, about 5000 ohms for 220-volt meters, etc. The resistance of the armature proper is about 1200 ohms for all voltages, so that the resistance of the compensating field and fixed resistance, R, must be about 1300 ohms for 110-volt meters, 3800 ohms for 220-volt meters, etc.

Customary full-load speeds of the spindle are from 25 to 50 r.p.m.; with such low speeds and weak fields (no iron is used in the motor) the value of the c.e.m.f. generated in the armature is insignificant, the entire voltage impressed being used up as IR drop.

In small watt-hour meters the dials read directly in kilowatt-hours, while in larger sizes a "dial" constant, always clearly indicated. must be applied to the dial reading to get kilowatt-hours.[1]

109. Mercury Flotation Watt-hour Meter.—In this type of meter, advantage is taken of the principle of Barlow's wheel. If a pivoted disc is placed between the poles of a magnet, as in Fig. 124, and current made to pass from one side of the disc to the other, the disc will rotate. Study of the figure will readily disclose the forces causing the rotation.

The essentials of a c.c. watt-hour meter of the mercury flotation type are shown in Fig. 125. A copper disc, D, slotted radially to prevent the current from spreading out over the disc, and supported on a spindle, S, is enclosed in an appropriate chamber, C, partially filled with mercury. A float, F, of hard wood on top of the disc, D, serves to give the moving system just a little excess buoyancy, causing it to exert a slight upward pressure against the jewel bearing, B, at the top. A guide bearing at the bottom serves to keep the spindle in alignment. The moving system of a watt-hour meter of the mercury flotation type is shown in Fig. 126.

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[1] For information on the testing of watt-hour meters see Continuous-current Experiment XIV, Volume III.

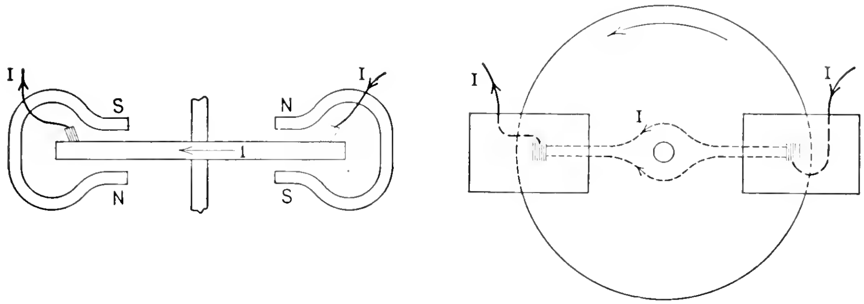


FIG. 124.—The principle involved in Barlow's wheel is the same as that employed in the mercury flotation type of watt-hour meter.

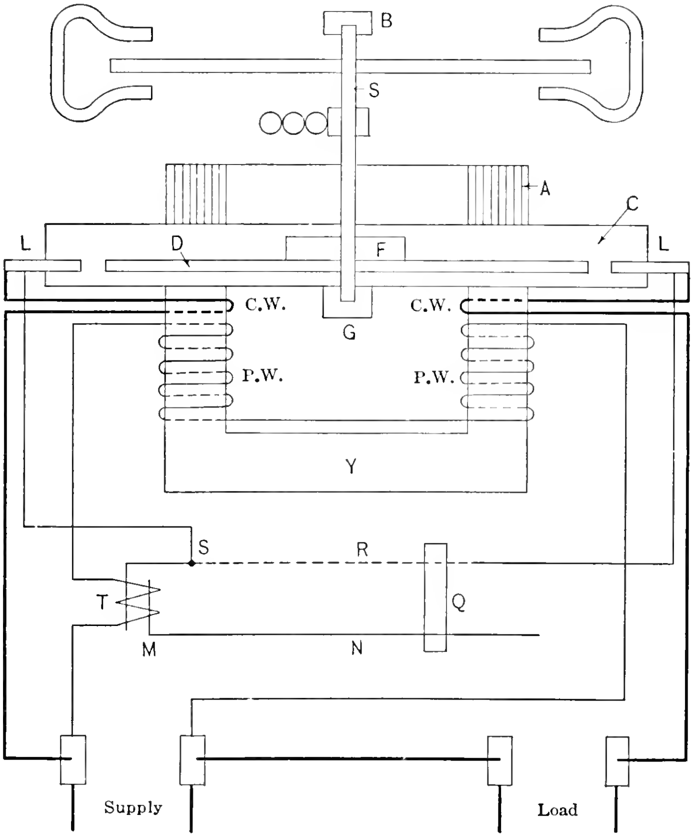


FIG. 125.—Arrangement of the essential parts of a mercury meter; the thermo-couple *T*, is used for the light load adjustment and the series turns *C.W.* serve to overcome the frictional effect of the mercury, which, of course, increases with speed.

Current is led in and out of the mercury chamber by means of the copper lugs *L* (Fig. 125), which are set 180° apart. The path of the current is from one lug, through the comparatively high-resistance mercury to the disc, across the low-resistance copper disc to the mercury and out from the other lug.

The magnetic circuit of the meter consists of the U-shaped piece, *Y*, built up of steel laminations and carrying the field windings, and the circular ring *A*, made up of steel ribbon.

The potential circuit of the meter is taken from the line terminals and, in most mercury flotation meters, has in series with its winding, *PW*, the heating element of a small thermo-couple, *T*. The load current is passed through one turn, *CW*, on one leg of the field magnet, through the mercury chamber and disc, and around another turn on the other leg of the magnet. The compounding action given by these two turns is desired to overcome the fluid friction of the mercury, which increases with the speed of the disc.

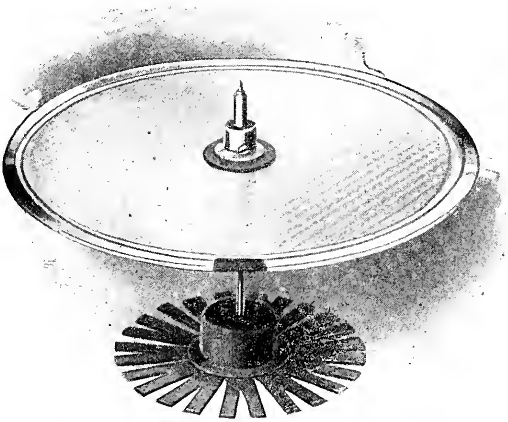


FIG. 126.—A view of the moving part of a mercury flotation type of meter; there are no windings or commutator to get out of order

The function of the thermo-couple is in connection with the light load adjustment of the meter. The heat produced by the heating element generates an e.m.f. across the two dissimilar metals of the couple and causes a current to flow, part of which passes from the point, *S*, over the resistance rod, *R*, and slider, *Q*, to the copper rod, *N*, returning to *M*. The other portion of the current flows from *S*, through the disc in the same direction as the load current, and returns to slider, *Q*. By adjusting the position of the slider, the proper amount of current can be sent through the disc to provide the necessary torque to overcome friction. When the slider is moved to the right, less of the resistance rod is in series with the circuit through the disc, and more in series with the local circuit; the starting torque is increased.

The damping device, consisting of an aluminum disc rotating between the poles of permanent magnets, is essentially the same as previously described.

Mercury watt-hour meters are generally operated with shunts, the full-load drop through the meter being quite low. Standard meters are built for 10 amperes, and external shunts are used for higher ranges. The resistance of the potential circuit is about the same as in Thomson meters, except that in meters below 130 volts the entire resistance is put into the field winding. Extra resistance is added in series for higher voltages.

A view showing the actual construction of a typical mercury flotation watt-hour meter is shown in Fig. 127.

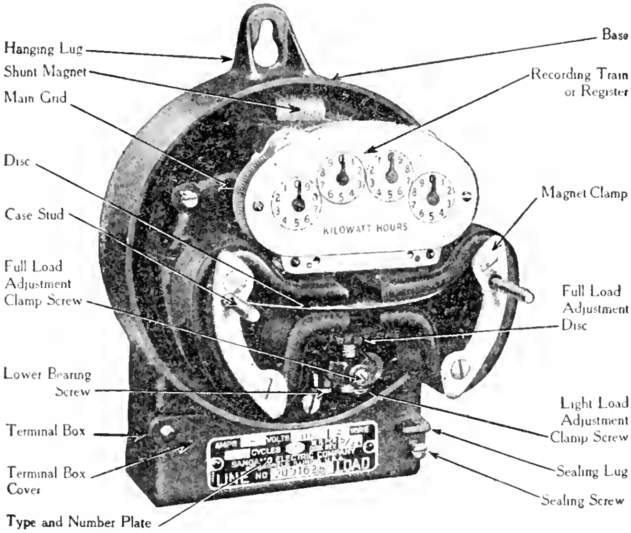


FIG. 127.—A Sangamo watt-hour meter, which is of the mercury flotation type.

110. Ampere-hour Meter.—The ampere-hour meter is used to measure quantity of electricity, and finds its application in the present wide use of storage batteries in central stations, electric vehicles, submarine vessels, lighting equipments, on steam railroad cars, etc.; in order that storage batteries may be operated to the best advantage, it is essential that a record be obtained of their input and output.

The watt-hour meter, being an energy meter, measures the product of voltage, current, and time, or voltage and quantity; in the ampere-hour meter, quantity, or the product of current and time, is measured. In order to make the torque of the motor element proportional to current only, a constant field strength is necessary, and is obtained by the use of permanent magnets. Any c.c. watt-hour meter may be transformed into an ampere-hour meter by replacing its electromagnets by permanent magnets. The mercury flotation type of meter, however, has proven itself more adaptable for use in ampere-hour meters and is most generally used.

The essentials of an ampere-hour meter of the mercury flotation type are shown in Fig. 128. It will be seen that the motor element is the same as that of the corresponding type of watt-hour meter, except that a permanent magnet, instead of an electromagnet, is used to provide the flux. Suitable pole shoes (shown hatched), guide the flux through the mercury chamber, *C*; the magnetic circuit is again completed by the circular ring, *A*, made up of steel ribbon.

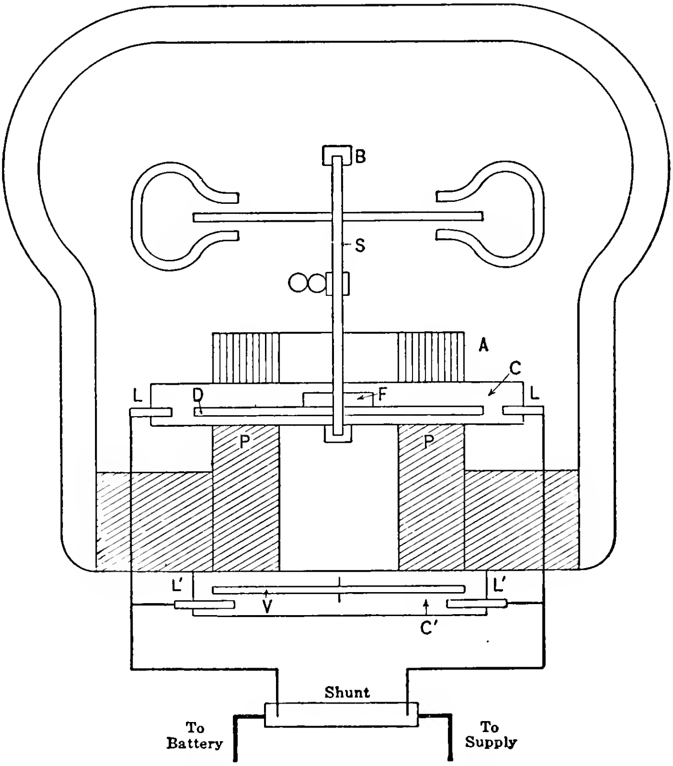


FIG. 128.—A mercury flotation type of meter using permanent magnet fields is suitable for an ampere-hour meter.

Where used with a storage battery, the ampere-hour meter runs in one direction when the battery is charging; on discharge of the battery the current naturally reverses, and the meter will run in the opposite direction. If a battery, completely discharged, were fully charged through the meter, this would register the entire ampere-hours put in and if the same battery is now fully discharged through the meter, the latter should show zero charge left in the battery at the end of discharge. Since only

60 to 90 per cent of the ampere-hours put into a battery can be recovered on discharge, on account of losses within the battery, the meter, in order to show full discharge, must run faster on discharge of the battery than on charge. This may be accomplished in several ways, one of the simplest devices for the purpose being a pivoted copper vane immersed in mercury, through which current is passed by suitable lugs, *L′*. The container, *C′*, is placed below the poles, *P*, of the permanent magnet (Fig. 128), so as to be in its stray field. The vane is electrically placed in parallel with the moving disc, *D*, so as to shunt off a portion of the current to the meter. When current flows through the vane, *V* (Fig. 129), it tends to rotate one way or the other until it comes up against suitable stops. The path of the current is then always from one lug, through a greater or lesser volume of mercury, to the vane, and out again in the same way from the other lug. The current lugs are so placed in relation to the vane positions that, when charging current passes through the meter, the vane moves nearest the lugs so that the current must pass only a small distance through mercury. With discharging current, the vane moves further from the lugs, forcing the current to pass through a longer mercury path. The resistance of the vane current path is thus greater with discharging current, so that a greater percentage of the entire meter current passes through the disc, *D*, and causes it to rotate faster than on charge, when more of the meter current is shunted off through the vane.

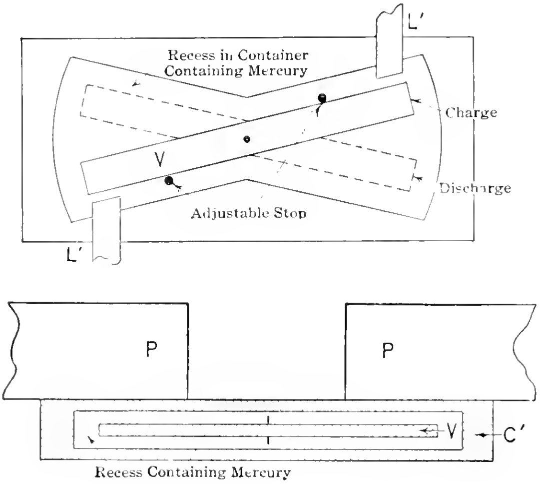
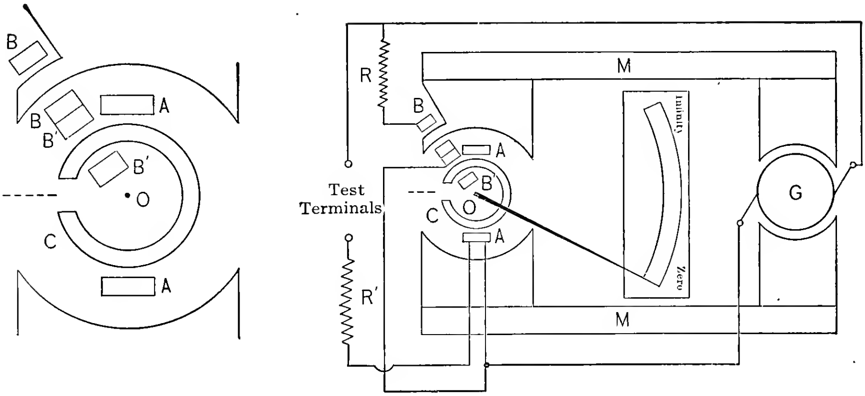


Fig. 129.—When an ampere-hour meter is used in connection with storage batteries an additional vane, movable in a mercury-filled chamber, is used to compensate for the difference in the ampere-hours used for charging and those available for discharge.

The requirements of the different types of service for which storage batteries are used vary widely, and ampere-hour meters are built with several other attachments, such as contacts which start a battery charging or discharging at predetermined values of charge, etc.; resets, which compensate for heavy rates of discharge or periodic overcharges, etc.; devices which compensate for light load accuracy, and loss of charge when a battery is not in use. A complete description of these devices is beyond the scope of this text, and the student is referred to the bulletins of the various manufacturers for further information.

111. The Megger.—The function of insulation in electrical machinery is, of course, to keep the current in the right path. To this end it must possess dielectric strength against breakdown under the static strain produced by voltage, and also resistance to the passage of leakage currents. If, by reason of moisture, temperature rise, dirt, etc., the resistance of the insulation is much decreased, the leakage currents will become appreciable. This increase will result in further rise of temperature and possibly charring, which further decreases the resistance, and, the effect being cumulative, a complete breakdown of the insulation may result.



(a) (b)

FIG. 130.—The construction and connection scheme of the megger; in sketch (a) the system of moving coils is shown more in detail.

It is therefore advisable that measurements of the resistance of the insulation of machines, cables and other apparatus, be taken periodically to determine its condition. Such measurements should precede a high-voltage test to determine the dielectric strength, inasmuch as trouble which can easily be located and remedied may cause breakdown if present while high voltage is applied.

An instrument much used for these periodic insulation tests is the *megger*, which gives direct readings of resistance. As diagrammatically shown in Fig. 130, it consists of two permanent bar-magnets, MM, between the poles of which, at one end, is the armature, G, of a small hand-driven generator, while at the other end is the moving system of the megger. The latter consists of three coils, A, B, and B', rigidly fixed together and capable of freely rotating about the axis, O. No restoring or controlling springs are used, so that the pointer may stand anywhere over the scale when the generator is not being driven. It will be seen that coil B slips over the pole tip of the upper magnet pole, and coils B' and A both slip over the C-shaped piece of soft iron.

Coil B is so wound that, with passage of current, its field is opposite to that of the magnet pole; a force is then developed tending to force the coil off the pole tip. Coil B' is similarly wound and, with passage of current, it tends to set itself in a position where minimum flux from the permanent magnets passes through it, or directly opposite the gap in the C-shaped piece of iron. Coils B and B', therefore, when energized, move the pointer counter-clockwise. Coil A is so wound that with passage of current it tends to move in a clockwise direction.

There are two circuits in the megger, the potential circuit consisting of the coils B and B', and the resistance, R, in series, and the current circuit through the insulation to be tested, across the test terminals, the resistance R' and coil A.

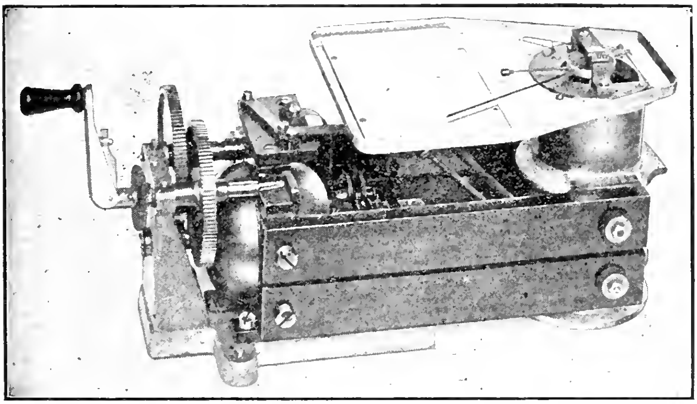


FIG. 131.—This cut shows the actual arrangement of the parts of the megger.

If the test terminals are open and the generator driven, current flows through coils B and B' and they set themselves as described above, opposite the gap in the C-shaped piece of iron along the dotted line. This corresponds to infinite resistance. If a sample of insulation of suitably high resistance is connected across the test terminals, current from the generator will flow over both circuits; that through coil A and the test insulation causes the moving element to move clockwise, while the current through coils B and B' produces a torque in the opposite direction. As the moving element moves clockwise, the coils B and B', moving into a stronger field, offer an increasingly stronger restraining torque until the torque due to coil A is balanced and the needle comes to rest over a point on the calibrated scale which correctly indicates the value of the resistance of the insulation being tested. Short circuit of the test terminals, corresponding to zero resistance, causes a current through coil A great enough to overpower the torque of coils B and B', and the moving element comes to rest as in Fig. 130.

Since both circuits are supplied by the same generator, any voltage change in the latter affects both circuits in the same proportion; the position of the pointer is therefore independent of the voltage or speed of the generator. However, for testing circuits or apparatus containing electrostatic capacity such as cables, machines with mica insulation, etc., it is advisable that the generator speed be constant; variable speed, causing variable voltage, results in charging and discharging currents, so that the readings vary. For this reason, in most meggers, the armature and crank shafts are connected through a special friction clutch; for crank speeds above a definite value the clutch slips, ensuring constant armature speed.

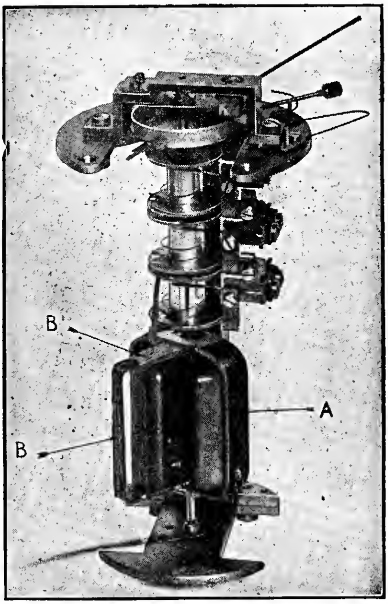


FIG. 132.—The arrangement of the coil system of the megger.

A view showing the actual construction of a typical megger is shown in Fig. 131, and the construction of the coil system is shown in Fig. 132.

112. The Oscillograph.—This is an instrument which permits the observation and recording of the forms of all kinds of currents and voltages; it has probably done more in advancing the exact study of electrical phenomena than any other piece of apparatus. Photographic records of wave forms, called *oscillograms,* will be freely used in this text to substantiate theoretical predictions.

The oscillograph, as ordinarily made, consists essentially of a two-strip suspension, mounted in a magnetic field, about as indicated in Fig. 133. The two wires, A–B, really form two sides of a single-turn loop,

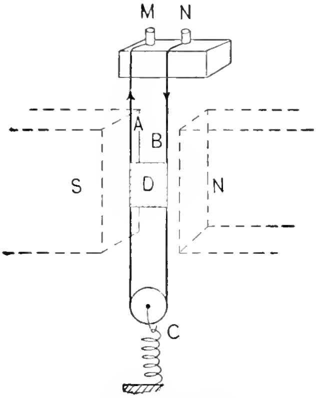


Fig. 133.—Elementary sketch of the essential part of the Dudell oscillograph. The mirror, D, oscillates if alternating current is sent through the wire suspension A–B.

Fig. 134.—Actual vibrator from a General Electric oscillograph. The two ends of the one-turn coil are connected at J–J_1. That part of the coil between the two ivory bridges is the real vibrating member, the mirror being fastened to the two wires midway between the bridges. In this picture one side of the coil has not yet been placed in its slots in the bridges and the mirror is not yet cemented in place.

its upper ends being fastened to terminals M–N, and its lower end being taken around a small pulley C, which holds the loop tight by being pulled downward by a suitable spring. Across the loop, at its middle point, is cemented a small mirror D. The plane of the loop is parallel to the magnetic field of the pole pieces, N–S.

Current, the form of which is to be determined, goes down one wire and up the other; as the wires are very light and flexible, the force set up moves one of them forward and the other backwards, thus producing a rotation of mirror D. Provided the changes in current are not too rapid, the position of the mirror, and hence that of a beam of light reflected from the mirror, will truthfully follow the variations of current through the strips. Alternating currents (currents which periodically reverse their direction of flow) may be accurately photographed if the frequency of reversal is not more than about a thousand per second.

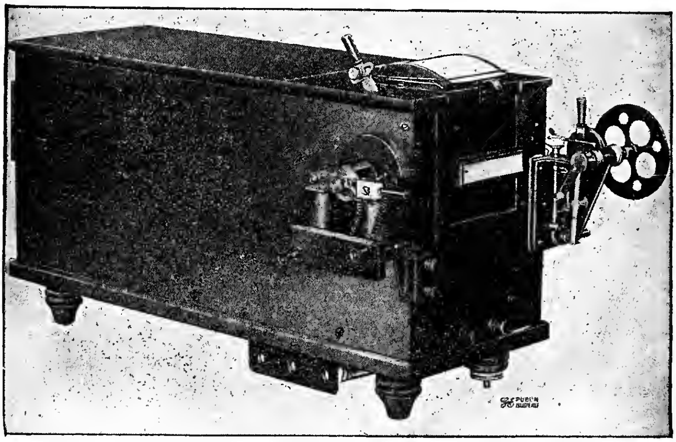


FIG. 135.—Showing the oscillograph complete, as made by the G. E. Co. This model contains three vibrator systems, so that three records can be taken at the same time. The revolving film holder fits on the front end, where the ground glass screen is shown in this cut.

To permit such rapid movements, the mirror is necessarily small; in the ordinary form of oscillograph, it measures one-hundredth inch wide and six-hundredths inch high. In Fig. 134, is shown a photograph of the vibrator of a General Electric oscillograph, from which an idea of the size of its parts may be gained. Three of these vibrators are generally built into one oscillograph; the mirrors can all be oriented so that their light beams come on the same film, and thus simultaneous records of three voltages or currents may be obtained. In Fig. 135, is shown a general view of an oscillograph.

113. Recording Meters.—Meters intended to keep a record of instantaneous values are called recording meters. Recording ammeters, voltmeters and wattmeters are much used in c.c. station work. The moving element of a recording meter carries a pen which draws a curve on a paper strip which is fed under the pen by suitable clock work. A very fine type of recording meter is shown in Fig. 136. The moving element is a kind of magnetic balance which carries the glass pen back and forth over the paper strip as the current through the instrument varies.

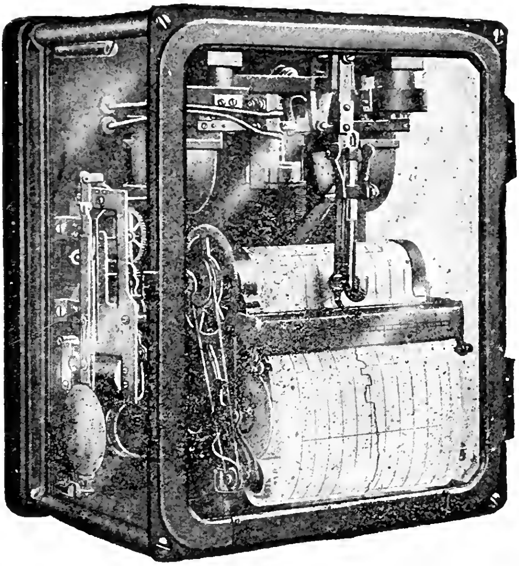


FIG. 136.—A high-grade recording ammeter. The paper record winds from one drum on to another, being long enough for many days' use.

The records from such meters are evidently valuable in showing the station manager how the load on his station varies, when peak load occurs and for how long, etc. The records of a recording voltmeter show how faithfully the voltage of a station is maintained at its rated value. In case of litigation involving the operation of the station, such records are, of course, invaluable.

CHAPTER VII

PARTS OF A DYNAMO-ELECTRIC MACHINE—FUNCTION MATERIAL, CONSTRUCTION

114. A dynamo-electric machine is a machine for converting mechanical energy into electrical energy or vice versa. It may be either a continuous-current machine or an alternating-current machine. In this text we shall deal only with c.c. (abbreviation for continuous current) machines. A general classification of the parts of any c.c. dynamo-electric machine, together with their functions, may be made as follows:

1. *Field Frame;* consists of the yoke, poles (and pole shoes if there are any); forms part of the magnetic circuit, carries the field coils and often forms the support for the bearings.

2. *Armature Core;* completes the magnetic circuit and carries the conductors which serve to generate the e.m.f. (if the machine is a generator) or to carry the current by which torque is developed (if the machine is a motor).

3. *Commutator;* rectifies the alternating voltages induced in the armature conductors and, together with the brushes, forms the electrical contact between the revolving conductors and the external circuit.

4. *Brushes and Brush Rigging;* the stationary portion for making a rubbing contact with the moving surface of the commutator.

5. *Field Windings;* set up the flux in which the armature conductors move.

6. *Armature Winding;* seat of the generated voltage in a generator or torque in a motor.

Each of these will be taken up separately, and reasons will be given for the different forms seen on various commercial machines, and for the superiority of certain materials as compared to others.

115. Field Frame.—Figure 137 represents the magnetic circuit of a simple *bipolar* machine, and Fig. 138 shows the magnetic circuit of a *multipolar* machine. By the term field frame we mean that part of the magnetic circuit made up of the yoke and poles (and pole shoes if there are any). In continuous-current machines, that part of the magnetic circuit constituting the field frame is stationary and is seen to be the part that carries the field coils through which current is passed to produce the magnetic field. Figure 139 shows a large multipolar machine; the field frame is split horizontally and the upper part fitted with an eye-bolt for lifting, to facilitate removal of the armature.

116. Yoke.—According to present practice, the yoke is either a casting or, as in the case of many small machines (less than 5 h.p.), is made of a steel plate bent into the form of a cylinder, with the butting edges welded together. Cast steel is preferred to cast iron because of its higher permeability, which allows the use of higher flux densities than are practicable with cast iron; because of this fact a field frame of steel, to carry a certain magnetic flux, is much lighter than would be the case if iron were used.

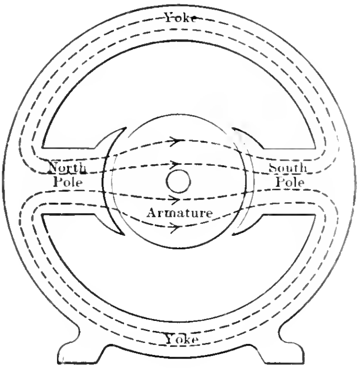


FIG. 137.

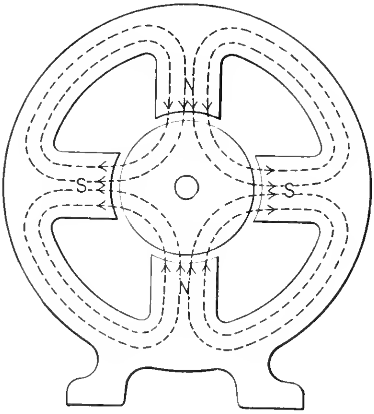


FIG. 138.

FIG. 137.—The general form of the magnetic circuit of a modern bipolar machine; the yoke is double, each part carrying half as much flux as the pole pieces and the armature.

FIG. 138.—For a multipolar machine the ring-shaped yoke is used as in the bipolar; the length of magnetic path in a multipolar machine is less than it would be in a bipolar machine of the same capacity.

117. Number of Poles.—The number of poles of a machine depends upon its capacity and the speed at which it is designed to operate. Generator speeds are largely determined by the type of prime mover to be used, and, in the case of motors, the speed is determined primarily by the speed requirements of the load which the motor is to serve.

Machines of 5 h.p. capacity and less generally have a bipolar frame, i.e., have only two-pole pieces (there must always be at least two); for generators or motors of ordinary speeds, above 10 h.p. in capacity, the multipolar form of field frame is generally used. In large generators, driven by slow-speed reciprocating engines, the number of poles may be as high as 24 or more. With the introduction of the high-speed turbine in place of the low-speed reciprocating engine, these large multipolar machines are being replaced by direct-connected machines of 2, 4 and 6 poles. More recent practice couples the turbine to the generator through a reducing gear, thereby permitting a higher turbine speed and more

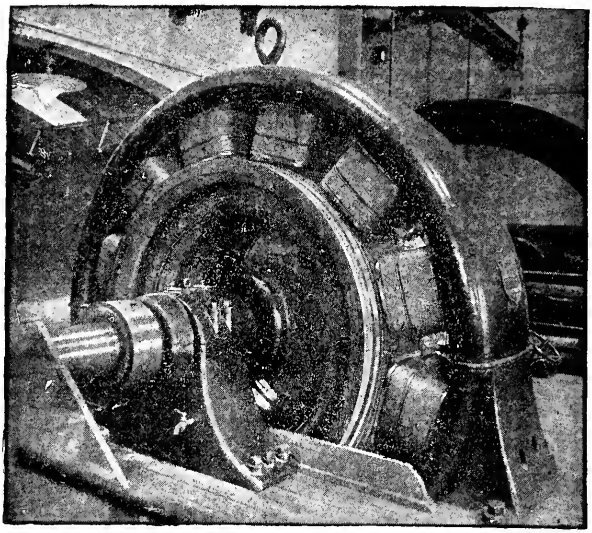


Fig. 139.—A modern multipolar generator of several hundred kilowatts capacity, direct connected to a steam engine; as the generator is revolving the armature spider spokes are not visible.

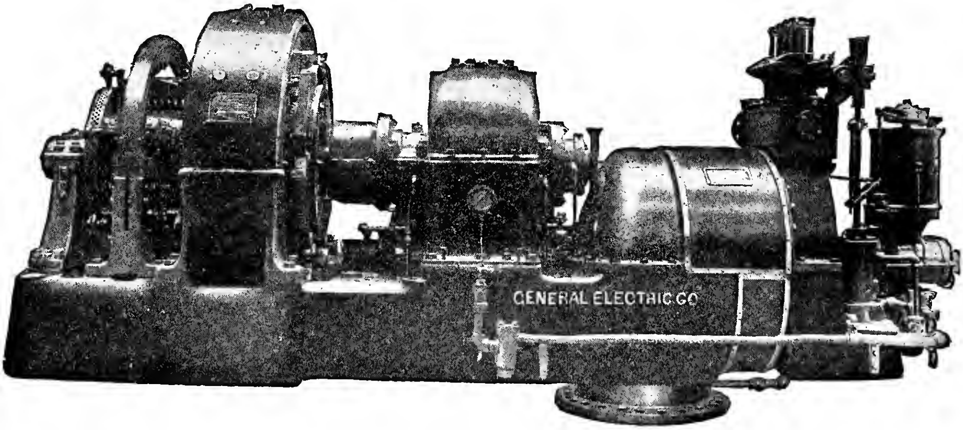


Fig. 140.—The modern tendency is to drive all generators by turbine, instead of by reciprocating engine; by the use of reduction gears best speeds are obtained for both generator and turbine. In this cut the turbine is at the right, the gear case is in the center, and the generator at the left.

economical use of the steam. Figure 140 represents a modern turbine-driven continuous-current generator in which a reducing gear is used.

118. Material for Poles.—The poles themselves are not cast as a part of the yoke except in the case of small, cheap machines. In discussing the design of field coils we shall show that it is advantageous to keep the cross-section of the pole as small as possible, and this makes advisable the use of laminated iron, because of its relatively high permeability. Most field poles are, therefore, built up of laminations, each about $\frac{1}{16}$ of an inch thick, and are bolted to the yoke. Figure 141 shows this construction.

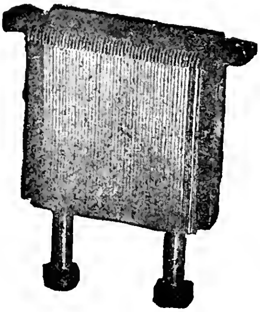


Fig. 141.—This shows the laminated construction generally used for field poles; the laminations, perhaps $\frac{1}{16}$ inch thick, are held together by several rivets which go through the laminations and are upset in the thick end plates.

119. Leakage of Magnetic Flux.—The function of the field coils and field frame is to produce a strong magnetic field in which the armature conductors may turn. Now all of the magnetic lines generated by the field coils will not pass through the armature core; part of them will go by various so-called *leakage paths* and this is called the *leakage flux*. In Fig 142 is shown the possible distribution of the leakage flux between two poles of a multipolar generator. The normal path of the flux is shown by the full lines, and the dotted lines show some of the leakage paths. In a well-designed machine, the proportion of leakage flux to total flux is kept low; perhaps it may be 20 per cent on a small machine.

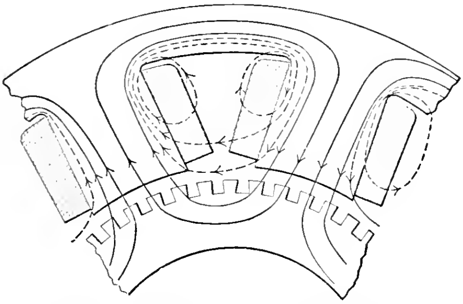


Fig. 142.—Not all of the flux set up by the field coils goes through the armature, where it is supposed to go. Flux which is distributed as indicated by the dashed lines is useless; it is called leakage flux.

120. Advantage of Multipolar Frame.—The reason for using a multipolar, instead of a bipolar, field frame on a large machine, may be seen from an examination of the length of the magnetic path in two machines of the same capacity, one bipolar and the other multipolar. In the bipolar machine a much longer path exists than in the multipolar, and the longer the magnetic path the more leakage there will be; also a larger and more expensive field coil is required for the machine having the long magnetic path. Difficulties from armature reaction (explained in Chapter VIII) occur to a greater extent in the bipolar machine than in the multipolar machine of the same capacity. There are other advantages of the multipolar frame which cannot be discussed here.

121. Pole Shoes.—Nearly all machines have the pole pieces fitted with pole shoes, the functions of which are to increase the effective area of the air gap and to hold the field coils in place. The pole shoes are almost always laminated, in order to reduce the loss and heating caused by eddy currents set up in the pole faces by the moving armature teeth. This effect will be further considered later on. Even when solid poles are used, the pole shoes themselves are usually separately built up of laminations and bolted to the ends of the poles next to the armature. When laminated poles are used, the pole shoes are generally made a part of the pole itself, the laminations of which the pole is built up being so shaped that when the pole is assembled, the lugs on the laminations form a pole shoe. This, the more general construction, is shown in Fig. 143, in which an assembled pole is shown.

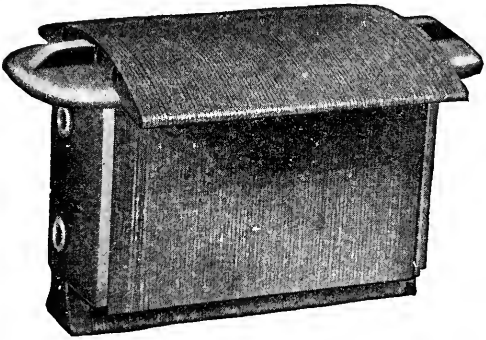


FIG. 143.—To form the pole shoe, the laminations are so formed that two projecting lugs are provided at the pole face; this pole piece happens to be for the revolving field structure of an alternator, hence the convex curvature of the pole face.

The two projections on such a pole are called the *pole tips*; that one which a point on the armature first comes under as the armature revolves is called the *leading pole tip*, and the other is called the *trailing pole tip*. The term pole tip is used even when the pole has no shoe; the edges of the pole itself constitute the pole tips in this case.

122. Air Gap.—One of the functions of the pole shoe is to increase the effective area of the air gap. The air gap (that part of the magnetic circuit between the pole face and armature core) constitutes the greater part of the reluctance of the magnetic path; in actual machines the air-gap reluctance constitutes about four-fifths of the total reluctance, and therefore four-fifths of the magnetomotive force of the field coil is used up in forcing the flux through the air gap. By lessening the reluctance of the gap the size of the field coil may be much reduced.

Now the reluctance of the air gap $=\frac{l}{\mu A}$, where $l=$ the length of the air gap (parallel to direction of magnetic lines) and A = the cross-sectional area of the air gap. For air, μ, the permeability, is equal to one.

The minimum length of the air gap being fixed from consideration of mechanical clearance, the only way to reduce the air-gap reluctance is to increase its area and this is what the pole shoe does. There are other factors affecting the length of the air gap besides that one mentioned here; as will be explained later the armature exerts a reaction on the magnetic field and the effect of this armature reaction is much exaggerated if the air gap is not of sufficient length. This armature reaction effect is really the factor controlling the length of air gap. On some high-speed alternators, for example, the air-gap length may be an inch or more, and evidently mechanical considerations would not necessitate an air gap of such length.

123. Commutating Poles.—Besides the main poles, most modern c.c. machines have another set of poles which are very narrow compared to the main poles and are placed midway between the main poles; these small poles are to help commutation and are called *commutating poles* (or sometimes *interpoles*). A machine equipped with commutating poles is shown in Fig. 144; the extra poles are seen to be very narrow in comparison with the main poles.

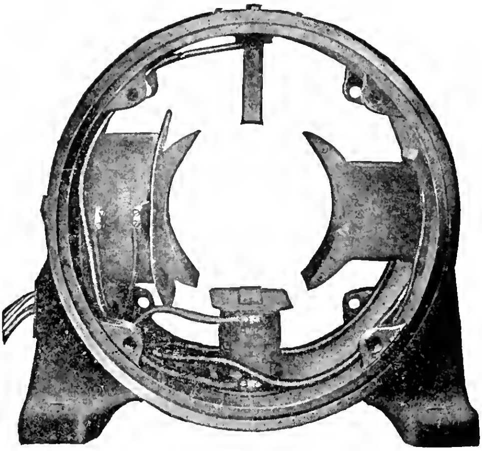


FIG. 144—Nearly all modern machines have in addition to the main poles a set of narrow poles situated midway between the main poles; they are to facilitate commutation and are called commutating poles, or interpoles.

124. Armature Core.—As previously stated, the magnetic circuit of any dynamo-electric machine may be considered as made up of two parts, one of which moves with respect to the other. The field frame, carrying the field windings, has already been discussed; the other part of the magnetic circuit is called the *armature core*. On this part of the magnetic circuit are placed the conductors which serve to generate the e.m.f. (if the machine is a generator) or to carry the current by which torque is developed (if the machine is a motor).

The armature core is always made of laminated iron, that is, the core is built up by placing on the shaft a number of thin discs of iron and clamping them tightly together. The reason for making an armature core in this way, evidently much more difficult and expensive than if a solid piece of cast steel were used, will now be considered.

125. Experiment to Show the Advantage of Laminating the Core.—Two armature cores, of exactly the same dimensions, were made for an experimental generator. One of the cores was of solid iron and the other was of thin iron plates clamped together; thin paper was put between every other plate. Then each armature core was in turn mounted in the field of the experimental generator and the power required to rotate these bare armature cores at 1200 r.p.m., with a certain strength of magnetic field, was measured. When the laminated core was tested it was found that the required power (excluding friction) was 9 watts; the core was left running for some time, but it remained cool. The solid core was then substituted for the laminated one and the power consumption was 350 watts; after running for a few minutes it became so hot that it had to be stopped, for fear of damaging the bearings. From this simple test we conclude that a solid armature core cannot be used in dynamo-electric machinery because, first, it requires so much power merely to rotate the armature core in the magnetic field, and second, the heat generated in the rotating core is so great that any windings on the armature core would be burned.

126. Losses in Armature Core. Hysteresis.—The heat generated in any piece of iron, rotating in a stationary magnetic field, is due to two effects, namely, *hysteresis* and *eddy currents.* It is evident that as the armature core rotates it is magnetized first in one direction and then in the opposite direction, as a point on the core moves from under a *N* pole to a *S* pole. In Chapter III the question of hysteresis loss in a piece of iron going through magnetic reversals was explained. There is, then, this hysteresis loss in an armature core, and this loss may be kept small by using, for the core, iron that has a very narrow hysteresis loop, such as a specially treated alloy steel. After the sheet steel has been punched out in proper shapes for the core construction, these laminations are heated to a dull-red heat and allowed to cool slowly. This process, called *annealing,* results in a steel having a very narrow hysteresis loop.

127. Loss Due to Eddy Currents.—Eddy currents (local currents in the material of the core itself) are produced in the rotating armature core because, as it moves through the magnetic field, an e.m.f. is generated in it (*any conductor moving through a magnetic field so as to cut lines of force has an e.m.f. induced in it*); the direction of this e.m.f. is parallel to the shaft i.e., lengthwise of the armature core. Under a *N* pole it is in one direction and under a *S* pole in the opposite direction. As the solid iron core may be considered as one large conductor it is evident that these e.m.fs. in the armature core will cause currents to circulate in the core. The direction of these currents is shown by Fig. 145, which represents a section of the core, taken parallel to the shaft. The direction of the e.m.f.

induced in the core is shown by the solid arrows, and the path of the eddy current is shown by the path formed by these arrows and the dotted lines.

128. Reduction of Eddy-current Loss by Laminating the Core.—Now, if the core is made of two parts, insulated from one another as in Fig. 146, the eddy currents will have to flow in two separate paths, as shown. The resistance of each of these paths is about twice as great as the resistance of the one path in the solid core, because, although the length of the eddy current path in Fig. 146 is about the same as that of Fig. 145 (more nearly true as thinner laminations are considered) the cross-sectional area of the path is about one-half as great as that of the solid core.

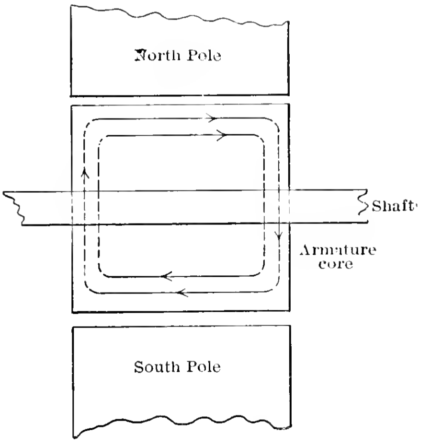


Fig. 145.

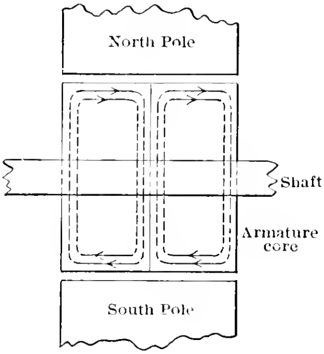


Fig. 146.

Fig. 145.—If the armature core was a solid cylindrical piece of iron, eddy currents would flow in the revolving core as indicated here.

Fig. 146.—By building the core of a lot of discs the heating produced by the eddy currents is much reduced; the loss due to eddy currents varies as the square of the thickness of the laminations.

The e.m.f. of each of the paths in Fig. 146 is one-half as great as for the path of Fig. 145, hence, each eddy current of Fig. 146 will be one-quarter as large as that of Fig. 145 (the voltage being halved and the resistance doubled). The I^2R loss of *each* path of Fig. 146 will therefore be one-eighth as great as that for Fig. 145, but as there are two eddy currents to consider in Fig. 146 it is evident that the eddy current loss of Fig. 146 is one-quarter as much as for Fig. 145. We thus arrive at the conclusion that *the eddy current loss in an armature core varies as the square of the thickness of the lamination.*

In commercial machines the laminations are generally 0.014 in. thick. The laminations may be insulated from one another on large machines by a coat of insulating paint on each lamination; on small machines the oxide coating, formed on the iron sheet while it is being annealed, is relied upon for insulation. Sometimes thin paper is put between every few laminations to increase the insulation.

129. Form of Laminations.—In small armatures the laminations are solid discs, merely having a hole in the center for the shaft. In multipolar machines it is unnecessary to use solid discs, because in a large core made up of solid laminations there would be more iron than is necessary for the magnetic flux. The paths of the flux in a multipolar machine are shown in Fig. 147. It is seen that the iron inside the dotted circle is useless as far as the magnetic circuit is concerned.

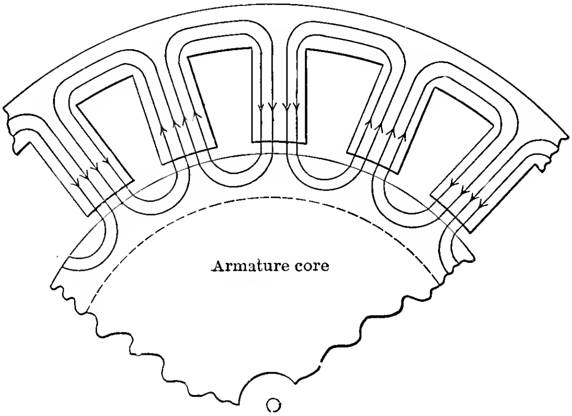


FIG. 147.—For large machines it would be wasteful to use solid discs for building up the armature core; the interior of the core would carry practically no flux. For this reason the discs used are ring shaped, thus making the core a hollow cylinder.

The discs for large machines, therefore, are generally made ring-shaped; the iron which is punched from the inside of the disc may be used for the armature of a smaller machine. If the armature is very large (say more than 2 feet in diameter) the rings will not be in one piece; each lamination will be made up of several pieces. In building up an armature core of such laminations, they are so assembled that the joints in one lamination do not come opposite those in the adjacent one.

130. Armature Spider.—The ring-shaped armature core must be fastened to the shaft in some way, and for this purpose the armature spider is used. The spider generally consists of a cast-iron hub, through which the shaft is fitted, and from which radiates a set of spokes; on the ends of the spokes are lugs which are fastened by dovetailing keys to the inside of the armature core. The armature punchings generally have dovetailed slots punched on their inside edge, in such a fashion that when the core is assembled they line up with one another and form a set of dovetailed slots in the inside surface of the armature core, into which the keys in the spider fit.

In assembling such an armature, the spider, fitted with keys on the lugs, is laid horizontally on the shop floor (see Fig. 155). Then the laminations, one at a time, are slipped over these keys all the way around; when a complete ring of laminations have been placed, the next layer of laminations is put on in a similar manner, but so placed that the two rings "break joints," i.e., the joints between plates in the adjacent layers do not come opposite one another.

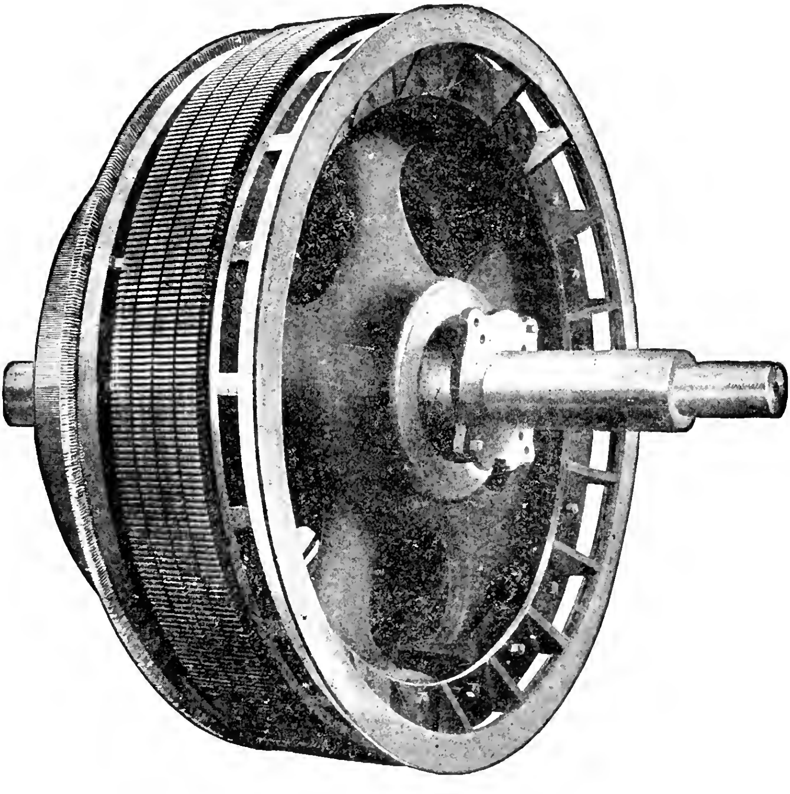


Fig. 148.—An armature core, completely assembled on its spider and shaft.

An armature core, assembled on its spider and fitted with its shaft, is shown in Fig. 148. This core is used in the construction of a large multipolar continuous-current machine.

131. Smooth and Slotted Cores.—In the early types of dynamo-electric machines, the periphery (outside surface) of the armature core was *smooth*, but in all modern machines the armature core is *toothed* (or *slotted*). In such a core there are a series of slots in the outer surface, running parallel to the direction of the shaft. These slots serve to hold the armature winding safely in place and also to reduce the effective length of the air gap of the machine.

By reference to Fig. 149 it is seen that in the smooth-core armature the minimum length of air gap is fixed by the distance required for mechanical clearance between the armature and the pole face plus the depth of the winding. In the toothed armature the minimum length of the air gap is determined by the mechanical clearance distance only; as the armature teeth are good conductors for the magnetic lines, the distance that most of the magnetic lines have to go through air is that between the outside of the teeth and the inside of the pole face only. The effective length of the air gap, however, is somewhat greater than the minimum length; it depends upon the depth and width of slots, but in any case is much less than it would be for a smooth-core armature.

FIG. 149.—The early machines had smooth cores, the winding being placed on the periphery and held in place by binding wires; modern machines always use slotted cores, the windings being imbedded in the slots.

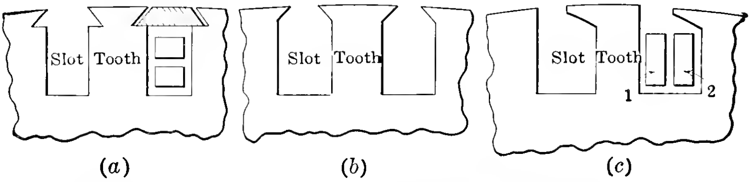


FIG. 150.—Various forms of teeth and slots; that shown in (*a*) is most generally used.

132. Shape of Slots.—The shape of the slot is generally determined by various factors in the design, and also, to some extent, by the kind of winding with which the armature is to be fitted. In Fig. 150 are shown representative forms of slots, that in (*a*) being known as an *open slot* with parallel walls, and those in (*b*) and (*c*) as *semi-closed slots.* Practically all but very small machines have open slots, the particular advantage of this type being that formed coils (previously bent up and insulated) can easily be inserted; they are held securely in the slot by a wooden wedge driven into place, as shown in the second slot of (*a*). (See also Fig. 200.) With open slots there are usually two coil sides per slot in the winding, the coil sides being placed in the slot one above the other, as indicated in (*a*). Several partially wound armatures with open slots are shown in Fig. 151; the method of placing the coils is apparent.

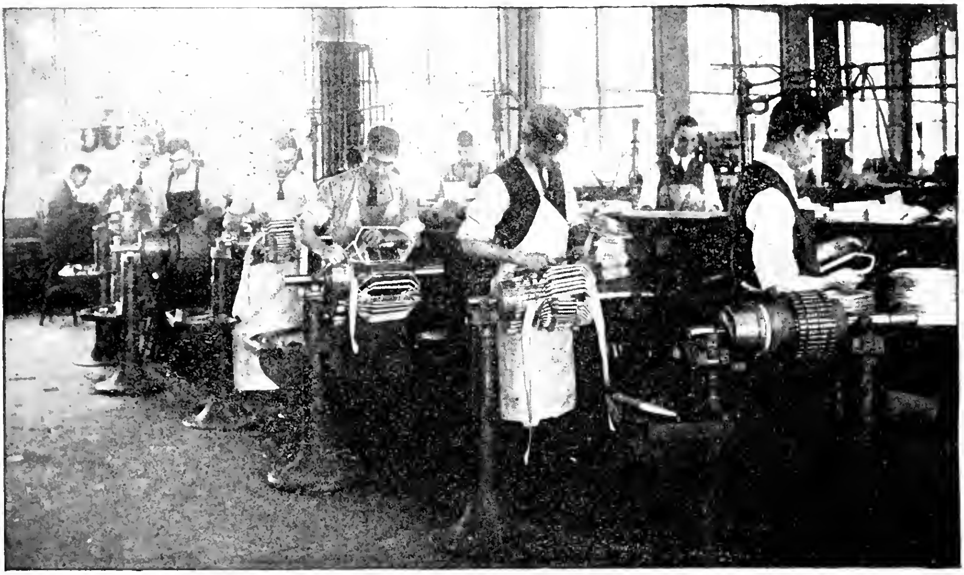


FIG. 151.—With open slots it is comparatively easy to wind the armatures; the coils, completely formed, can be pushed into the slots as they are being done on the armatures in this picture.

The semi-closed slot has the advantage of affording better protection and support for the coils than does the open slot; also, as may be seen by reference to Fig. 152, the effective area of the air gap is much larger when a semi-closed slot is used. This decreases the magnetic reluctance of the machine, so that smaller field coils may be used. When open slots are used, the effective area of the air gap (i.e., the area of that part of the air gap through which the magnetic field goes) is not much more than the area of the teeth situated under a pole face, as seen by Fig. 152*a*. When the semi-closed slot is used, the flux utilizes practically all of the area of the air gap; the reluctance of the gap in (*a*) Fig. 152 is much greater than that of (*b*).

It is evident that a formed coil could not be forced into slot (*b*) of Fig. 150 very easily. The coil may be fitted into such a slot by having it only partially formed; that part of the coil which is to be inserted into the slot is left untaped, so that the wires forming the coil can be separated and pushed into the slot one by one. This is generally done in the case of small, low-voltage machines, where the individual conductors of the coil are wire of small size and therefore flexible. It could not be done when the armature conductors are large and stiff. The insulation of such a coil cannot be as good as that of formed coils used with open slots; this will be evident when formed coils are considered.

An armature having slots of the shape shown in (*c*) Fig. 150, may be wound with a completely formed coil if there are to be two coil sides per slot in the winding. The coils lie in the slot side by side, as indicated. The coil marked (2) is first pushed in through the narrow opening in the slot and then pushed to the right to make way for coil (1) which is then put in place. The process of winding an armature having slots of this form is, however, more difficult than that of winding one with open slots.

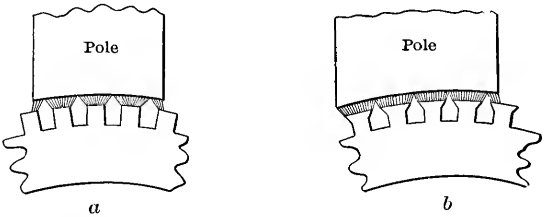


FIG. 152.—A core with partly closed slots gives a larger useful area to the air gap, as shown by the flux distribution in these two diagrams.

133. Pole-face Losses Produced by Slotted Cores.—With the tufting of the flux opposite the teeth, as shown in Fig. 152*a*, any element in the pole face is subjected to considerable variation in its flux density, as the tufts of flux move across the pole face. Accordingly, voltages will be induced in the pole faces which in turn set up eddy currents, as represented in Fig. 153*a*, by the dotted lines. Considering such an element of a supposedly solid pole face, as indicated by the dotted lines, it is evident that the amount of flux through this area varies periodically as the teeth and slots alternately go by. This increase and decrease in flux will produce alternating currents having a frequency equal to the number of teeth passing the pole face per second. By the use of laminated pole faces, these eddy currents are confined to the narrow paths available in each lamination, as suggested by Fig. 153*b*.

In addition to the heat generated by the eddy currents in the pole faces, there is also a small amount produced by hysteresis, this being caused by the periodically changing flux density.

134. Ventilation.—The question of *ventilation* of electric machinery will be taken up here because there are some features of construction of the armature core which are incorporated in the design with the idea of keeping the armature well ventilated.

One of the factors determining the capacity, or rating, of any dynamo-electric machine is the safe permissible temperature rise. Suppose we have a generator which the manufacturers have given a rating of 50 kw. and, when the machine is tested, it is found that the armature becomes so hot, when carrying a load of 50 kw., that there is likelihood of the insulation on the armature becoming damaged. It is evident that 50 kw. is more load than the machine can safely carry continuously, and we say the manufacturer has *over-rated* the generator. As we know the heat generated in the armature windings will be less if the load is less, we now test it at 40 kw. load and, after running at this load for a sufficient length of time to have reached constant temperature, the armature is fairly cool—say not more than 30° C. above room temperature.

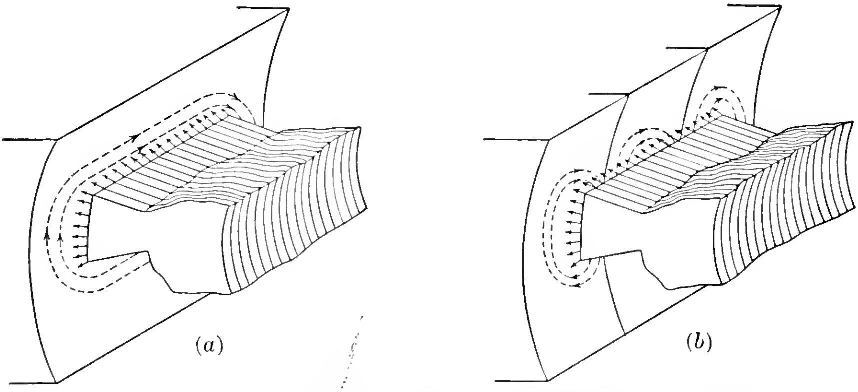


Fig. 153.—If the pole face were solid it would have large eddy currents flowing in it, in the paths shown by the dashed lines in (*a*); by laminating the pole face the eddy current paths are broken up and the heating they produce is much diminished, just as it is in the armature core. Actually the poles are of thin laminations instead of the few thick ones shown (for the sake of simplicity) in (*b*).

It has been determined by the standardization committee of the American Institute of Electrical Engineers that for ordinary machines the safe maximum temperature allowable in the armature is from 90° C. to 125° C., depending upon the kind of insulation used. As the load of 40 kw. causes the armature temperature to reach only 70° C., the machine can evidently carry more load with safety—and as 50 kw. load overheated it, the safe capacity must be somewhere between 40 kw. and 50 kw.

One way of keeping down the temperature rise of a machine is by continually forcing cool air through it, thus carrying off the heat; or we may say that the temperature of the machine may be kept down by properly ventilating it. Heat is generated in the armature core by the hysteresis and eddy currents and it is necessary to carry off this heat to keep down the core temperature. For this purpose ventilating ducts are built into the armature core. As the laminations are being assembled, spacers are introduced about every 3 inches of core. These spacers are placed radially, so that air passages, about $\frac{1}{4}$ inch wide, are formed, through which air can circulate from the inside periphery of the core to the outside. As the armature revolves, cool air is drawn in through the ends of the core by the spider and is thrown by centrifugal force through these ventilating ducts out against the pole faces, etc. Figure 154 shows how these ducts are placed in an armature core and the paths taken by the air as it passes through them; Fig. 155 shows a partially assembled core, in which the air ducts may be seen and on the top of the core may be seen one of the spacing laminations which has been specially formed with radial ridges in it about $\frac{1}{4}$ inch high. These ridges serve to properly space the laminations and so to form the air ducts as the core is built up.

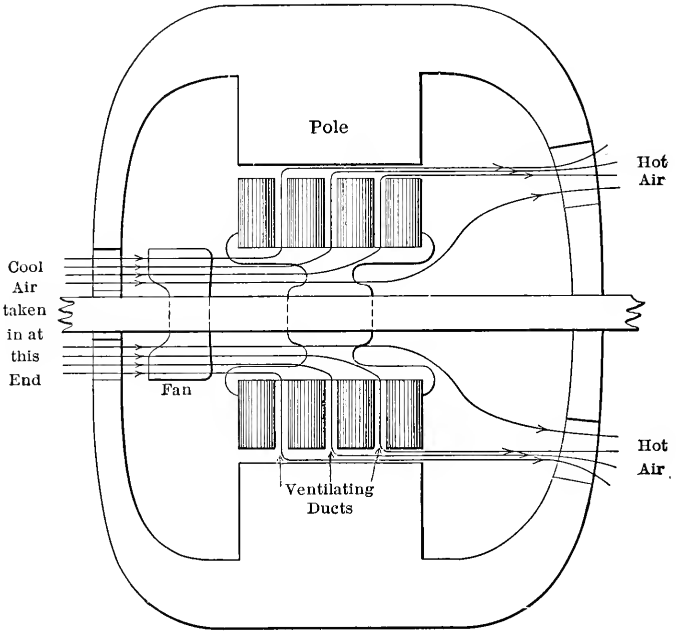


FIG. 154.—Except in the smallest sizes, armature cores are so assembled that air ducts are formed; air circulates through them as shown here, keeping the core much cooler than it would otherwise be.

135. Commutator—Function.—The e.m.f. generated by a coil of wire as it rotates in a magnetic field is an alternating one, i.e., the e.m.f. is first in one direction and then in the opposite direction, a complete reversal taking place as a coil-side moves by one pair of poles. Now, even if an armature winding is made up of a great many coils, connected together in any fashion whatever, the e.m.f. generated in this winding as it rotates in the magnetic field is an alternating one, and if the ends of the winding are connected to insulated rings on the shaft, and brushes bearing on the rings connected to the external circuit, the e.m.f. which such an armature winding impresses on the external circuit is an alternating one. The current which would flow in such an external circuit would be an *alternating current;* the current would flow through the circuit in one direction for a short time, would stop flowing altogether, and would then reverse and flow in the opposite direction, etc. Such an alternating current is useful in many classes of service; but for many other classes of service, such as electroplating, electric railways, etc., there is required a current which runs continuously in the same direction through the circuit.

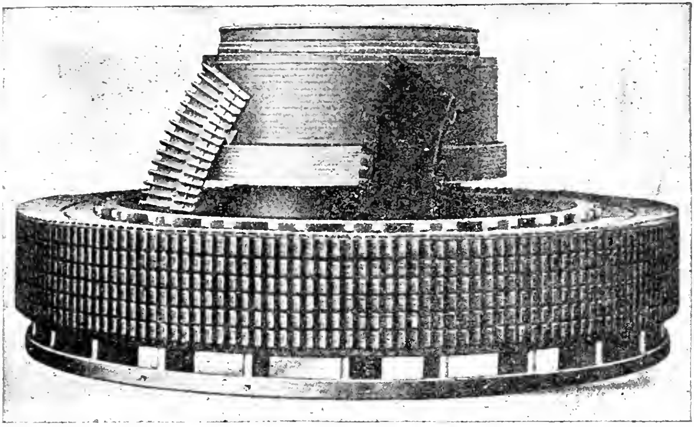


FIG. 155.—The air ducts are formed by specially shaped laminations; the corrugated laminæ, a set of which is used for about every three inches length of the core, give proper spaces for the air circulation from inside to outside of the core.

The current through a circuit will not be continuously in one direction (generally called a continuous current, abbreviated c.c., or direct current, abbreviated d.c.) unless the machine which is forcing current to flow through the circuit impresses on the circuit an e.m.f. which continuously acts in one direction.

The elementary generator having slip rings and brushes connecting its windings to the external circuit, could not do this. The *commutator* is a device which serves to reverse continually the connection of the armature coils to the external circuit so that, as the e.m.f. in the coils

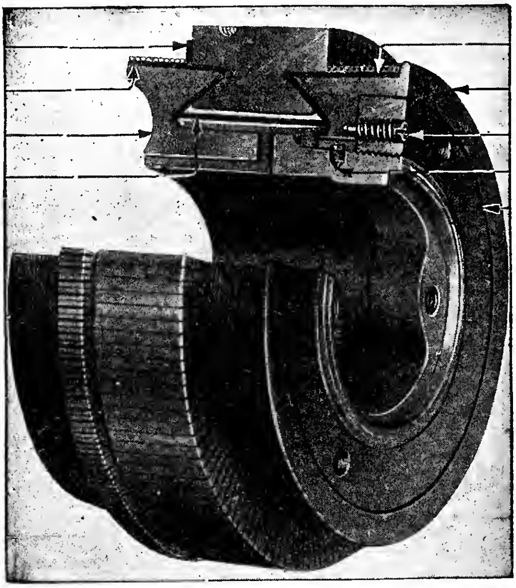


Fig. 156.—Cross-section through a small commutator showing how the spider serves to hold the copper segments.

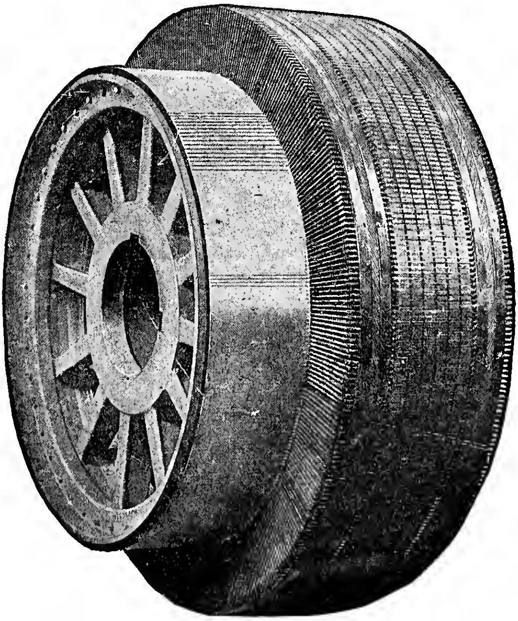


Fig. 157.—A completely assembled armature, showing the commutator in place and connected to the armature coils by the radial risers.

reverses, the connection of the coils to this external circuit reverses also, thus maintaining on the external line a uni-directional e.m.f.

136. Construction of the Commutator.—The function and action of the commutator will be taken up in detail in Chapter VIII; here we shall consider only the mechanical features of the commutator. It consists essentially of a set of copper bars of such a shape that when assembled, they form a hollow cylinder. As they are assembled, insulation is put between adjacent bars so that every bar is well insulated from every other. The commutator for a small machine is shown in Fig. 156, and Fig. 157 shows an assembled commutator connected to the armature winding of a large railway generator.

In taking up the construction of the commutator we shall consider the *bars* themselves, the *insulation* used in the commutator, and the *commutator spider,* which serves to clamp the bars together and hold them on the armature shaft.

137. Form of Bars.—The bars themselves are always made of copper, because it is a good conductor, is easy to shape properly, and wears well. Generally, these bars are stamped from copper rods, the cross-section of the rods being trapezoidal, so that as the bars are assembled side by side they form a cylindrical ring. The bar for a medium-sized commutator is shown in Fig. 158. The end view, in (*b*), Fig. 158, shows the trapezoidal form of the bar; the angle between the two sides depends upon how many bars are to be used in the commutator. If there were to be 360 bars in the complete commutator this angle would be about 1°; if there were to be 720 bars it would be about one-half of a degree, etc.

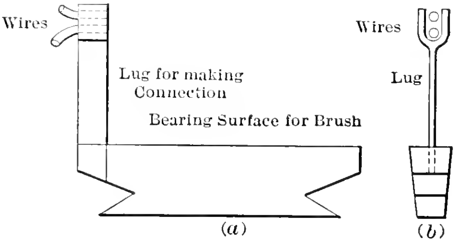


Fig. 158.—In large commutators risers are frequently sweated into the copper bars; these serve to facilitate the connection of the coils. The bars themselves are trapezoidal in shape as shown in (*b*), so as to build up compactly into a cylindrical form.

138. Insulation.—As the bars are assembled, sheets of insulation must be placed between every bar and its neighbor; insulation must also be used on the ends of the bar, to keep it from contact with the spider, which clamps the bars together. For the insulation between bars a special grade of mica is always used; a grade of mica must be employed which has about the same wearing qualities as the copper bar itself. If the mica should be too tough and wear away more slowly than the copper, *high mica* would result, i.e., the mica insulation would soon project above the copper bars and would cause sparking at the commutator, due to imperfect brush contact, as may be seen in Fig. 159*a* (exaggerated). In more recent practice, the mica is cut away about $\frac{1}{16}$ of an inch below the surface of the commutator, obviating the necessity of using mica of definite wearing qualities; such an "*under-cut*" commutator is represented in Fig. 159*b*.

The ability of mica to stand high temperature without deterioration makes it preferable to any such insulation as fiber, oiled cambric, etc. These substances are good insulators when kept cool and dry, but commutators frequently become very hot when the machine is operating, and also sparking is likely to occur where the brushes bear on the commutator; under such conditions these insulators would become charred and thus spoiled in so far as their insulating qualities are concerned.

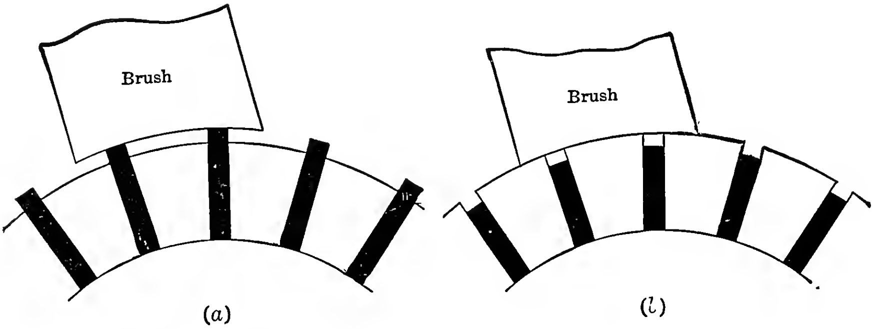


Fig. 159.—In (*a*) is shown what happens when the mica does not wear away fast enough; in (*b*) is shown the modern practice of "under-cutting" the mica to obviate the trouble caused by the "high" mica in (*a*).

Where the spider clamps the assembled copper bars, the bars must be insulated from the spider. Ring-shaped pieces, with V-shaped cross-section, are required for this purpose. As mica sheets are very brittle and could not be bent into the proper shape for such use, *micanite* is generally used. This is made by cementing very thin, small sheets of mica together by some such flexible binder as shellac. The thin flakes of mica are laid in the bottom of a tray to an even depth (perhaps one-sixteenth of an inch) the thin shellac poured in and allowed to drain off. By a proper drying and compressing process a flexible sheet of micanite is obtained, which, when warmed, may be formed into any desired shape.

139. Spider.—The commutator spider is made of cast iron in two or three pieces. The assembly view of a small commutator is shown in Fig. 160. The spider, shown in cross-section, is seen to consist of a sleeve, *a*, that fits on the armature shaft, a loose end ring, *b*, and the lock nut, *c*. The sleeve, *a*, is pinned to the shaft when the commutator has been placed in its proper position. The micanite rings are shown at *d*. If the commutator has been properly assembled the two surfaces marked *e* receive all the pressure as the lock nut, *c*, is tightened; in this way the lock nut serves not only to tighten the commutator bars endwise, but also to squeeze them tightly together sidewise by forcing them into a cylinder of smaller radius.

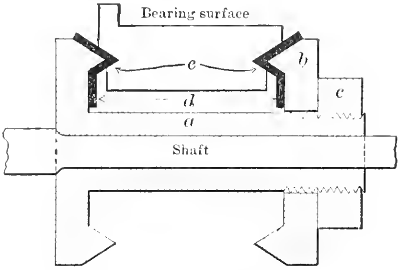


FIG. 160.—Assembly scheme of a small commutator; as the nut *c* is tightened, pressure should all occur on the surfaces *e* so that the bars are forced into a compact cylinder.

140. Number of Bars.—The number of bars to be used in a commutator depends principally upon the voltage for which the machine is designed. The function of the commutator is to change the alternating e.m.f. of the armature coils to a uni-directional e.m.f. on the external circuit. If too few bars are used in the commutator, the line e.m.f. will be uni-directional but will not be constant in value, i.e., it will be a pulsating e.m.f. If as many as 12 to 16 bars are used between brushes, these pulsations are so small as to be negligible. This may be seen by comparing Figs. 161 and 162. Figure 161 is an oscillogram of the e.m.f. of a 110-volt generator with 12 bars between brushes; Fig. 162 shows the e.m.f. of a 110-volt generator with 24 bars between brushes. The oscillograms were taken with both machines running at the same speed, as may be seen from the timing wave, the parts of which are 1/60 second apart.

If a two-pole machine had only 8 bars in its commutator, the amount of variation in the line e.m.f. would be about 4 per cent; hence, this number of bars would be too small for ordinary use. For an electroplating generator, however, the pulsation would not be disadvantageous, and plating machines generally have very few commutator bars. The number of bars to be used on railway generators, lighting generators, etc., is fixed by the rule that *the voltage between adjacent bars should not be greater than 15 volts.* Now, as the full voltage of the machine exists between adjacent sets of brushes, this means that the minimum number of bars

$$= \frac{\text{voltage of machine}}{15} \times \text{number of sets of brushes.}$$

This rule is only approximate for many reasons. The voltage between adjacent brushes is not distributed uniformly among the commutator bars between them. Between a pair of adjacent bars on the commutator, midway between two sets of brushes, the e.m.f. may be 25 volts, and at the same time two bars near one set of brushes may have a difference in voltage of only one or two volts. The above formula considers only the *average voltage* between bars and so gives the *minimum* number of bars that should be used.

141. Effect of Using too Few Bars.—Between adjacent bars of the commutator the mica insulation is generally about 0.02 inch thick; such a thickness of mica will stand a pressure of perhaps 10,000 volts, so that the limit of 15 volts per bar is evidently not fixed by the dielectric strength of the mica. The difficulty which arises if the voltage per bar gets too high is caused by *current leaking over the surface of the mica insulation.* The surface of the mica becomes soaked with oil, and if this oil carbonizes it becomes a fair conductor. Even if there is no carbonized oil on the surface of the mica, there will always be more or less dust rubbed into it, and also the whole surface of the commutator may become coated with a thin layer of oil and dust. If the voltage per bar is too great under

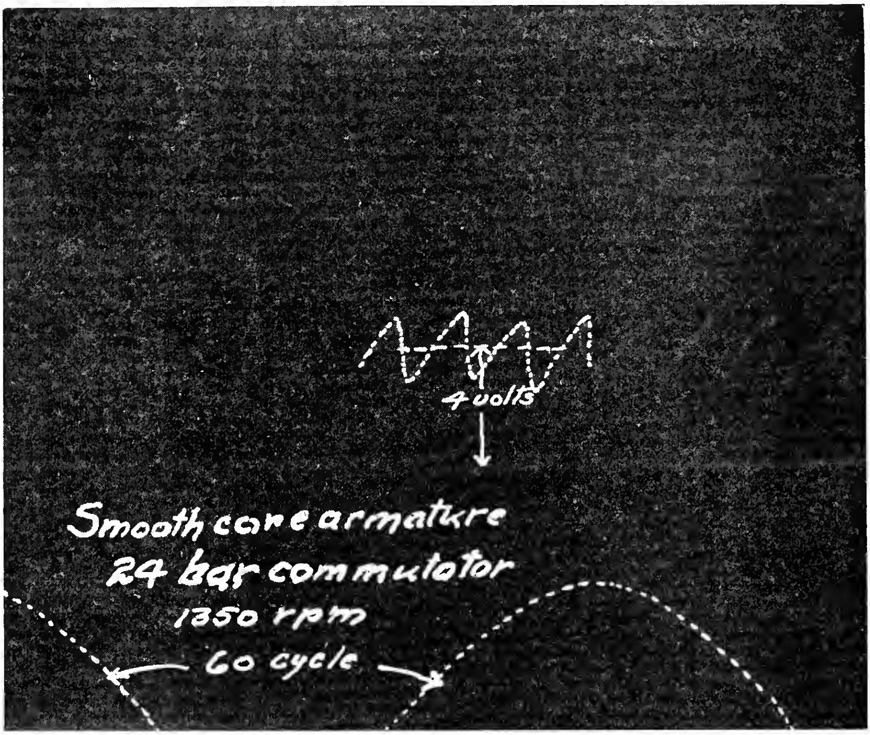


FIG. 161.—If a machine has but few coils and commutator bars the voltage is not constant, but has noticeable fluctuations, called commutation ripples. This is an oscillogram of the voltage of a smooth core machine having 12 coils in series between brushes, generating 110 volts. All but about four volts was balanced out by putting a storage battery in series with the armature, before connecting it to the oscillograph.

such conditions, current leaks over the surface of the commutator from one brush to the next and a "flash-over" may occur; i.e., a short circuit may occur between a pair of brushes, the short-circuit current following the surface of the revolving commutator, burning brushes, studs, etc.

142. Brushes and Brush Holders.—As the commutator, to which the armature coils are attached, revolves with the armature, it is necessary to have some stationary conductors for making a rubbing contact with the moving surface of the commutator, the external circuit being connected to these stationary conductors. Such conductors are called *brushes*; they are generally made of *carbon blocks*, but may sometimes be made of *copper leaves*, *copper gauze*, etc. The choice between copper and carbon depends entirely upon the voltage for which the machine is designed A low-voltage machine, such as is used in electroplating, must be equipped

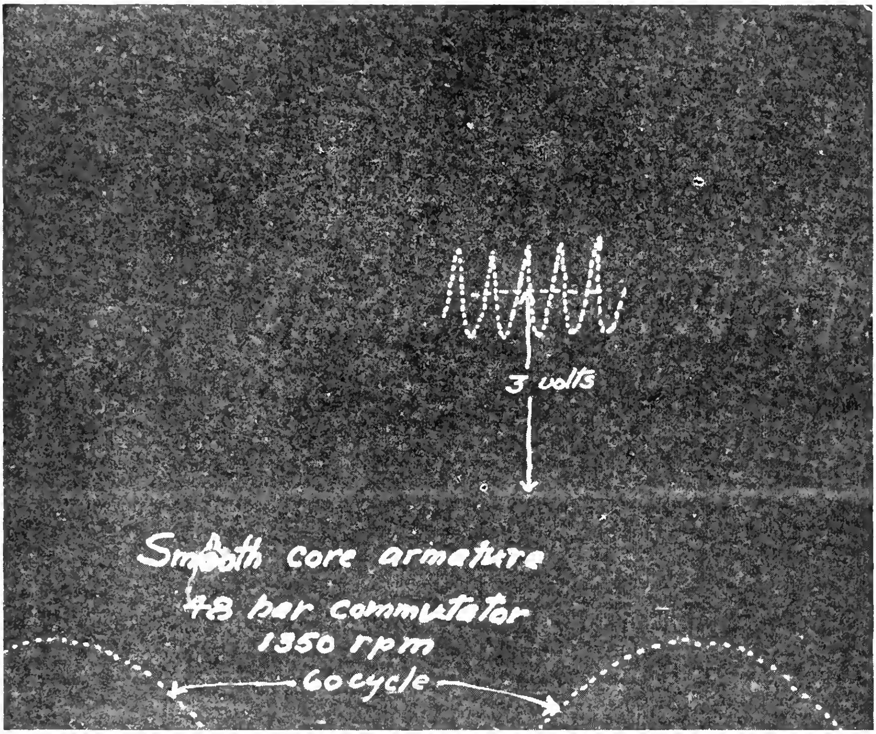


FIG. 162.—This voltage wave is taken from a smooth core machine having 24 coils between brushes. This machine was generating 110 volts, the same as that shown in Fig. 161, but most of this voltage was balanced out by storage battery as for Fig. 161 Notice that the ripples are twice as frequent for this machine as for that of Fig. 161, and that the amount of fluctuation is much less than for the machine having one-half as many coils and commutator bars.

with copper brushes; carbon brushes would not serve at all. For machines of voltages of 100 and more, carbon brushes must always be used. The reason for this will appear later.

143. Contact Area and Safe Current-density of Brushes.—The brushes and commutator form a moving contact surface, across which all of the current which flows from the armature to the external circuit must pass. To keep this contact surface in good condition, i.e., smooth and free from dirt, requires more care than any other task connected with the operation of dynamo-electric machinery. The area of the brush where it comes in contact with the commutator surface is called the *contact area* of the brush.

The current which can be carried safely by a square inch of contact area depends upon the two materials forming the contacting surfaces. If a copper brush is bearing on the commutator surface and the brush has 1 square inch of contact area, 150 to 200 amperes may safely be carried from the commutator by the brush; if the brush is made of carbon and has 1 square inch of contact area, not more than 40–60 amperes may be safely carried. If these values are exceeded, the brush and commutator will get too hot, this being due to the resistance of the brush contact, as will be discussed later.

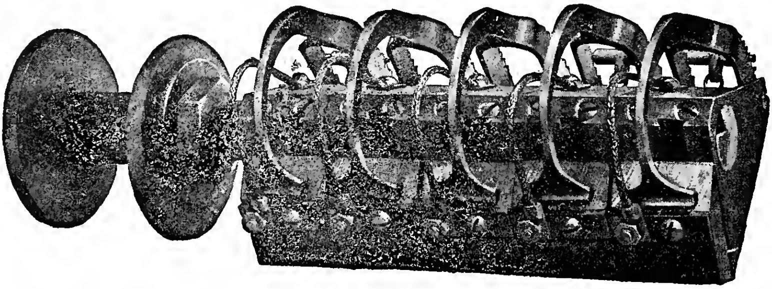


FIG. 163.—To obtain flexibility in a carbon brush it is generally made up of several small brushes, independent of each other, and having its own spring to keep it in place.

If either the commutator or brush surface becomes rough, the contact area is very much diminished. If a piece of dirt works into the contact surface of a brush and so makes a projection on this surface, it is evident that the brush will touch the commutator only at this projecting place, unless the brush is very *flexible*.

144. Flexibility of Brushes.—A brush made of copper leaves or wires is very flexible, while one made of a carbon block is not flexible at all. Therefore, if a slight projection on the commutator lifts off from the commutator one part of a copper-brush contact surface, the rest of the brush may stay in contact with the commutator owing to its flexibility. But if one corner of a carbon brush is lifted from the commutator, the whole brush leaves the commutator surface and the circuit is practically opened.

If the machine is delivering current to the external circuit when this happens, sparking will occur at the commutator surface, and if this occurs for any length of time the surface of the commutator becomes so roughened as to be unserviceable. In so far as flexibility is concerned, therefore, copper is preferable to carbon. The non-flexibility of the carbon brush is partly overcome by using, instead of one big brush, several small brushes, each capable of movement separately. Then, as one of these small brushes is lifted from the commutator accidentally, no open circuit is produced because the rest of the small brushes are still making contact. Figure 163 illustrates this construction; on the one brush-holder stud there are mounted five separate brushes, each of which is free to move by itself.

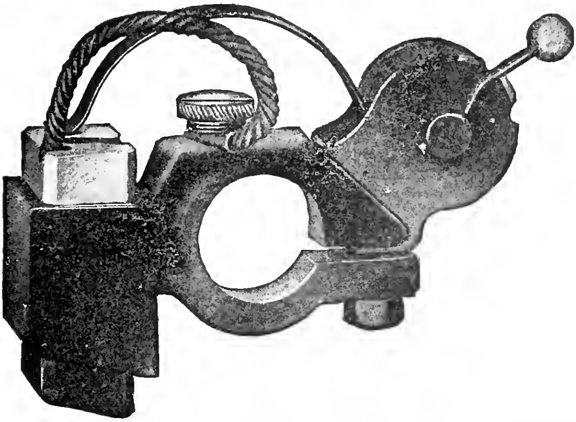


Fig. 164.—To lead the current from the brush to the brush holder a flexible cable (called a pig-tail) is generally used. This is firmly imbedded in the carbon brush on one end and clamped securely to the brush holder on the other.

145. Brush Holders.—As the brush (whether copper or carbon) wears away with use, some arrangement must be made to feed the brush continually toward the commutator, so that the proper pressure between the brush and the commutator is always maintained. The device by which this is accomplished is called the *brush holder*. It is a sort of clamp, through which the brush is capable of motion, and is mounted on the *brush-holder stud*. This motion of the brush is produced by a spring fastened tightly on one end to the brush holder; the other end of the spring presses directly on the brush itself. There may be several brush holders on the same stud. On large machines, as many as twelve or more may be used; there are seldom less than two, because one carbon brush has no flexibility and sparking is likely to result if only one is used.

In order that there may be good electrical contact between the brush and the brush holder, and also to prevent the carrying of current by the spring, a copper pig-tail is used. If the steel spring carries too much current it will become hot and lose its temper. The pig-tail consists of many strands of fine wire braided into a single conductor, which is firmly moulded into the brush at one end or clamped thereon, and screwed to the brush holder at the other, as in Fig. 164.

146. Number of Sets of Brushes.—There must be at least two brush-holder studs, one at which the current leaves the machine and another at which it enters. With the type of armature winding commonly employed, as many sets of brushes (i.e., groups of brushes on the same stud) are required as there are field poles. A 12-pole generator would have 12 brush-holder studs, mounted rigidly on the *brush yoke;* the studs would be equally spaced on the yoke and insulated from it; every other stud would be connected together and the two sets of studs so formed would be connected to the two terminals of the generator.

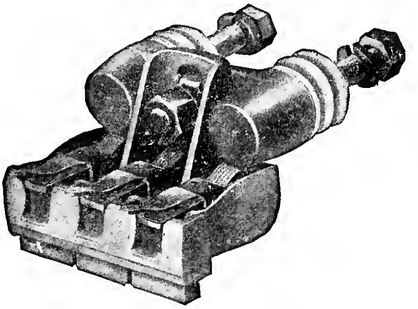


FIG. 165.—Brushes and brush holder for a railway motor. Because of the small space available in such an enclosed motor, the holder is more compactly designed than is that shown in Fig. 164.

In Figs. 164 and 165 are shown two types of brush holders, and Fig. 166 shows the complete brush rigging of a small four-pole c.c. generator.

147. Superiority of Carbon for Brushes.—It has been said that carbon brushes are used on all machines except those designed for electroplating, and this in spite of the fact that copper is a better conductor than carbon and only requires about one-quarter as much contact surface for a given current. There are two important reasons why carbon is preferred to copper for brushes: First, the mechanical wear on the commutator is much less with carbon than with copper; second, it is practically impossible to obtain sparkless commutation when copper brushes are used, on any but low-voltage machines.

148. Sparkless Commutation.—The commutation is said to be sparkless, or "black," when no sparking takes place at the contact surface between the brush and commutator. It is very important that sparkless commutation be obtained, because under the action of sparking at the brush contact, the commutator very quickly roughens, which makes the sparking worse, and so the machine is soon rendered unfit for service.

The explanation of this effect (i.e., sparking produced by copper brushes) will be taken up in a later chapter. It must be borne in mind here, however, that the sparking with copper brushes is not due to such causes as rough commutator, etc.; no matter how smooth the commutator may be, or how well fitted the brushes may be, this sparking cannot be eliminated.

149. Pressure of Brushes.—The springs on a brush holder are adjustable, so that the pressure exerted by a brush on the commutator may be varied as desired. If too little pressure is used, the contact is not good, and the electrical resistance of the contact becomes high; hence the I^2R loss at this place becomes too great and the brush will overheat. If too much pressure is exerted by the brush on the commutator, the power used up by mechanical friction of the brushes becomes too great, and the brushes and commutator will get hot from this cause. It has been found that with carbon brushes the best results are obtained when the springs are adjusted to give a brush pressure of about 1.4 pounds per square inch of contact surface; this value may, however, be anywhere between 1

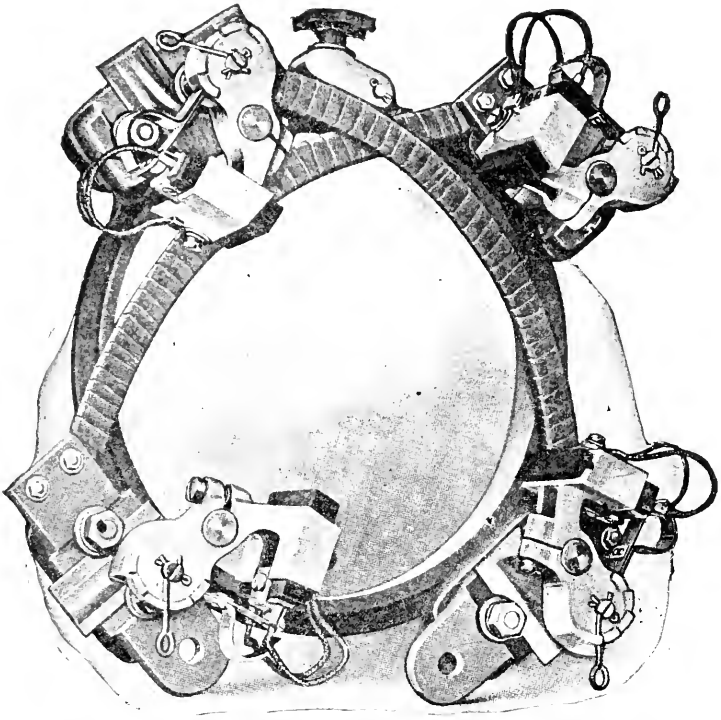


FIG. 166.—Complete brush rigging for a small four-pole machine.

pound and 5 pounds, depending upon the quality of the brush, peripheral speed of the commutator, etc.

150. Resistance of Brush Contact.—The resistance of the contact surface of a carbon brush and commutator is a variable depending upon the current density at the contact surface. As the current density increases, the resistance decreases; the variation of the resistance with the current takes place in such a manner that the *IR drop at the contact surface is nearly constant, and not dependent upon the current.* While this *IR* drop is slightly different with different types of brushes and with the different grades of carbon employed, it is safe to assume that on the average c.c. machine, with the commutator in good condition, the drop is about *one volt per brush contact.* As there are always two brush contacts in series, the *total brush contact resistance drop in any c.c. machine is from two to three volts.*

151. Brushes on Low-voltage Machines.—It is because of this contact resistance drop that carbon brushes are never used on machines of very low voltage. We shall show in a later chapter that the efficiency of any electric generator must be less than the ratio of the terminal voltage to the generated voltage.

$$\text{The terminal voltage} = E_g - IR_a, \quad \ldots \ldots \quad (92)$$

where E_g = the generated voltage of the machine;

IR_a = the total " drop " in the armature circuit (always greater than the brush-contact resistance drop);

If carbon brushes are used on the machine, efficiency is less than

$$\frac{E_g - 2}{E_g}. \quad \ldots \ldots \quad (93)$$

Now, if E_g is, say, 6 volts, the efficiency of the generator would have to be less than 66 per cent; as a matter of fact, it would probably be about 40 per cent. On higher-voltage machines this drop of 2 volts at the brush contacts is not such an important factor in determining the efficiency.

152. Field Windings.—The field windings of a dynamo-electric machine serve to force the flux through the whole of the magnetic circuit. The field coils are always placed on the poles; the m.m.f. produced by these field coils must be sufficient to force the magnetic flux through the poles, yoke, air gaps, and armature core. If the length, area, and *B–H* curve of each portion of the magnetic circuit are known, the number of turns required in one field coil and the current necessary can easily be calculated.

153. Magnetic Field Calculation.—The method used for calculating the ampere turns required to set up a given total flux in a magnetic circuit has already been given in section 52. It was there pointed out that the practical method is to use the *B–H* curve instead of the method involved in Eq. (24).

The first step in designing the magnetic circuit is to choose suitable flux densities for the various parts. The following table gives the limits for the values generally used in American practice:

TABLE V

Part	Material	Kilolines per Square Inch	Kilolines per Square Centimeter
Field yoke	Steel	70–100	10.8–15.5
Field yoke	Cast iron	35– 70	5.4–10.8
Pole core	Steel	70–100	10.8–15.5
Air gap	Air	40– 70	6.2–10.8
Armature teeth	Steel laminations	90–125	14.0–19.4
Armature core	Steel laminations	60– 90	9.3–14.0

Let us consider a bipolar field frame of the shape given in Fig. 167. The magnetic circuit consists of the yoke, two poles, two air gaps, and the armature core, all in series. The field is to be excited by two coils, shown in section at *A* and *B*. These two coils give m.m.fs. which act in the same direction on the magnetic circuit. We shall figure the ampere turns required for the whole circuit as though only one coil were to be used; one-half this number will be the proper number of ampere turns per coil.

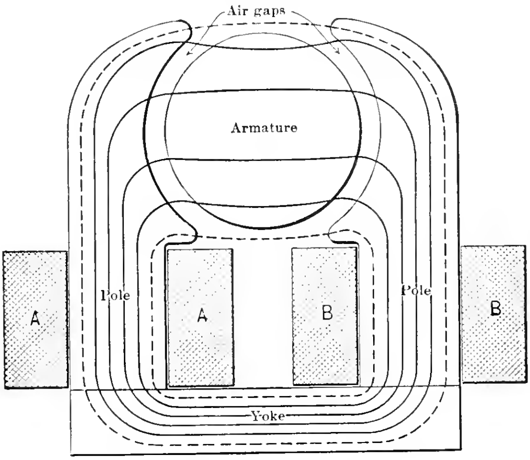


FIG. 167.—An old-fashioned bipolar field frame, showing where the field coils were placed.

The $B-H$ curves given in Fig. 49 (page 50) are to be used in this calculation; these curves are plotted between *flux density* and *ampere turns per centimeter length* of the magnetic materials to be used.

Suppose the necessary flux through the armature core is 1,600,000 lines. There must be more flux than this through the poles and yoke, because of the *leakage lines*. If the leakage factor is taken as 1.25, the flux through the yoke and poles is 1,600,000×1.25=2,000,000 lines.

The cross-sectional area of the different parts of the magnetic circuit would be given. The average length of the magnetic path in each part may be estimated. From the flux and cross-section, the density in each part is calculated, and then from the magnetization curves the ampere-turns per centimeter are obtained; this quantity, multiplied by the length of path in that part of the circuit, gives the required ampere-turns for that part. The sum of the ampere-turns required for each part gives the number required for the complete magnetic circuit, and one-half this number is to be put in each field coil.

The number of ampere-turns required for the air gaps cannot be obtained from the curve sheet, but is easily calculated as was shown on page 51.

In air, $B=H$, and hence, from Eq. (23).

$$B=\frac{0.4\pi NI}{l}, \qquad (94)$$

and

$$NI=\frac{Bl}{0.4\pi}=0.796Bl. \qquad (95)$$

The ampere-turns required per centimeter of air gap is therefore equal to the flux density in the air gap divided by 0.4π, or multiplied by 0.796. In calculating the magnetic circuit, it is convenient to tabulate the data as shown below:

Part	Material	Average Length in Centimeters	Area in Square Centimeters	Flux	Flux Density	Ampere-turns per Centimeter	Ampere-turns
Yoke	Cast iron	35	350	2,000,000	5,720	28	980
Poles	Cast steel	30 each	200	2,000,000	10,000	6.5	390
Air gap	Air	0.2 each	250	1,600,000	6,400	5095	2038
Armature	Silicon steel	25	150	1,600,000	10,650	3	75

Total ampere-turns required 3483

Ampere-turns to be used per coil $=\frac{3483}{2}=1742$

We shall next figure the necessary ampere-turns for a small modern bipolar machine with a slotted armature and double yoke. In this yoke the flux from the pole divides, half going one way and half the other. The flux, in going through the armature teeth, is very dense, and although the length of path is small it is better to figure this as a separate part of the magnetic circuit, instead of treating the teeth as part of the armature core.

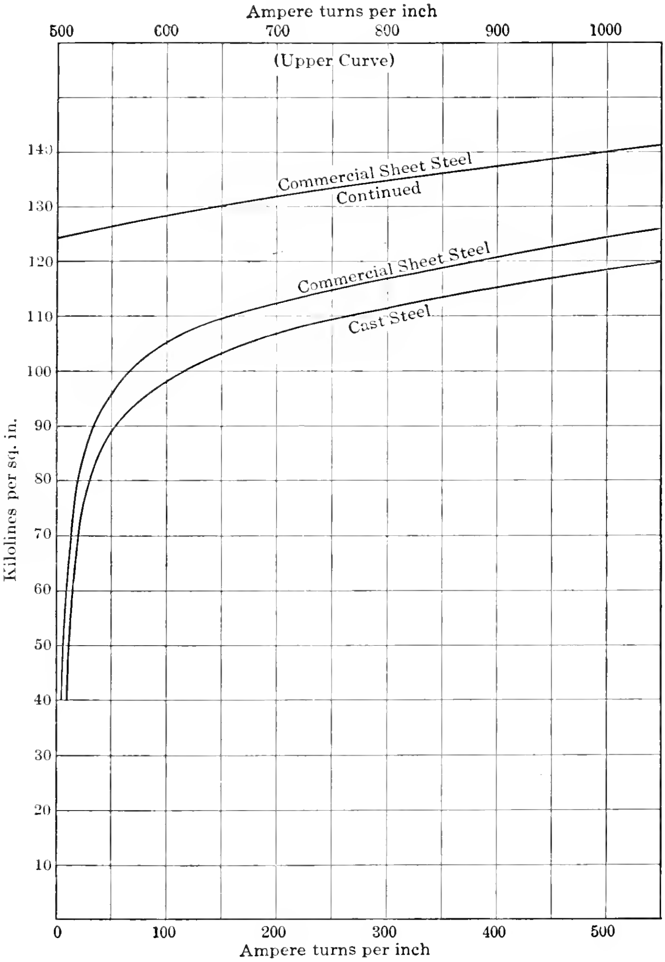


FIG. 168.—Magnetization, or B–H, curves, for the two materials almost universally used in modern machines; the scales are given in English units, most American designers using English, rather than metric, units in their work.

Although it is more scientific and logical to deal with dimensions in centimeters and densities expressed in lines per square centimeter, it is the custom among American designers to work in inches and express densities in lines per square inch. For this reason, another set of magnetization curves, plotted between *flux density in lines per square inch* and *ampere-turns per inch length,* is given in Fig. 168; for the problem now to be considered, we shall express the dimensions in inches and use these curves.

A diagram of the magnetic circuit to be used is given in Fig. 169 showing the location of the field coils, leakage lines, etc. The poles are made of laminated steel and are bolted to the cast-steel yoke. The area of the teeth is taken as being approximately equal to the cross-sectional area of as many teeth as lie under the pole face. Practically all the flux leaving the pole face crowds into the teeth, very little of it going down to the armature core by the slots. Although incorrect, we shall consider the area of the air gap as equal to that of the pole face; the useful area is actually somewhat less, owing to the effect of the slots.

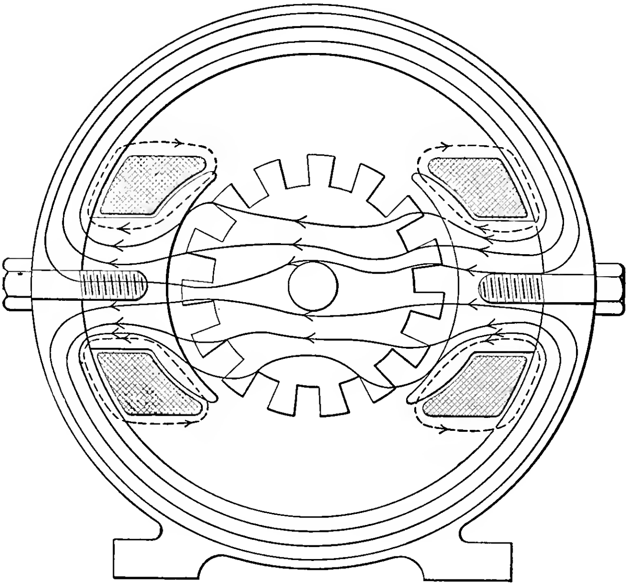


Fig. 169.—Cross-section of the essential parts of a modern bipolar machine, showing the paths of the leakage flux in the dashed lines.

The leakage factor for the machine is assumed to be 1.15; if the required armature flux is 1,200,000 lines, the flux through the poles and yoke is then 1,380,000 lines.

To determine the ampere-turns required per inch of air gap, Eq. 94 may be converted; since $B=B'\ (2.54)^2$ and $l=l'\times 2.54$, where B' is in lines per square inch and l' is in inches we have

$$\frac{B'}{(2.54)^2}=\frac{0.4\pi NI}{l'\times 2.54},$$

and

$$NI=\frac{B'l'}{0.4\pi\times 2.54}=0.313B'l'. \quad \ldots \ldots \quad (96)$$

The ampere-turns required per inch of air gap is therefore equal to the flux density in lines per square inch multiplied by 0.313.

Tabulating our data as before, and using the magnetization curves, of Fig. 168, we have:

Part	Material	Length in Inches	Area in Square Inches	Flux	Flux Density	Ampere-turns per Inch	Ampere-turns
Armature core....	Commercial sheet steel..	6.0	19	1,200,000	63,160	9	54
Teeth.....	Commercial sheet steel..	0.5 on each side	12	1,200,000	100,000	66	66
Air gap....	Air.........	0.1 each	20	1,200,000	60,000	18,780	3756
Poles.....	Commercial sheet steel..	4 each	14	1,380,000	98,570	60	480
Yoke.....	Cast steel....	20	10each	1,380,000	69,000	20	400

Total ampere-turns required.. 4756

$$\text{Ampere-turns per pole}=\frac{4756}{2}=2378$$

We shall next figure the necessary ampere-turns for a modern 300 kw., 6-pole generator, which is to furnish 275 volts when running at 900 r.p.m. A diagram of its magnetic circuit is given in Fig. 170, from which it will be seen that the pole flux divides, one-half going each way in both the yoke and the armature core. As any one complete magnetic circuit involves a pair of poles, we shall determine the ampere-turns required per pair of poles. The leakage factor for the machine is taken as 1.18; if the required air-gap flux is 8,800,000 lines, the flux through the poles and yoke is then 10,400,000 lines.

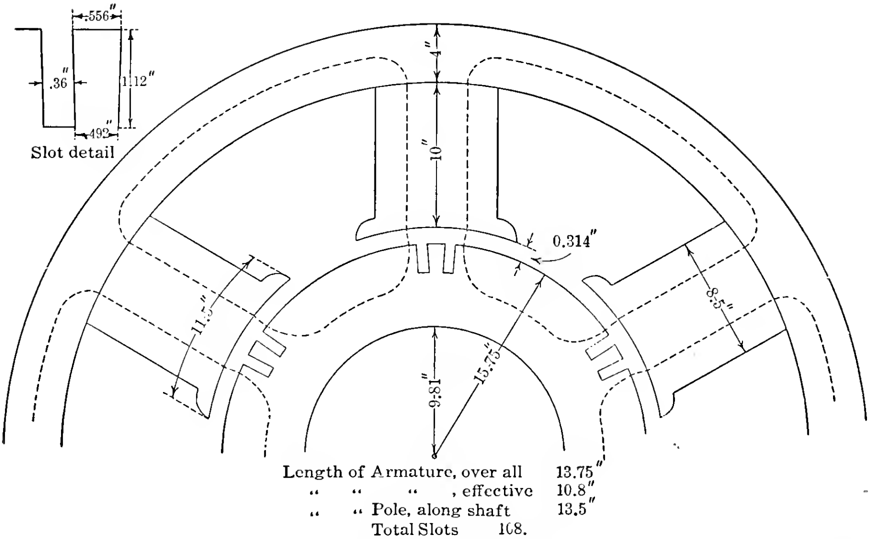


FIG. 170.—Giving the essential dimensions of the magnetic circuits of a modern multipolar machine; this is the kind of a sketch the designer works from.

Tabulating our data as before, we have:

Part	Material	Length in Inches	Area in Square Inches	Flux	Flux Density	Ampere-turns per Inch	Ampere-turns
Armature core....	Commercial sheet steel	15.0	52	4,400,000	84,610	25	375
Teeth.....	Commercial sheet steel..	1.12 each	70	8,800,000	125,900	550	1,232
Air gap....	Air..........	0.314 each	155	8,800,000	56,770	17,769	11,159
Poles.....	Commercial sheet steel..	10 each	115	10,400,000	90,430	35	700
Yoke.....	Cast steel....	32	61	5,200,000	85,250	40	1,280

Total ampere-turns required.. 14,746

$$\text{Ampere-turns per pole} = \frac{14746}{2} = 7{,}373$$

These three examples will serve to show how the number of field ampere-turns is roughly determined. In an actual design other factors have to be taken into account; the effect of the armature m.m.f. on the field strength has to be considered and the problem is somewhat more complicated than we have indicated.

It is to noticed that when *the flux* of a machine is spoken of, the flux per pole is always intended. In case the machine is bipolar, this flux is the total flux of the machine, but if a multipolar machine is being considered the *total flux* of the generator will be equal to the flux per pole multiplied by the number of pairs of poles.

154. Proper Size of Wire for a Shunt-wound Field.—The size of wire to be used in winding the shunt field coils is now to be determined. A *shunt* field is wound of relatively small-sized wire and is shunted across, or connected in parallel with, the armature. Generally, all of the field coils of the machine are connected in series and whatever voltage exists across the armature is the source of e.m.f. from which the field current is to be taken.

The average length of one turn of the field coil is estimated from the known size of the field pole and the assumed depth of the coil. Suppose that the length of one turn close to the field pole was 10 inches, and that we assumed a winding depth of 1 inch; the length of an outside turn would be in the neighborhood of 18 inches (if the pole were of rectangular cross-section) so that the *average length* of a turn would be 14 inches.

Let E = the voltage of the field current supply;
NI = the total ampere-turns required on the machine
= NI per pole × number of poles;
l = the average length of one turn, in feet;
r = the resistance per foot of the proper sized wire; this is to be determined;
I = the current through the field circuit;
R = the resistance of the field circuit;
N = the total number of turns on all coils in series.

Then,

$$R=Nlr, \quad \ldots \ldots \quad (97)$$

and

$$I=\frac{E}{R}=\frac{E}{Nlr}; \quad \ldots \ldots \quad (98)$$

multiplying both sides by N

$$NI=\frac{NE}{Nlr}=\frac{E}{lr},$$

or

$$r=\frac{E}{lNI}. \quad \ldots \ldots \quad (99)$$

Now NI has been calculated, E is known, and l has been approximately determined from the assumed size of the field coil. Therefore, r, the resistance per foot, of the proper sized wire to use, is determined. From the wire table, the diameter of the wire and its cross-sectional area in circular mils can be obtained.

Knowing the proper sized wire and the number of required ampere-turns, the proper number of turns is now to be determined. Here we at once notice a peculiarity in the problem. *So far as the magnetizing effect of the coil is concerned, with a given supply voltage, it makes no difference how many turns are used for the field coil; no matter how many turns are used, the m.m.f. of the coil will be the same.*

Suppose that we put on 1000 turns and that this number of turns has a resistance of 50 ohms. If the voltage of the field-current supply is 100 volts the field current will be 2 amperes and the ampere-turns of the coil will be 2000. If, now, 2000 turns are used, the resistance of the coil increases to 100 ohms, the field current decreases to one ampere and the ampere-turns in the coil will be 2000 as before.

155. Proper Number of Turns.—There must be some method of determining the proper number of turns to use, *and this is fixed by the safe allowable temperature rise in the coil.* It is apparent that if, for example, only one turn were used in the field coil, the proper number of ampere-turns would be obtained, but the current through the one turn would be so great that the wire would melt. As the number of turns is increased the required current decreases; when the number of turns has been increased to such an extent that the current which flows through the coil does not heat the wire more than 55° C. above room temperature, and never to a temperature in excess of 95° C. (the limit fixed by the A.I.E.E.), we may say that the field coil has been properly designed.

The process of predicting the temperature rise from the power used up as heat in the coil (I^2R loss in the coil) and the calculated radiating surface, requires judgment and experience, because some radiating surfaces are more effective than others, etc.

It has been found, however, from the results of many tests, that a safe figure to use in fixing the number of turns is obtained by allowing 1200 *circular mils per ampere* in the winding. As the proper sized wire has already been found, the cross-section in circular mils is known. Suppose it was 2400 circular mils; this wire could safely carry 2 amperes, so that if 3000 ampere-turns were required per pole the proper number of turns $= 3000/2 = 1500$ per coil. The figure 1200 circular mils per ampere is good only for field coils where the wires are packed tightly together and the heat cannot easily escape. In armature windings, and for series field coils, where the ventilation is better, it is customary to allow 750 circular mils per ampere; and if the wire were stretched in the open, probably 200–300 circular mils would be a sufficient allowance.

156. Series Field Winding.—In practice, to improve the operating characteristics of electrical machines, an additional field winding is fitted to each pole, this extra winding, consisting of a few turns, being called the *series* field. It derives its name from the fact that it is connected in series with the armature instead of being shunted across it, as in the shunt-field winding.

157. Construction of Field Coils.—Shunt field coils are usually made of double cotton-covered (d.c.c.) wire, or, particularly in the case of small machines, of enamel-covered wire. The wire is wound either on a metal frame arranged to slip over the pole core (Fig. 171), or else on a properly shaped wooden form from which the coil is taken off and taped, and then dried and impregnated with some insulating compound; when cold the coil is ready for assembly on the machine (Fig. 172).

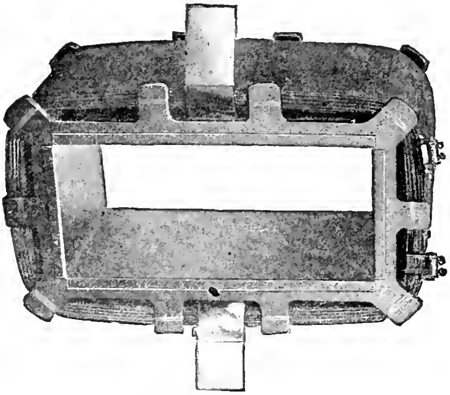


Fig. 171.—Field coil on a metal spool, showing the air ducts for ventilating the coil.

Series and commutating field coils, since they carry larger currents, are made either of large-sized insulated wire or of copper strap wound on the spool edgewise, the turns, in the latter case, being separated by paper or by distance pieces of insulating material (Fig. 173).

When a shunt and a series coil are used together on the same pole, the series coil is generally wound over the shunt coil; such a *compound* field coil is shown in Fig. 174.

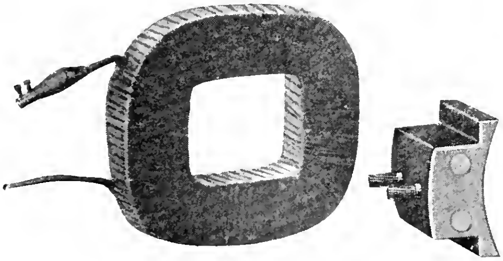


Fig. 172.—Showing how a shunt-field coil is made, and the pole over which it fits; when made without a metal frame like this the coil is taped strongly to give the requisite rigidity.

In larger machines it becomes necessary to afford some ventilation for the field coils. When metal frames are used, they can be made with a double wall, permitting air to circulate between the coil and the pole core, as in Fig. 171. In the case of compound field coils, a space may be left between the shunt and series coils; the use of copper strap, wound edgewise, with distance pieces, serves the same purpose.

The terminals of the field coils are usually brought out on opposite sides to facilitate connections between coils on adjoining poles.

158. Armature Windings.—The question of armature windings is a very broad one, and only an elementary outline and analysis of some of the various forms employed in different c.c. generators can be given here. By the term "armature winding," is meant the group of conductors placed on the armature core, which revolve with it and cut the magnetic field. The different ways in which these conductors may be placed on the armature core and interconnected, leads to innumerable winding schemes. The armatures of motors and generators are wound in exactly the same way; the same factors have to be kept in mind, no matter whether the armature is to be used for one purpose or the other. We shall speak of the different windings as generator windings, but they serve just as well for a motor armature.

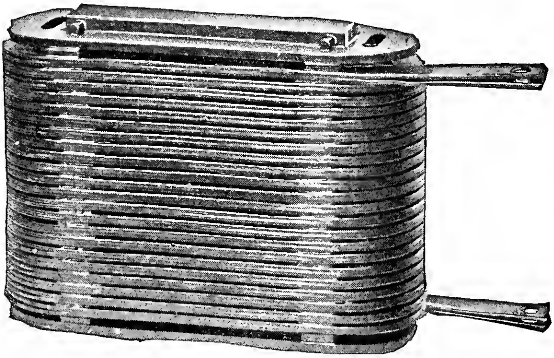


FIG. 173.—A commutating pole wound with heavy copper strap, the different turns being separated by spacers of insulating material.

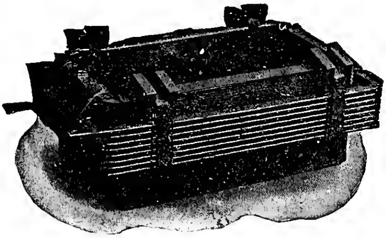


FIG. 174.—The compound field winding if frequently made with turns sufficiently large that they may be fitted right over the shunt coil; ventilating space is generally left between the two coils.

The armature windings for a.c. generators are quite different from those intended for a c.c. generator; we shall consider here only the c.c. windings, the first two general divisions of which are the *ring winding* and the *drum winding.*

159. Ring Winding.—In the ring winding, the coils are wound around the armature in such a way that half of the coil is on the *inside of the armature core.* This may be seen by reference to Fig. 175 which shows the armature of an early type of generator. The winding of these armatures is not as easy as the winding of a drum armature, nor can a ring winding be insulated as well; but a more important objection is the amount of "dead wire" on the armature. The conductors on the inside of the core do not cut any flux as the armature revolves, and hence they can generate no e.m.f. This extra wire is detrimental to the machine, not only because of the extra cost for wire but because the *resistance of the armature winding is much larger* than it should be; this high resistance means high I^2R and a high heat loss in the generator. As a result, the efficiency is lowered.

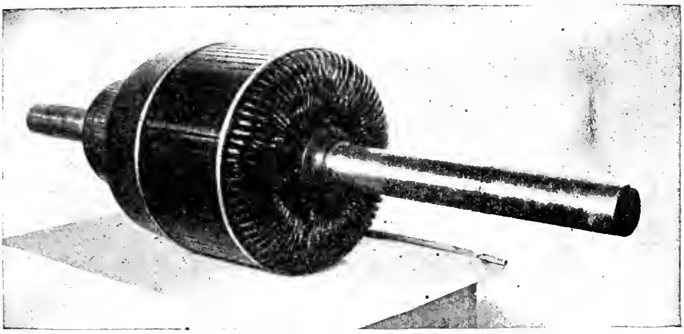


Fig. 175.—A ring-wound armature, practically never used in modern machines; the coils wind right through the center of the armature core.

For the reasons given above the ring winding is used very little at present. It would not require further discussion, were it not for the fact that the drum winding is not easy to show by a diagram. In the analysis of the action of a drum winding therefore, the drum winding will often be pictured as a ring winding. The ring winding is easier to represent, and the deductions made regarding its action apply equally well to the drum winding.

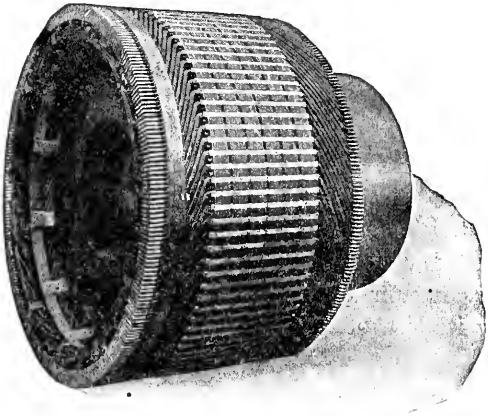


Fig. 176.—A drum-wound armature; both sides of the coils are fitted on the outer periphery of the core.

160. Drum Winding.—The drum winding is distinguished by the fact that all of the armature conductors *are on the outside surface of the armature* *core;* the coils do not thread through the inside of the armature as they do in the ring winding. The drum winding *does not contain as much inactive wire,* therefore, as the ring winding; also, the operation of winding the armature is much more easily accomplished with the drum than with the ring type and results in a better insulated armature. These are two of the advantages which make the drum winding so universally used. A typical medium-sized drum-wound armature is shown in Fig. 176; it may be seen that all of the winding is placed on the periphery of the armature.

161. Open and Closed Windings.—The next distinction to be made is between the *open* and the *closed winding.* In Fig. 177 are shown two ring-wound armatures; (*a*) represents the winding of a Thomson-Houston arc light generator, which was once used very extensively for arc lighting. The armature had three coils only, and these coils were connected to each other and to the commutator as shown in the figure. This is called an *open-circuit winding* in contrast to (*b*), which shows an eight-coil, ring armature with all of the eight coils connected in series with each other, forming by themselves a closed circuit. Such is called a *closed-circuit winding* and is the type used for all modern c.c. machines. The winding shown in (*a*) quite evidently does not form by itself a closed circuit; in fact, as the armature revolves each coil is entirely out of the circuit twice per revolution.

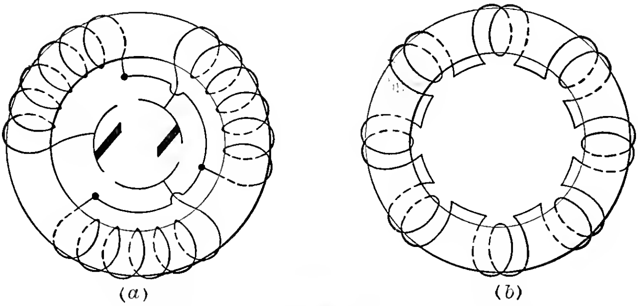


FIG. 177.—Two simple-ring windings; (*a*) is an open circuit winding and (*b*) is a closed circuit winding.

162. Series and Multiple Circuit Windings.—The next classification is made according to the number of paths by which current can flow through the armature winding to the external circuit. In one type of winding, called the *series, two circuit,* or *wave winding,* there are two paths through the armature, no matter how many poles the generator may have. This type of winding is used principally for railway motors; it is seldom used for generators designed for lighting, power purposes, etc.

In another type of winding, there are as many paths through the armature winding as there are poles on the generator; this is called the *multiple circuit, or lap winding.* By far the greater percentage of machines are equipped with this type of winding.

From this classification we see that the most important type of winding for c.c. armatures is the closed circuit, drum, multiple-path winding and we shall later consider two or three examples of this type. When the generator is bipolar, the series winding and the multiple-circuit winding have the same number of paths, namely, two. Two paths is the smallest number which can be obtained with a closed-circuit winding.

In discussing armature windings, we shall call any conductor in which an e.m.f. is induced an *inductor;* the rest of the conductors making up the winding we shall call simply conductors. The wires on the outer side of the coil of a ring winding are inductors, because when the armature rotates these wires move through the magnetic field and an e.m.f. is induced in them. Those wires which are on the inside of the armature core and on the ends are merely conductors.

163. Superiority of Multiple-circuit Winding for Generators.—One reason why the multiple-circuit winding is used so much more than the series, or two-circuit winding will appear when we consider the voltage generated by a given number of armature inductors, first when connected so as to give a two-path winding and then when connected to give a multiple-path winding.

Suppose we have a 12-pole generator having 1500 active inductors on the armature altogether. The voltage generated per inductor might be 2 volts. If these inductors were arranged in a series winding, all of them would have to be used in only two paths, so that there would be 750 inductors in series in each path. The voltage generated per path would be 1500 and this would be the voltage of the machine. Now, this would be much too high for an ordinary c.c. generator, as the highest voltage ordinarily used in c.c. service is 600, which is the voltage of an ordinary railway system.

Suppose now that the inductors were arranged in a multiple-circuit winding; as the machine has 12 poles there would be 12 paths in the winding and therefore the number of inductors in series per path would be 125. The voltage per path, which is the same as the voltage of the generator, would be 250, which would be right for a lighting generator.

A machine of the size we have in mind could not use a smaller number of inductors than 1500, because in that case the inductors would have to be of such large cross-section that they could not be bent readily, and therefore the operation of winding the armature would be too difficult. Of course the two windings would have the same capacity in kilowatts; the multiple-path winding which generates one-sixth as much e.m.f. as the wave winding could deliver six times as much current, making the output of the two windings the same.

The series winding has, however, certain advantages, and most motors are wave wound. For one thing, a series winding requires only two sets of brushes, no matter how many poles the machine has. In certain cases, notably railway motors, the inspection and adjustment of brushes is thereby made much easier than it would be with a multiple-circuit winding with many sets of brushes. Also, if the bearings of a machine become worn, letting the armature down so that it is eccentric with respect to the pole pieces, the flux from the lower poles into the armature becomes greater than that from the upper poles. If the winding of such an armature is multiple-circuit, such an inequality in flux produces overheating and sparking at the commutator; if the armature is series wound no such effect exists. As motors are small compared to generators, the idea brought out above, about the voltage of a winding and the number of inductors, is not important when the machine in question is a motor, moreover, motors are generally installed in some corner and are rarely inspected for bearing wear, etc. Hence the preponderance of series windings for motors.

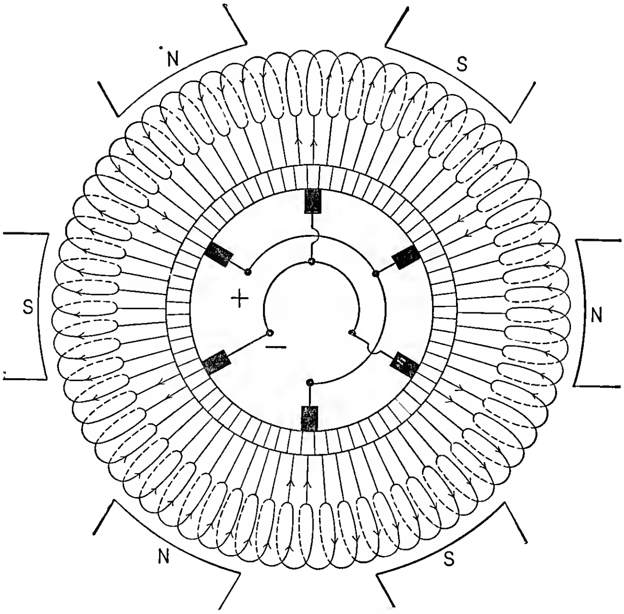


FIG. 178.—A six-path ring winding suitable for a six-pole generator.

The six-path ring winding shown in Fig. 178 shows how the six paths are formed in a multiple-path winding; also how the voltage of the machine is the same as the voltage generated in one path.

In a bipolar machine the multiple circuit and series windings are just alike except for the end connections, where the coil connects to the commutator. The difference between the two styles really appears only in a multipolar machine.

164. Examples of Simple Bipolar and Multipolar Armature Winding.—In Fig. 179 is shown a lap winding in the most elementary form. Twelve inductors are to be arranged in a closed circuit winding and *must be so connected together that all conductors in series in a path give an e.m.f. in the same direction in that path*; we might state it differently by saying that in either path there must be *no opposing* e.m.fs.

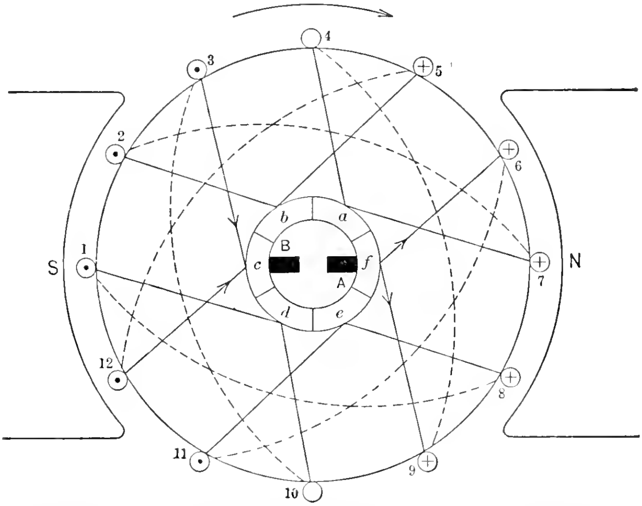


FIG. 179.—Elementary bipolar winding to show how the inductors are connected together and to the commutator bars. On the actual machine the brushes rest on the outer surface of the commutator, but they are here shown on the inside for clearness in the diagram. The dots and crosses in the inductors signify voltages, not currents

In order to do this, a certain order of "picking up" the inductors must be followed. If two inductors under the same pole are joined at the same end of the armature, the e.m.fs. of the inductors will be in opposition in the circuit and will neutralize each other; but if two inductors under opposite poles are connected at the same end of the armature, their e.m.fs. will be added together. In the example shown, starting at bar *c*, we pick up, in order, inductor 12 under the south pole and inductor 5 under the north pole, returning to bar *b*. From bar *b*, we go back to pick up inductor 2 under the south pole, and so on.

If the two paths in Fig. 179 are followed out, it will be seen that, in going from one brush to the other through either path of the winding, all inductors give an e.m.f. in the same direction. This winding is made up of six coils of one turn each; it will be noticed that the commutator has as many segments as there are coils on the armature. *This will generally be the case;* if a machine has 64 coils there will be 64 commutator segments, no matter how many turns there are per coil or how the coils are connected.

The two ends of any one coil .connect to adjacent commutator bars in the multiple-circuit or lap winding. In Fig. 179, for example, the coil made up of inductors 4 and 9 has its terminals connected to bars *f* and *a*, which are adjacent. Suppose the armature has revolved one-twelfth of a revolution from the position shown in Fig. 179 in the direction indicated by the arrow; the brush, *A*, will at that time rest half on segment *a* and half on segment *f*. Hence, coil 4–9, the terminals of which are connected to segments *a* and *f*, is temporarily short-circuited by the brush. At the same time, brush *B* is short-circuiting coil 3–10.

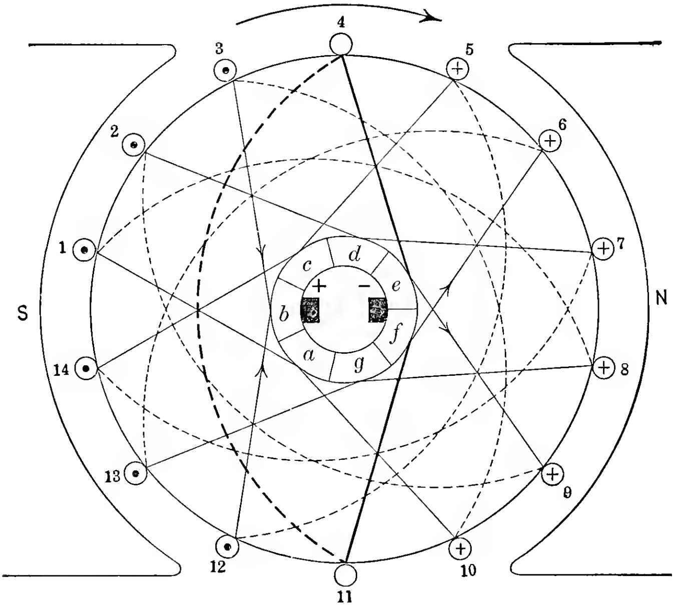


Fig. 180.—Another elementary winding showing the coil which is short-circuited by the brush; this coil is shown in the heavy lines.

In Fig. 180 is shown another elementary bipolar winding with seven coils. With the armature in the position shown, the coil 4–11 is short-circuited by the negative brush; one-fourteenth of a revolution later, in the direction indicated by the arrow, coil 3–10 will be short-circuited by the positive brush, and so on.

In Fig. 181 is shown a four-pole machine having 24 inductors on a lap-wound armature. There are therefore four paths and four brushes are required. (In these figures of armature windings the brushes are shown on the *inside* of the commutator for clearness only. Of course they really bear on the *outside* surface.) There is one turn per coil, and so twelve coils; the commutator has twelve segments.

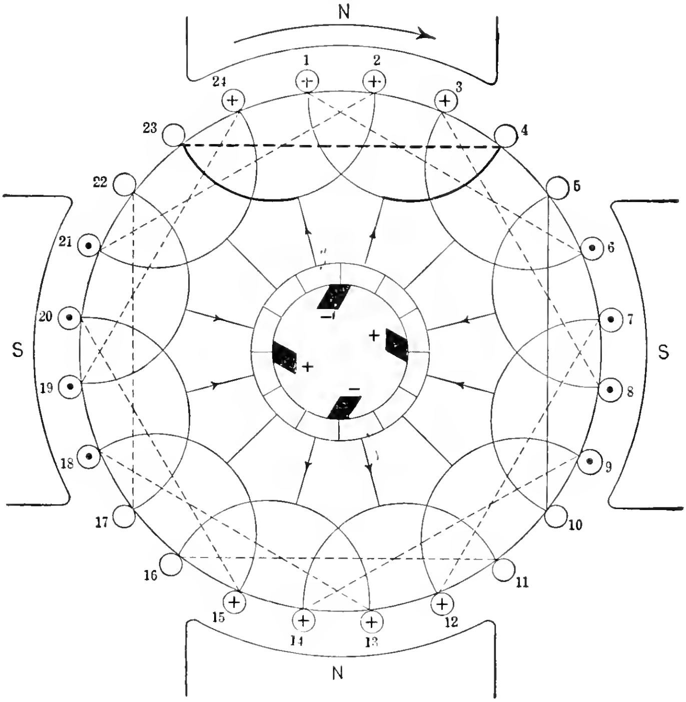


FIG. 181.—A four-pole multiple-circuit winding; an actual winding is never as simple as this one.

Figure 182 shows a four-pole wave winding, there being 22 inductors and only two brushes and two paths. The method of connecting the different inductors together gives the wave winding. Instead of going from 9 to 12 and then back to 7, as in the lap winding of Fig. 181, this wave winding continually goes forward, i.e., from 9 to 14 and then to 19. If the winding were spread out on a flat surface, it would have a wave appearance.

The ends of a coil are connected to two commutator bars spaced by an angle slightly less or greater than the angle occupied by a pair of poles.

165. Brush Position and E.M.F. in Short-circuited Coils.—If, at the time a coil is short-circuited by a brush, an appreciable e.m.f. is being generated in the coil, a large current will circulate through the path made up of the coil, the two-brush contacts, and part of the brush itself. *This short-circuit current,* as it is called, will produce disastrous sparking at the brush contact if it is large; hence *the brushes must be so placed on the commutator that the coil which they short circuit occupies such a position in the magnetic field that there is no e.m.f. induced in it while it is short-circuited.* (This statement will be qualified later when discussing commutation.) Evidently the position of the brushes shown in Figs. 179 to 182 satisfies this requirement, because at the time a coil is short-circuited, the inductors are moving in the interpolar space, where the flux is practically zero.

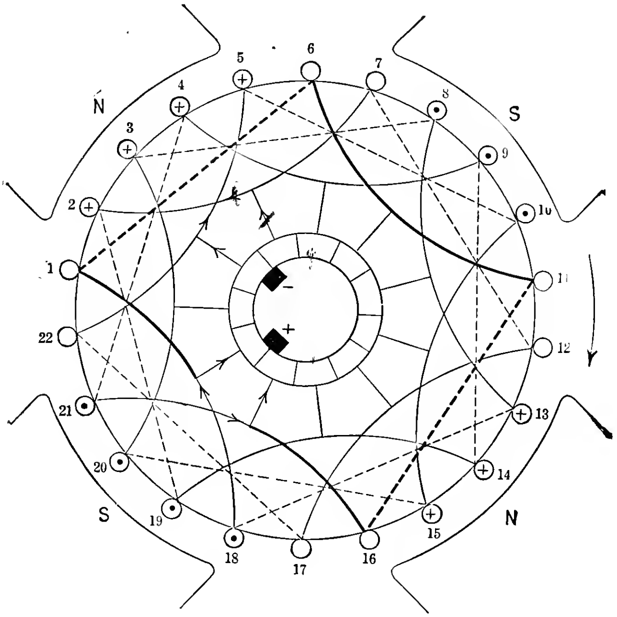


FIG. 182.—This is a wave, or two-circuit, winding for a four-pole machine; in the heavy lines are shown the two coils, connected in series, which are short-circuited by the brush.

166. Commercial Windings.—None of the windings given so far represent actual practice. In a slotted armature there are always at least two inductors per slot and these inductors are not parts of the same coil; they form sides of two different coils. A winding with two coil-sides per slot is usually called a *two-layer* winding, and one with only one coil-side per slot is called a *single-layer* winding. The former is the usual type of winding, using as many coils as there are slots.

In order to build up the voltage of a machine, it often is necessary to use more than one turn per coil (i.e., more than two inductors per coil). All the turns are usually taped together to form a *single* coil. In a two-layer winding there are then two coil-sides, in each slot.

Figure 183 represents a two-layer lap winding of more than one turn per coil. It will be seen that the left-hand coil-sides (odd numbered) form the upper layer and the right-hand coil-sides (even numbered) the lower layer. The kinks in the coils are for the purpose of allowing the end connections to pass each other neatly and compactly.

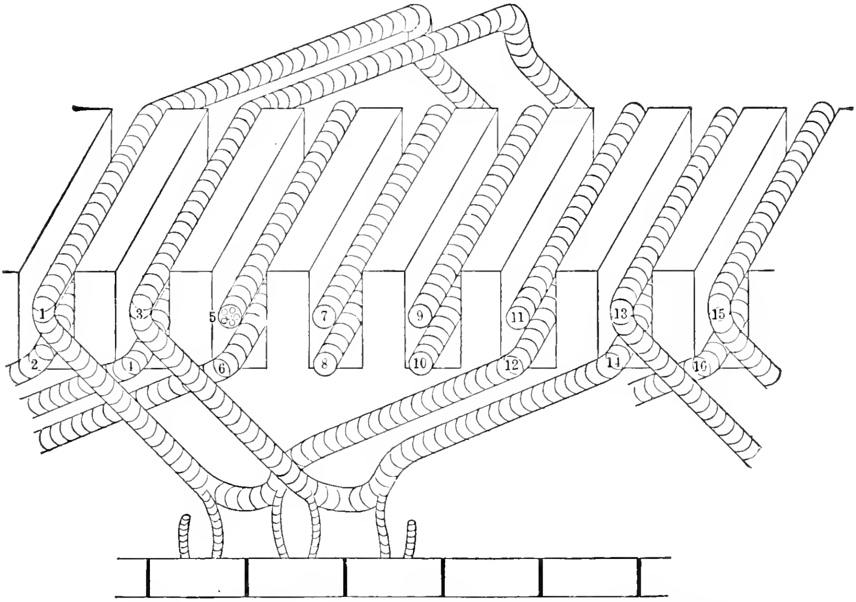


FIG. 183.—Formed coils are generally so shaped that one side of a coil fits in the bottom of one slot and the other coil side fits in the top of a slot distant from the first by nearly the distance between poles. The ends of the coils are so formed that they fit neatly one into the other.

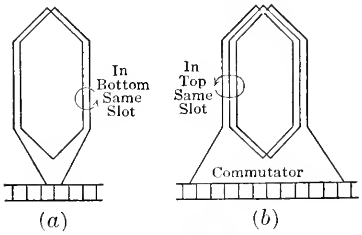


FIG. 184.

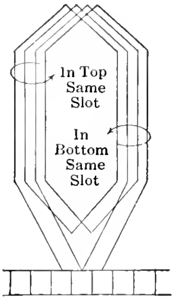


FIG. 185.

FIG. 184.—Coils generally consist of more than one turn; (*a*) shows a two-turn coil for a lap-winding and (*b*) shows a three-turn coil for a wave winding.

FIG. 185.—Showing how a double coil is connected to the commutator bars.

In Fig. 184, coils of several turns per coil are represented, that marked (*a*) showing a coil of 2 turns for a lap winding, and that marked (*b*) a coil of 3 turns for a series or wave winding.

If there are as many coils as there are slots, there must be the same number of commutator bars. Frequently, however, two single coils may be taped together to form a *double* coil and inserted into the slots as before. Such an arrangement requires as many commutator bars as there are single coils, or twice the number of slots, as represented in Fig. 185.

In Fig. 186 is shown a two-layer multiple-circuit, or lap, winding, for a four-pole machine having 24 slots on its armature. As is customary in winding diagrams, only one turn per coil is shown even though there are more, as we are interested only in the end connections of the coils. It is evidently a lap winding, as one coil "laps over" its neighbor. The proper position for the brushes is fixed by locating a coil in which no e.m.f. is generated and then placing one brush on the two adjacent segments to which the coil ends are attached. The other three brushes are located symmetrically with respect to this one.

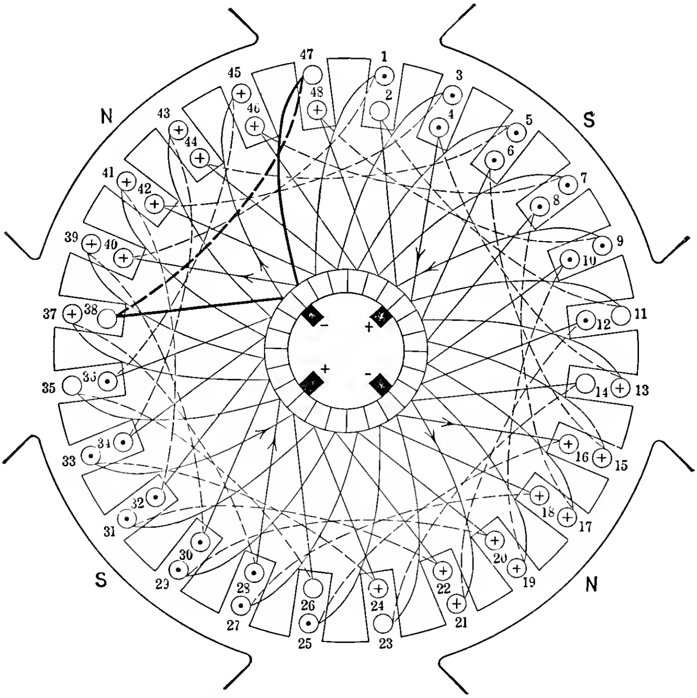


FIG. 186.—A two-layer multiple-circuit winding for a four-pole machine. One short-circuited turn is shown in heavy lines.

In Fig. 187 is shown a two-layer series, or wave, winding for a four-pole machine having 31 slots. In this diagram the armature coil-sides are shown radially, the two coil-sides in the same slot being represented close together and the lower coil-sides in broken lines. This winding requires only two brushes, 90° apart (shown black), as may be seen by tracing it through, as follows:

Negative brush {
42–27–12–59–44–29–14–61–46–31–16–1–48–33–
18–3–50–35–20–5–52–37–22–7–54–39

55–8–23–38–53–6–21–36–51–4–19–34–49–2–17–
32–47–62–15–30–45–60–13–28–43–58–11–26
} Positive brush

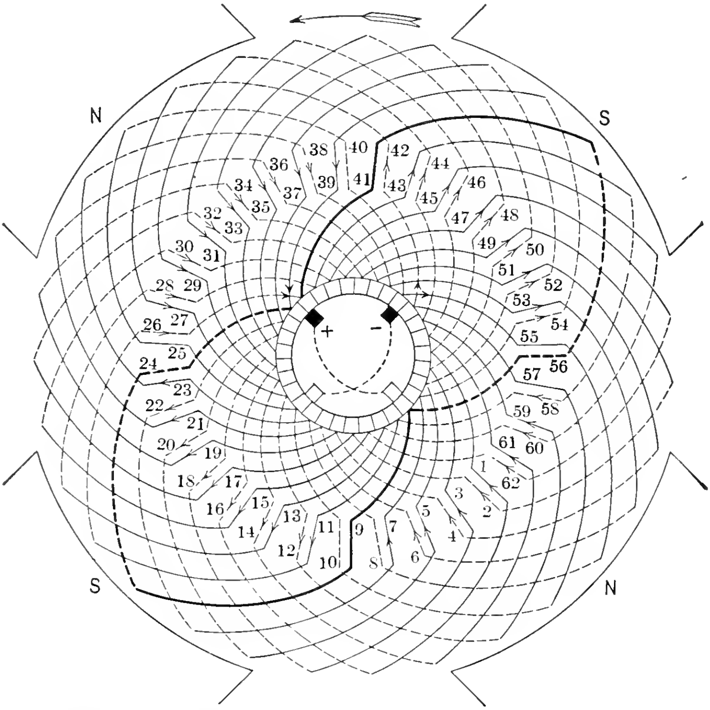
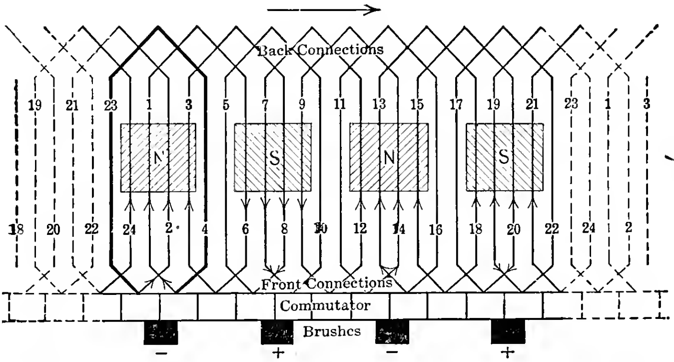


Fig. 187.—A two-layer wave winding for a four-pole machine, in a commonly used method of representation.

It will further be seen that the positive brush short-circuits two coils in series, namely, 41–56 and 9–24 (shown heavy) and the negative brush short-circuits coils 57–10 and 25–40.

If additional brushes are put on the machine, they can be placed 90° from the others, as indicated in outline. The two positive brushes are separated electrically only by the short-circuited coils 41–56 and 9–24, which, lying in the interpolar space, are generating no voltage. Similarly, the two negative brushes are joined by turn 25–40, the additional negative brush now also short-circuiting turn 55–8. The two positive brushes and the two negative brushes are thus practically in parallel respectively.

167. Pitch.—The term pitch is frequently used in connection with armature windings. The pitch of a coil is measured by the number of inductors passed over between coil-sides; the pitch so determined is called the *back pitch* (referring to the back of the armature). Thus, in Fig. 181 inductors 1 and 6 form a coil, and the back pitch is 5, that number of inductors having been counted over in connecting inductors 1 and 6.



$y_b = 5$

$y_f = 3$

FIG. 188.—A so-called developed winding; this shows about the appearance the winding of Fig. 181 would have if it were stripped off from the armature and laid out flat.

In Fig. 182 the back pitch is 5, and in Fig. 186 it is 9. The *front pitch* (or pitch at the commutator end) is formed by the number of inductors counted over between the two connected to the same commutator bar. In Fig. 181 the front pitch is 3; in Fig. 182 it is 5; and in Fig. 186 it is 7.

The *average pitch* is the average of the front and back pitches and, in any winding, may be equal to, or slightly less than, the number of inductors between centers of adjacent poles. When the average pitch is thus equal to the total inductors on the armature divided by the number of poles, the winding is said to be a full-pitch winding. If this is not the case, there will very likely be some inductors in every path which generate an e.m.f. opposite to that generated by the rest of the inductors in the path. While this is a disadvantage, there is an advantage gained by having *short end connections* when the pitch is less than the number of inductors between pole centers, and so this *fractional pitch*, as it is called, is generally used. If the number of inductors between the centers of consecutive poles is 12, a suitable average pitch would be 10, the front pitch being 9 and the back pitch 11.

168. Developed Windings.—Instead of representing an armature winding as we have done in Figs. 181, 182, 186 and 187, a so-called *developed winding* may be given. In such a diagram, the winding of the armature is represented in a plane surface, just as though the winding had been peeled off from the armature and laid on a flat surface; the poles are represented in their proper positions and the commutator is shown in developed form also. The windings of Figs. 181, 182 and 187 are shown in developed form in Figs. 188, 189 and 191.

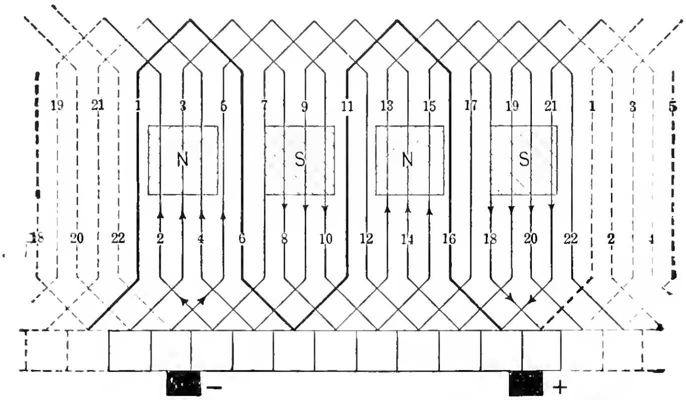


Fig. 189.—The winding of Fig. 182 shown in developed form.

$y_b = 5$

$y_f = 5$

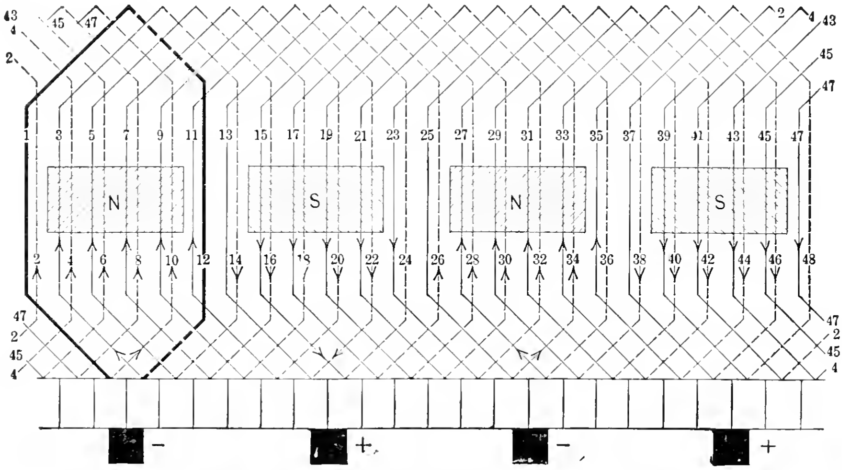


Fig. 190.—A developed four-pole multiple-circuit winding similar to that shown in Fig. 186; in this winding, however, the back pitch is 11 and front pitch is 9.

169. Equalizer Connections or Equipotential Connections.—It has been shown that in multiple circuit windings there are as many paths in

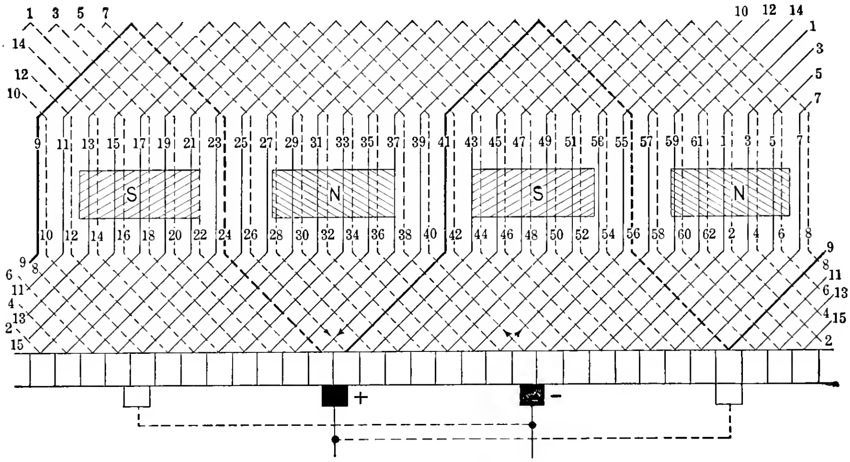


FIG. 191.—The winding of Fig. 187 shown in developed form.

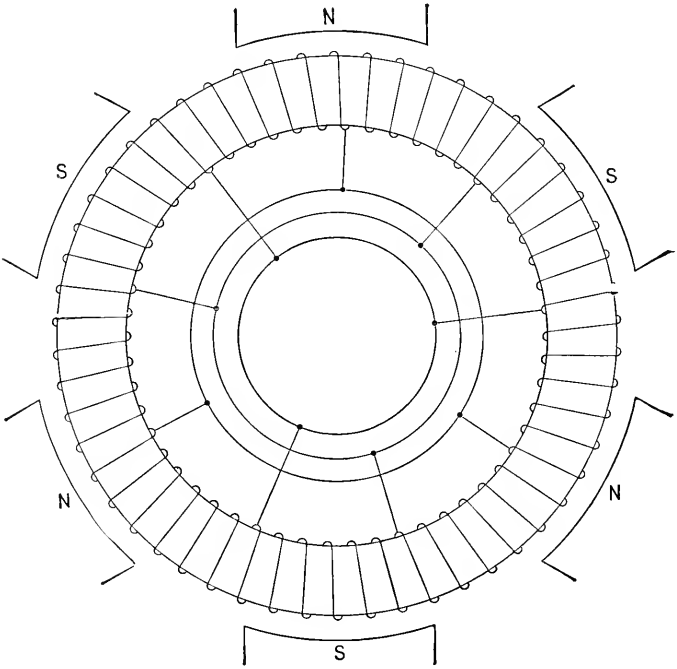


FIG. 192.—Equalizer connections are frequently put on armatures fitted with multiple circuit windings; they serve to distribute the current equally between the different paths.

parallel as there are brushes, and further, that each circuit is always under the influence of the same pole. In accordance with Kirchhoff's laws, if each circuit is to carry its proportionate share of the total armature current, it must at all times generate the same e.m.f. and have the same resistance as all the other paths. If, with no external load on the armature, one path were to generate more e.m.f. than another, an equalizing current would flow through them over the connecting brushes, resulting in unnecessary heating of the winding and sparking at the brushes. With load on the armature, the external current will divide unequally between

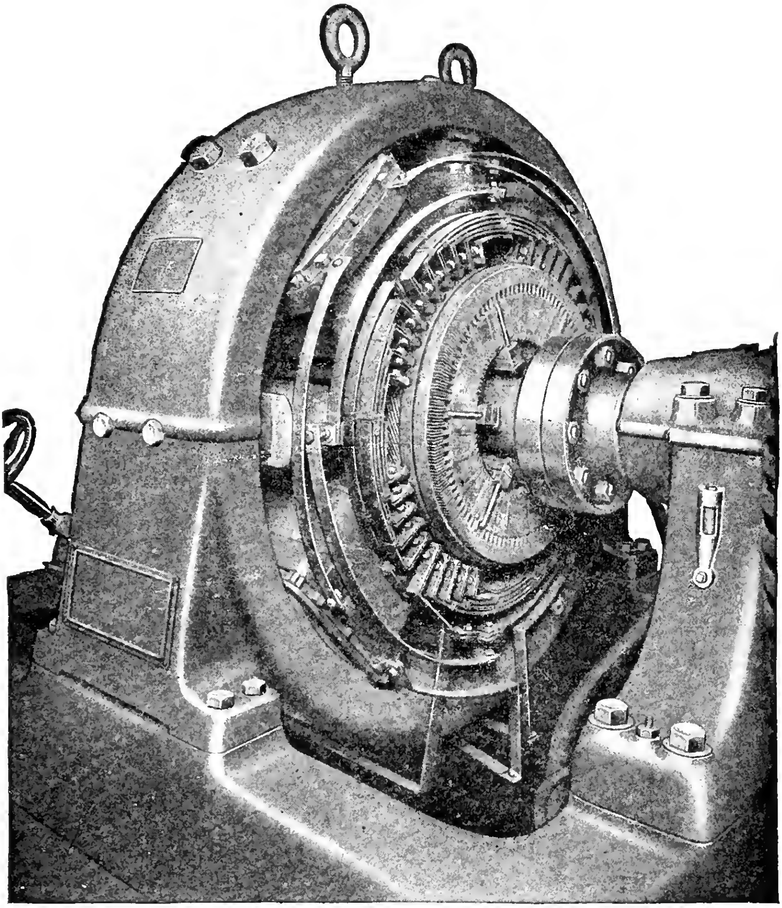


Fig. 193.—This is a 500-kw. 1200-volt generator equipped with many equalizer connections which may be seen below the ends of the armature coils. This machine is also equipped with compensating windings in the pole faces, referred to in section 205.

brushes of the same polarity and also between the parallel paths of the armature winding. Unequal distribution of current between brushes of the same polarity will overload some of them, thus limiting the possible output of the machine, and may also result in sparking.

Unequal generated e.m.fs. in the parallel paths of the armature winding may be due to various causes: (1) Slight inequalities in the air gap, caused by irregularities in construction or by wear of the bearings; such irregularities in the air gap cause some of the poles to carry more flux than others. (2) If the various parallel magnetic paths of the machine are not all alike in construction, even though the air gaps are identical, the flux per pole will be different; such inequalities may be caused by poor joints between the pole cores and the yoke or, in large machines, by poor joints between the upper and lower portions of the yoke.

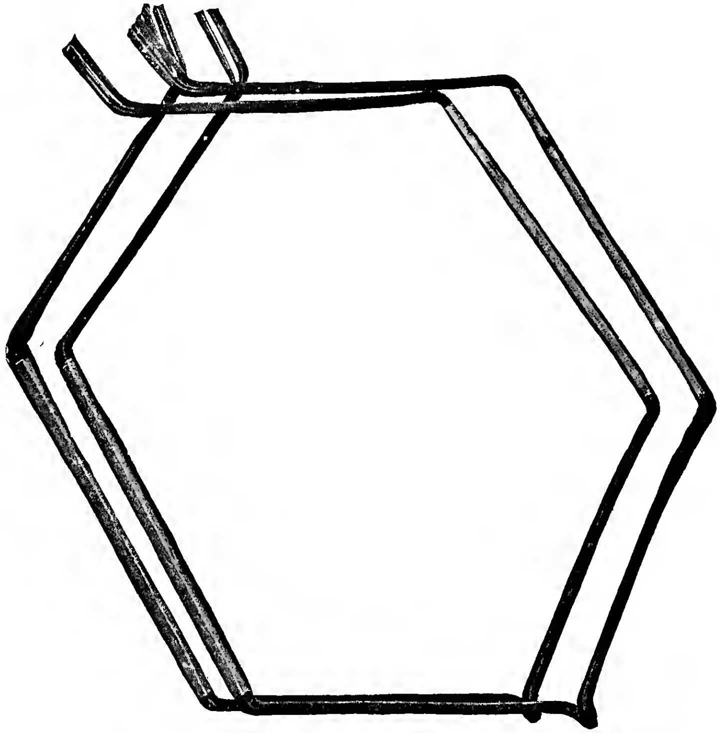


FIG. 194.—Formed coils of single turns (bar winding) for a multiple-circuit winding. The extra insulation put around the coils-sides where they fit into the armature slots is easily seen. It is to be noticed that two bars in parallel are used in the coil, instead of one bar only; this to make the coil more flexible.

By careful workmanship, the effects of constructional defects may be minimized, but to overcome the effect of unequal current distribution between brushes, *equalizer connections* or *equipotential connections* are used. These consist of low-resistance ring-shaped conductors, which are tapped into the winding at points which are theoretically at the same potential. If the points are not actually at the same potential, equalizing currents will flow over the equalizer connections from one armature circuit to another and thus balance the currents carried by the brushes. In Fig. 192 is shown a ring winding for a six-pole machine, with three equalizer connections.

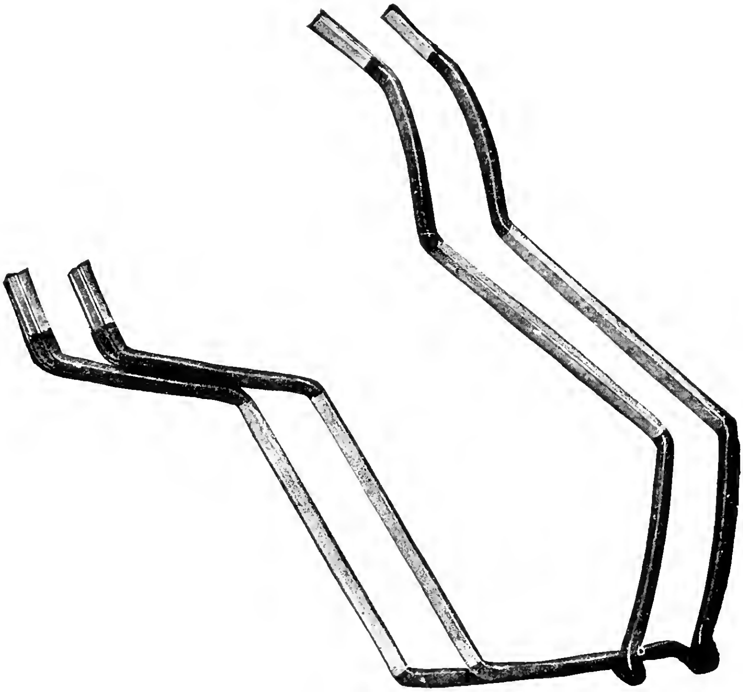


FIG. 195.—Bar winding coils for a series, or wave, winding; note the different form of ends on these coils compared to those of Fig. 194.

Equalizer connections may be placed either at the back of the armature, as shown in Fig. 193, or under the end connections of the winding between the core and the commutator. In large machines the equalizer connections are sometimes referred to as *equalizer rings*.

In series windings, no equalizer connections are necessary, since each of the two parallel paths through the armature is acted upon by all of the poles.

170. Current Capacity of a Winding.—The current capacity of an armature is fixed by the safe allowable rise in temperature, just as the safe current for a field coil is fixed. The armature is much better ventilated than the field coils, however, because the fan action of the armature comes into play as it revolves. It has been found that the safe current-density allowable in armature conductors is reached by providing between 600 and 900 circular mils cross-section of conductor per ampere, the lower figure to be taken for the machine having the better ventilation. If an armature winding is made up of four paths, and the conductor comprising each path is No. 10 B. & S. wire, the safe current capacity *per path* is between 11.1 amperes and 16.7 amperes—say 14 amperes. As there are four paths in parallel on the armature, and as each path can safely carry 14 amperes, the armature may safely furnish 56 amperes to the external circuit. If the same sized wire was used for a wave winding, in which there would be only two paths, the safe capacity of the armature would be 28 amperes.

Fig. 196

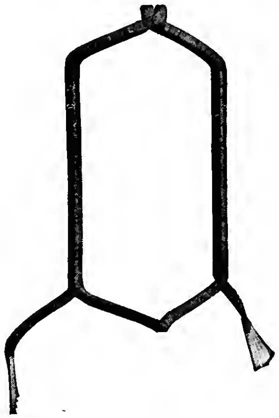


Fig. 197

Fig. 196.—This coil is for a multiple-circuit winding, and has several turns; the end connections of the coil are brought out close together.

Fig. 197.—This coil is for a series winding, as may be seen from the separated end connections; there are several turns in this coil.

171. Formed Coils.—So far we have said nothing about the actual process of winding the armature. The older machines were all *hand-wound*, but practically all modern machines are wound with *formed coils*. Suppose that an armature is to be wound with a coil pitch (i.e., back pitch) of 10 slots, and that there are to be six turns per coil. Instead of actually winding six turns of suitably insulated wire in the proper slots (hand winding) the wire may first be wound upon a wooden form, shaped in such a fashion that when the formed coil is taken off this wooden form it is of the right shape and size to be slipped, complete, into the proper slots.

Formed coils for use on c.c. machines are shown in Figs. 194 to 197. In Figs. 194 and 195, single-turn coils are shown; these are usually of copper bars, and the winding formed by them is often spoken of as a *bar*

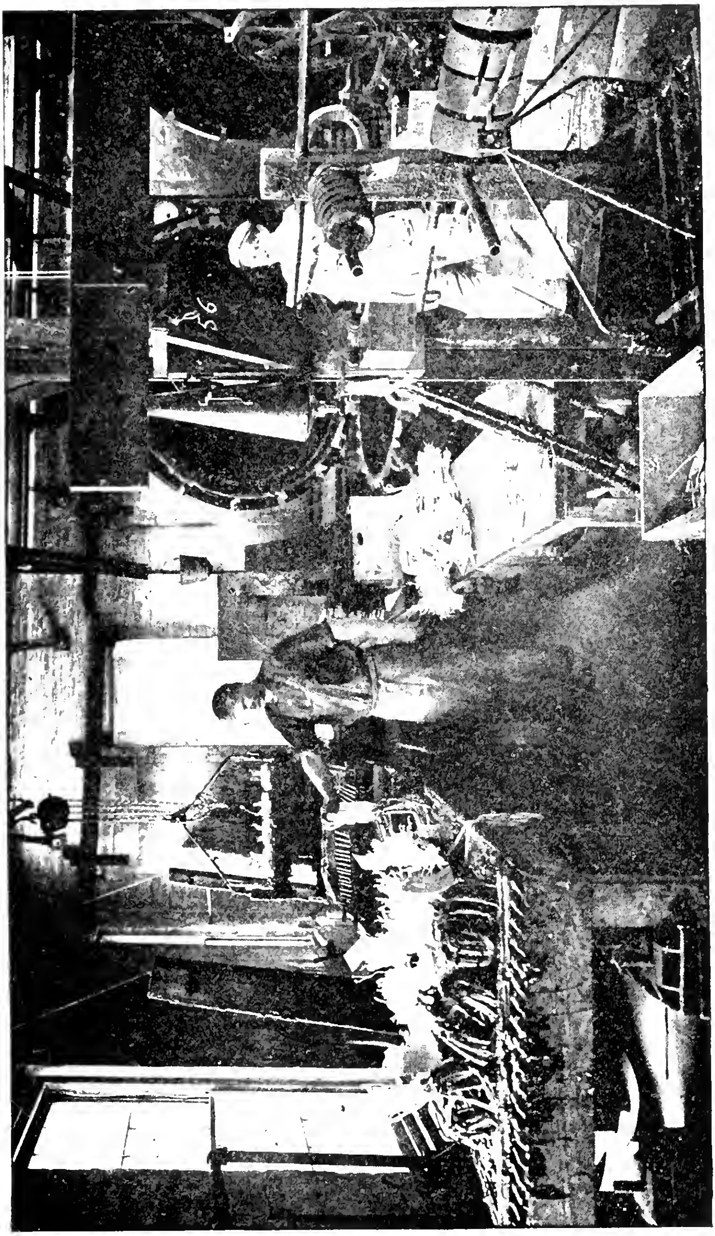


FIG. 198.—After coils have been wound and taped, whether for armature windings or for field coils, they are sent to the impregnating room, where they are dried out and then thoroughly impregnated, by the vacuum process, with a good insulating compound. The impregnating tank may be seen at the right, with its cover lifted up.

winding. In Figs. 196 and 197 there are several turns per coil. The ends of the coils are so bent that those of neighboring coils fit snugly against each other when they are assembled on the armature core.

In Figs. 194 and 196 the front connections of the coil are so formed that they lie close together; such coils are used for lap windings in which the two ends connect generally to adjacent commutator bars. The front connections of the coils shown in Figs. 195 and 197 are bent widely apart, these coils being designed for wave windings.

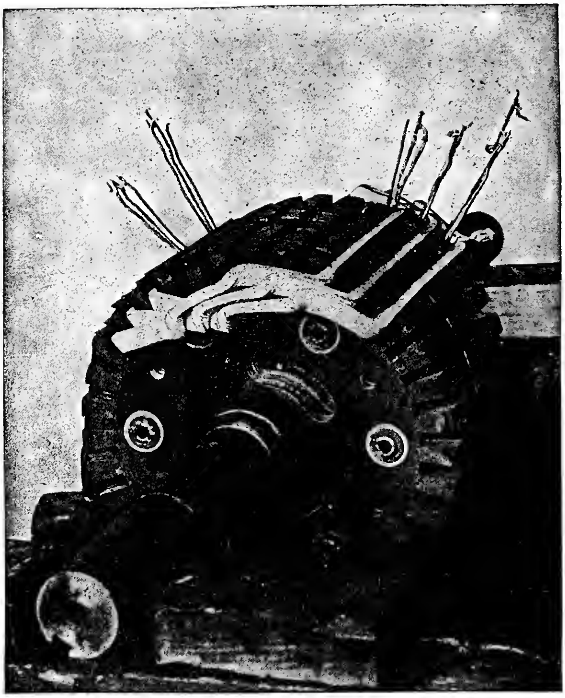


FIG. 199.—In the left-hand side of this armature, partly wound with formed coils, can be seen the heavy paper linings put into the slots before the coils are forced into place. Notice how the ends of the coils, having a peculiar twist at the ends, fit snugly together.

172. Insulation of Armature Windings.—After the wire coil is taken from the wooden form, it is taped tightly with insulating tape, the ends are cut to the proper length for connecting to the commutator and are properly bent, so that when it is placed on the armature the front pitch will be the desired amount. The amount of insulation put on these formed coils before they are ready for use on the armature depends upon the voltage for which the winding is designed. In a high-voltage machine several layers of cotton tape and oiled cambric are bound tightly around the coil and then the coil is impregnated, by the vacuum process, with some good insulating compound.

Figure 198 shows several sets of such coils after they have been taken from the impregnating tank; some impregnated field coils may be seen also. Fig. 199 shows an armature core which is partially wound with these formed coils. In Fig. 199 can also be seen the method of preparing the slots before the coil is forced into place. A sheet of tough paper (fiber or fish-board) is bent into the form of a trough and pushed into the slot, and the formed coil is pushed and hammered down into the slot inside this paper trough. This insulating trough extends beyond the ends of the armature core, so that as the formed coil is hammered into place there may be no risk of sharp edges on the iron core cutting through the insulation of the coil and so *grounding it* on the armature core.

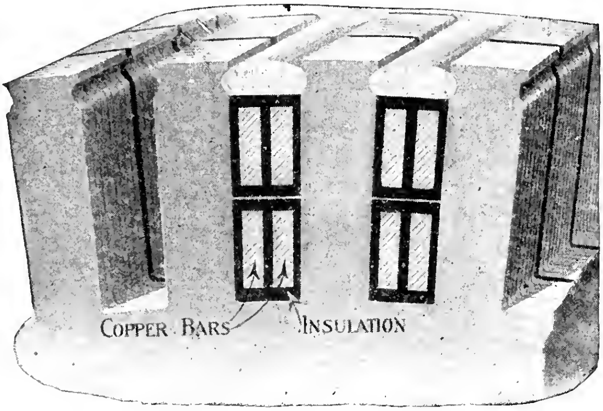


FIG. 200.—A cross-section through a slot, showing how the coils fit in place, how they are insulated, and how the wooden wedges keep them tight.

173. Shape of Conductor.—In small machines, the armature conductor is generally a round wire, which can easily be bent and shaped in sizes up to about No. 4 B. & S; if it is necessary to use larger cross-section that No. 4, several smaller wires may be used in parallel. If the cross-section necessary were larger than that of a No. 4 wire, a specially shaped conductor would probably be used. A round wire does not use the space in a slot very economically, and so a rectangular shaped conductor is generally used on large machines. Fig. 200 represents a cross-section through the slot of a high-voltage machine, showing how such copper bars fit into a slot, with the insulation around them, and a wooden wedge fitting tightly into grooves in the sides of the slot near the top; this serves to hold the winding in place and prevent it throwing out as the armature revolves.

On small machines no wooden wedge is used; after the coil sides have been hammered snugly into place, a strip of stiff, insulating material is slipped into the top of the slot and then binding wires are wound around the armature periphery. Several bands of such binding wire are generally used, as shown by Fig. 201, which represents the armature of a small c.c. generator.

Fig. 201.—A completely finished armature ready to be fitted on its shaft; the windings are held tightly in place by the binding wires, several bands of which can be seen. Two ventilating ducts can be seen in the core.

As the coils are being placed on the armature, each end is made fast to its proper commutator bar and, when all the coils have been put in the slots, the end connections are hammered snugly into their proper places. They are then soldered to the commutator bars and the armature is completed.

PROBLEMS

1. An armature core is in the form of a hollow cylinder of 10 inches inside diameter and 18 inches outside. Its axial length is 20 inches. If used in a four-pole field and revolved 900 r.p.m., the flux density in the air gap being 10,000 gausses, what is the probable hysteresis loss in ergs per second, and in watts? See section 59 for data on hysteresis loss.

2. If the speed of the above machine is increased 100 per cent and the flux density is cut down 50 per cent, the voltage generated by the armature will be the same. By what percentage is the hysteresis changed? Increased or decreased?

3. The loss in the armature of a certain turbo-generator is 7500 watts. If the temperature of the ventilating air at input is 20° C. and upon emission is 55° C., how many cubic feet of air per minute must be supplied to the core?

4. A 110-volt, 500-kw. generator has six sets of brushes. What must be the contact area of one set?

5. A bipolar machine, similar to that shown in Fig. 167, is to have 1,900,000 lines through the armature core. The leakage factor is 1.21. The cross-section of the yoke is 280 sq. cm., of poles 175 sq. cm., of air gap 190 sq. cm., and of armature core 180 sq. cm. Yoke and poles are of cast steel and armature of sheet steel. Length of yoke is 30 cm., of each pole 28 cm., of each air gap 0.15 inch, and of armature core 26 cm. How many ampere-turns are required per coil?

6. In a magnetic circuit similar to that given in Fig. 169, the leakage factor is 1.26, and the flux required through the core is 1.2×10^6 lines. Armature core and poles are sheet steel; yoke is cast steel. Core is 8 inches long and 22 square inch in cross-section; teeth are 0.6 inch long and useful cross section is 12 square inches. Air gap is 0.18 inch long and 18 square inches cross-section; poles are 5 inches long and 14 square inches cross-section; yoke is 24 inches long and 12 square inches actual cross-section. How many ampere-turns are required per pole?

7. Using a machine similar to that shown in Fig. 170, the magnetic calculations of which are there given in the accompanying text, how many ampere-turns would be required per pole, if the required air-gap flux per pole were 9.6×10^6 lines?

8. If the average length of turn for the machine of Problem 6 above is 22 inches, and the voltage available for field excitation is 95 volts, what size wire should be used and how many turns of it are required per pole?

9. The machine of Problem 7 above is a 275-volt generator. If the field rheostat is allowed 15 per cent of the armature voltage, and the average length per turn of the field winding is 58 inches, what size wire should be used, and how many turns of it must be used per pole? What is the power lost in the field circuit?

10. Figure the probable hysteresis loss in the armature core of the machine worked out in the text on page 180.

11. If the increase in field called for in Problem 7, over that worked out for the machine in the text, is to be supplied by a series field, how many series turns are required per pole? (The rating given, 300 kw. and 275 volts, really gives the full-load current available for the series turns.) What cross-section should the series turns have? If the series winding is made of copper strap $\frac{1}{4}$ inch thick, how wide should it be?

12. What would be the full-load I^2R loss in the series field, if the average length per turn were the same as that of the shunt field?

13. If there are six paths in parallel in the machine of Problem 7 above, what should be the size of wire used for the armature coils?

14. Give the winding diagram for the armature of a four-pole machine, multiple-circuit winding, with 36 inductors and 18 commutator bars.

15. Give winding diagram for the armature of a four-pole machine, series winding, with 52 inductors and 13 commutator bars. (In this, as well as in the previous problem, the number of bars has purposely been made too small for a commercial winding, in order to simplify the winding diagram.)

CHAPTER VIII

THE CONTINUOUS-CURRENT GENERATOR

174. E.M.F. of a C.C. Generator.—In calculating the voltage generated by any machine, we start from the fundamental rule that *if a conductor cuts a magnetic field at the rate of* 10^8 *lines per second, it generates one volt of e.m.f.* Therefore, to find the e.m.f. generated by any machine it is only necessary to calculate how much flux the armature winding cuts per second. First, a single coil armature will be considered, and then the more complicated windings will be taken up.

175. E.M.F. Form for an Elementary Generator.—Consider a ring-wound armature with only one coil, the two ends of the coil being connected to two slip rings, as in Fig. 202. These rings are represented one inside the other; really they would be of the same size and would be placed side by side on the armature shaft. As the coil moves from A to B it is cutting the magnetic field, and as the field is supposed to be of uniform density under the pole face, the e.m.f. generated will be constant during this time. In moving from B to C *no flux is cut,* hence no e.m.f. is generated. From C to D the coil is again cutting flux, but now the e.m.f. will be *in the opposite direction* to the e.m.f. generated when the coil was moving from A to B, because the flux is in the same direction as it was

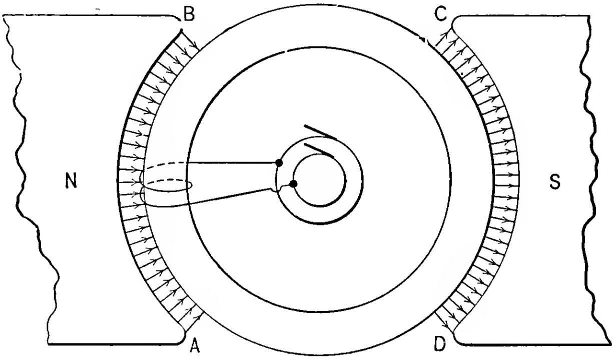


FIG. 202.—Conventional representation of a single coil on a ring armature, revolving in a bi-polar field of practically uniform density.

before, but the motion is in the opposite direction. Then, as the coil moves from D to A, no e.m.f. is generated.

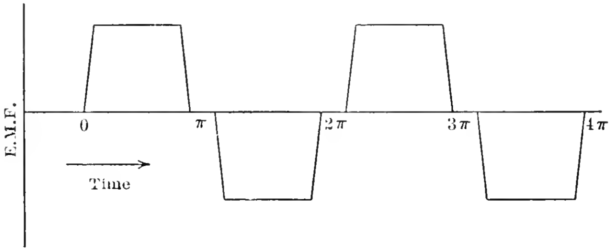


FIG. 203.—The coil shown in Fig. 202 would generate an e.m.f. wave of about the form shown here; actually, however, the corners of the wave would not be so sharp, as it is impossible to make the flux in the air gap change from a high value to zero value so abruptly.

If the *e.m.f. wave* be represented by using the position of the coil for the X axis and the magnitude of the generated e.m.f. for the Y axis, a curve will be obtained, such as is shown in Fig. 203. Calling the position zero when the coil is at A, it is evident that when the coil has revolved 180° the negative part of the e.m.f. begins; and, of course, when the

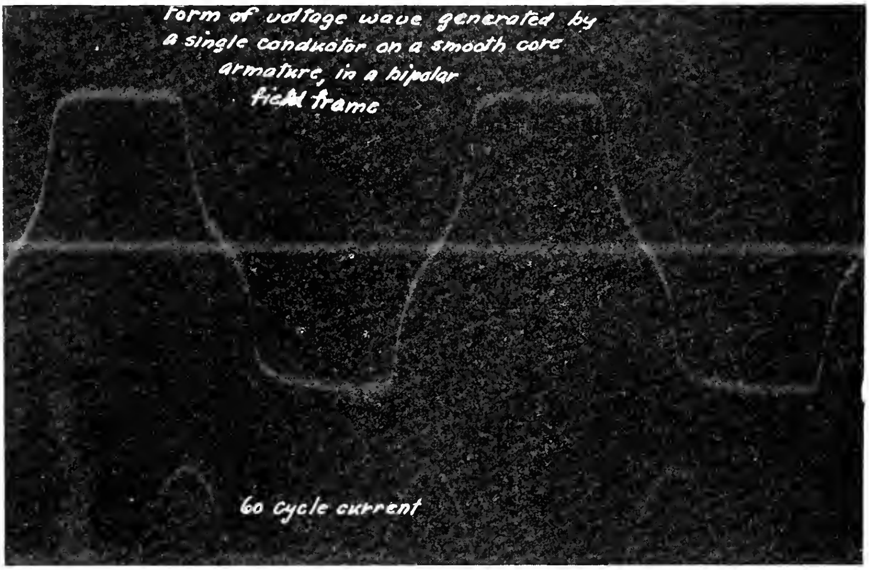


FIG. 204.—An oscillogram of the voltage generated by a single conductor on a smooth-core armature in a bipolar field. If the armature had been slotted there would have been ripples on the wave as may be seen from Fig. 235, which gives a wave similar to that above, using however a slotted-core armature.

coil has rotated 360° (i.e., back to A) the cycle of events begins over again. It is to be noted that since the flux in Fig. 202 is practically uniform in the air gap, the e.m.f. wave is flat topped.

Such a machine gives, then, an e.m.f. which alternates in direction once every time the coil moves by a pole; and, if an external circuit (such as a load of lamps) were connected to the brushes, an alternating current of the same shape as the e.m.f. wave of Fig. 203 would flow in it. An oscillogram of the e.m.f. of such a one-coil winding is given in Fig. 204; it is seen to be approximately the same shape as given in Fig. 203.

176. Action of Commutator.—To make the simple machine of Fig. 202 give a current which, *in the external circuit, is uni-directional,* it is necessary to connect the ends of the coil to the two segments of a two-part commutator and to so place the brushes on the commutator (diametrically opposite to each other) that, *at the same time that the e.m.f. of the coil reverses, the connection of the coil to the external circuit reverses.* When this is done, a pulsating, uni-directional current, as shown in Fig. 205, flows in the external circuit.

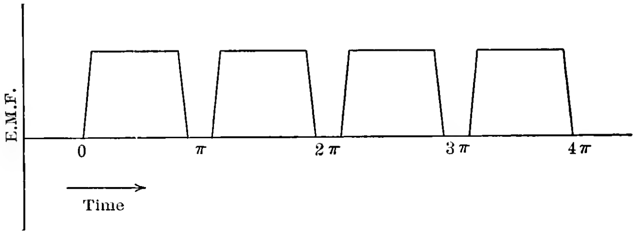


FIG. 205.—If the e.m.f. wave of Fig. 203, were reversed in polarity every other alternation, the e.m.f. would be uni-directional, but not uniform; the coil of Fig. 202, connected to a two-part commutator, would give, on the line, a voltage of about the form shown here.

The commutator and proper position of the brushes to give such a current in the external circuit are shown in Fig. 206, where the brushes are shown inside the commutator for clearness of representation. They really bear on the outside surface of the commutator. It will be seen that as the coil passes from a position under the north pole to a position under the south pole, its connections with respect to the brushes are reversed.

A single coil does not form a closed winding and would never be used. If another coil is wound on the armature in the same direction as the first, and if the coils are connected as in Fig. 207, we have a two-coil, closed-circuit winding. The direction of the induced e.m.f. is marked on both coils by arrows, and it is seen that the e.m.f. of both coils acts in the same direction *as far as the outside circuit is concerned,* but that the two e.m.fs. oppose each other in the local circuit made up of the two coils only. In this two-circuit winding *the e.m.f. of the generator is evidently the same as the e.m.f. per path, but the current capacity of the machine is equal to twice the current capacity per path.* The e.m.f. wave form of the armature, shown in Fig. 207, would be exactly the same as that shown in Fig. 205, which was for a single-coil armature.

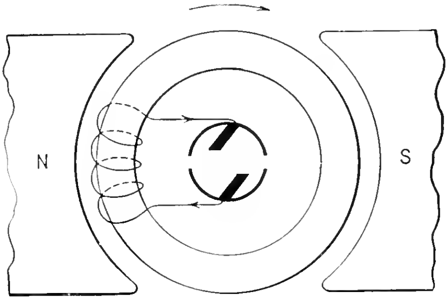


FIG. 206.—The connection of the coil of Fig. 202 to a two-part commutator is shown here; brushes, 180 degrees apart, rest on the commutator and serve to connect the rotating coil to the outside circuit.

The e.m.f. wave of Fig. 205 is not suited for ordinary purposes of lighting, running motors, etc. A uniform, non-pulsating e.m.f. is desired, and this is the purpose of making an armature with many coils and many commutator bars. Consider a four-coil, closed-circuit armature with a four-part commutator, connected as shown in Fig. 208. As the armature revolves it is evident that every coil will generate a wave of e.m.f. of the same shape and magnitude as that of any other coil, but that these waves will be behind one another, a time equal to that required for one-quarter of a revolution of the armature. Moreover the e.m.f. of the

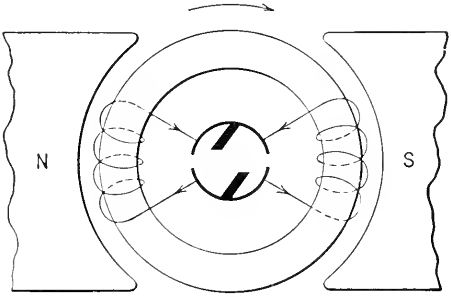


FIG. 207.—If two coils were used with a two-part commutator, they would be connected as shown here; being in parallel for delivering current to the brushes.

machine is obtained at any time by adding together the e.m.fs. of the two coils that are in series with each other at that time in the path considered. In Fig. 208, for the instant shown, coils 1 and 4 are in series to form one path through the armature winding, and coils 2 and 3 are in series to form the second path; the two paths are obviously in parallel.

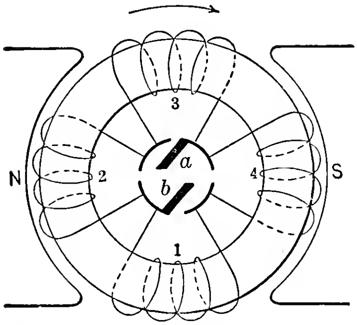


FIG. 208.—A four-coil winding would require a four-part commutator, and there would be two coils in series in each path of the armature winding.

177. E.M.F. Form of a Multi-coil Armature.—The e.m.fs. of the different coils are shown in Fig. 209 (*a*), the curves being numbered to correspond with the coils. Thus at time = 0 (Fig. 208) coils 1 and 3 are generating zero voltage, while coil 2, moving under the north pole, and coil 4, moving under the south pole, generate voltages which are in the same direction with respect to the external circuit. An eighth of a revolution later, as indicated in Fig. 210, coil 1 moves under the north pole and coil 3 moves under the south pole. It will now be seen that coils 1 and 2 are both generating an e.m.f. in the same direction and that they are in series to form one path

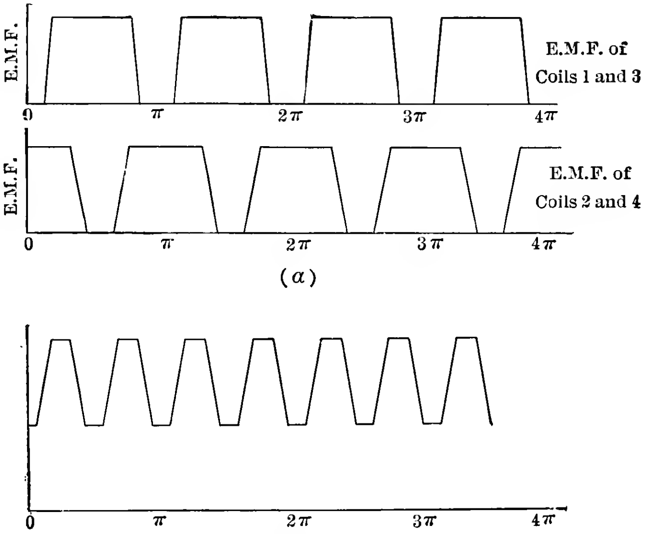


FIG. 209.—The four-coil, four-part commutator, armature gives on the external circuit a voltage more nearly uniform than that delivered by the two-coil armature.

between brushes, the line voltage being the sum of the two coil voltages.

The e.m.f. generated by the other half of the armature winding is obtained by considering coils 3 and 4, of Fig. 210. It will be found that they give, between the brushes, an e.m.f. exactly equal to that given by coils 1 and 2 and in the *same direction with respect to the external circuit.* The line e.m.f., obtained by adding the coil e.m.fs., is shown in Fig. 209 (*b*). It is seen that although the line e.m.f. is not yet regular, it never goes to zero value as with the one coil winding (Fig. 205) and that the fluctuation is only 50 per cent of the maximum value.

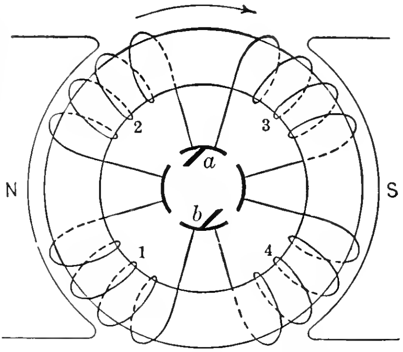


Fig. 210.—At the time shown in Fig. 208, only one coil is generating voltage in either path of the winding, the other coil in each path being in the inter-polar space and so generating no voltage; after the armature has rotated a little, as shown here, both coils in either path are active in generating voltage.

By considering in the same way an 8-coil armature with an eight-part commutator, it will be found that, as the number of coils is increased, the pulsation in the line e.m.f. continually decreases, and, on an ordinary commercial machine having perhaps 20 coils between brushes, the variation is scarcely perceptible and can only be detected by some sensitive instrument like the telephone. Besides getting smaller in magnitude as the number of coils is increased, the fluctuations of e.m.f. also increase in rapidity until on commercial generators they are in the neighborhood of 1000 or more per second.

We see, then, that *a multiple-coil armature, equipped with* a *multiple-part commutator, will produce a uni-directional line e.m.f. of practically constant magnitude;* the machine is therefore called a *continuous-current* or *direct-current generator.* The e.m.f. in the individual coils is alternating in direction; the commutator cannot change this, but it does so change the connection of the coil to the line that what is, in the coil, an alternating e.m.f. becomes uni-directional on the line.

178. Method for Calculating the E.M.F. of a Generator.—In determining the e.m.f. of a generator it is only necessary to calculate the rate at which flux is being cut by all the conductors connected in series in one path of the winding. We shall use the term *active inductors* to indicate those conductors on the armature which are cutting flux at the time considered. Evidently all the inductors on an armature are not active, because while some of them lie under the pole face, generating an e.m.f., others must be situated in the interpolar space where there is no flux to be cut and hence they can generate no e.m.f. Generally, 60 to 70 per cent of the inductors of a machine are active at any one instant.

Also, we have to consider the fact that some of the active inductors lie in a weaker field than others. The field may be considered in two parts; that directly under the pole face where the field has normal density, and that near the edge of the pole or pole shoe, where the density of the field is less than normal. This part of the field is called the *pole fringe.* If the voltage of the machine is obtained by calculating the e.m.f. generated by each of the inductors in a path, and then adding these e.m.fs. together, it is necessary to find the voltage generated, (*a*) by inductors under the pole face and, (*b*) by inductors in the pole fringe, and then to add the two voltages so obtained.

179. Example of E.M.F. Calculation for a Bipolar Generator.—We shall first consider a bipolar machine wound with 44 coils of 8 turns each of No. 13 wire. The total length per turn is 2 feet, of which 1 foot is active. Assume that the peripheral speed of the armature is 3000 feet per minute, that 60 per cent of the inductors are active (i.e., lie under the pole face at the same time), and that the flux density in the air gap is 10,000 lines per square centimeter, the pole fringe not considered. It is desired to find: (1) the generated e.m.f.; (2) the safe current capacity; (3) the armature resistance; (4) the full load terminal voltage.

The first thing to determine is the *active length of inductor per path.* This must be a two-path winding, hence we have:

Coils per path = 22;

Turns per path = 176;

Length of inductor per path = $176 \times 1 = 176$ ft.;

As the flux density is given in lines per square centimeter we shall work in the metric system;

Length of inductor per path = 5370 cm.;

Active length of inductor per path

$= 5370 \times 60$ per cent $= 3220$ cm.;

The velocity of the inductors is 3000 feet per minute

$= 1525$ cm. per sec.;

Flux density = 10,000 lines per sq. cm.;

Flux cut per second $= 3220 \times 1525 \times 10{,}000$

$= 491 \times 10^8$ lines.

Hence the generated voltage per path=491 volts, and this is the generated e.m.f. of the machine.

A formula in the derivation of which the voltage is considered from the standpoint of *average voltage* of an inductor, rather than its instantaneous value, is easier to apply than the method suggested in section 178; in this method the electromotive force of a generator is derived as follows:

Let Z = total number of conductors on armature;
p = number of field poles;
m = number of parallel paths in armature winding;
ϕ = total flux per pole, entering armature;
N = revolutions per minute of armature.

There are evidently Z/m conductors in series, per path. Then flux cut by one conductor in one revolution $= p\phi$.

$$\text{Flux cut per second by one conductor} = \frac{p\phi N}{60}.$$

$$\text{Average voltage generated by one conductor} = \frac{p\phi N}{60 \times 10^8}. \quad . \quad (100)$$

$$\text{Voltage generated per path} = \frac{p\phi N}{60 \times 10^8} \times \frac{Z}{m}.$$

As the voltage per path is the voltage of the machine we have the formula:

$$\text{E.m.f. of generator} = \frac{p\phi NZ}{m \times 6 \times 10^9} \text{ volts.} \quad . \quad . \quad . \quad (101)$$

In many cases the data for a problem are given in such a way that this formula is more convenient in obtaining an answer than the method used in the previous problem.

180. Allowable Current.—In a well-ventilated armature it is safe to allow one ampere per 600 circular mils cross-section of the armature conductor. As a No. 13 wire is 5178 cir. mils in section, the safe current to allow is about 8.63 amperes per path. As there are two paths in parallel in this armature, and as each can safely carry 8.63 amperes, the armature can carry about 17.3 amperes. If we had supposed a poorly ventilated armature and allowed 900 circular mils per ampere, the safe current would have been 11.6 amperes.

181. Calculation of Armature Resistance.—The resistance of the armature is obtained by calculating the resistance per path and then dividing by the number of paths in parallel in the armature. In the machine above there are in each path 22 coils of 8 turns each and each turn is 2 feet long. The length of wire per path is therefore 352 feet. The resistance of 352 feet of No. 13 wire (at 50° C.) $=0.786$ ohm. As there are two paths in parallel the armature resistance is $\frac{0.786}{2}$ or 0.393 ohm.

182. Calculation of Terminal Voltage.—The full load IR drop in the windings is therefore

$$0.393\times 17.3=6.8 \text{ volts (say 7 volts).}$$

The drop at the brush contacts (carbon brushes) assumed

$$=2 \text{ volts.}$$

Therefore the total drop in the armature with full load current

$$=7+2=9 \text{ volts.}$$

The full-load terminal voltage (voltage at brushes)

$$=491 \text{ volts (generated)} - 9 \text{ volts } (IR \text{ drop})$$
$$=482 \text{ volts.}$$

183. Calculations for a Multipolar Generator.—Next consider a 12-pole, lap-wound generator having 240 coils on the armature, 4 turns per coil, each turn consisting of two No. 8 wires in parallel. The length per turn is 4 feet and the length of inductor per turn is 20 inches. The armature is 4 feet in diameter and makes 200 r.p.m. Fifty per cent of the inductors lie in a field of 9000 lines per square centimeter, and 20 per cent of them lie in the pole fringe where the average flux density is 5000 lines per square centimeter. Find the same quantities as in previous problem.

To calculate the generated e.m.f., the method of procedure is to find: (1) the length of inductor in the denser field (A) and calculate the e.m.f. generated by it; (2) the length of inductor in the weak field (B) and calculate the e.m.f. generated by it. The sum of the voltages generated by inductors (A) and (B) gives the total generated voltage.

As the machine is lap wound and has 12 poles, there must be 20 coils per path.

The length of inductor per path

$$=20\times 4\times 20\times 2.54=4070 \text{ cm.;}$$

The length of inductor in the dense field

$$=4070\times 50 \text{ per cent}=2035 \text{ cm. } (A);$$

The length of inductor in the pole fringe

$$=4070\times 20 \text{ per cent}=814 \text{ cm. } (B);$$

The peripheral speed $=\dfrac{4\times 200\times 2.54\times 12\times \pi}{60}=1275$ cm./sec.

E.m.f. generated by inductors (A)

$$=1275\times 2035\times 9000\times 10^{-8}=233 \text{ volts.}$$

E.m.f. generated by inductors (B)

$$=1275\times 814\times 5000\times 10^{-8} = 52 \text{ volts.}$$

Total e.m.f. per path $=285$ volts

The conductor of which the winding is formed is a double No. 8, and so has a cross-section of 33,020 circular mils.

Allowing 600 circular mils per ampere gives a capacity per path of 55 amperes. As there are 12 paths in parallel and each can carry 55 amperes, the whole armature has a capacity of $12\times 55=660$ amperes. The length per turn is 4 feet, therefore the length per coil is 16 feet. There are 20 coils in series in one path, therefore the length per path = 320 feet.

The resistance of 320 feet of No. 8 wire is (at 50° C.) 0.224 ohm.
The resistance of 320 feet of double No. 8 is therefore 0.112 ohm.
Hence the armature resistance is $0.112/12=0.00934$ ohm;

$$\text{Full load } IR \text{ drop}=660\times .00934=6.16 \text{ volts.}$$

Allowing in this case 3 volts for the drop at the brush contacts gives a full load IR drop in the armature of about 9 volts, so that the full load terminal voltage $=285-9=276$ volts.

In these two sample problems the data were not taken from actual machines. The voltages obtained are not those of commercial machines; certain voltages have been more or less standardized, by usage, for certain classes of service. The voltages ordinarily used are 110–125 and 220–250 volts for lighting generators, and for railway generators 550–600 volts. Very special machines may be built for voltages as high as 10,000 volts, but these are seldom used. Generators intended for electroplating are generally built for 10 volts or less.

184. Commutation.—The function of the commutator and its construction have been taken up in previous paragraphs, and mention has been made of the fact that there may be sparking at the contact surface of the brush and commutator. The reasons for this sparking will now be given, the superiority of the carbon brush over the copper brush will be shown, and the purpose of the commutating poles will be discussed.

The direction of the current through any coil is reversed as the commutator bars, to which the coil ends are attached, move under the brush. This must be so, because before the coil reaches the brush, as at (*A*) Fig. 211, the current is in one direction and when the coil is at (*B*) the current is in the opposite direction in the coil. This reversal of current takes place just as the coil is short-circuited by the brush, as in position *C*, Fig. 211. The reversing of the direction of current in the coil as the coil moves under the brush is called commutation. If this reversal takes place with no visible sparking at the brush contact, the machine is said to have black, or sparkless, commutation.

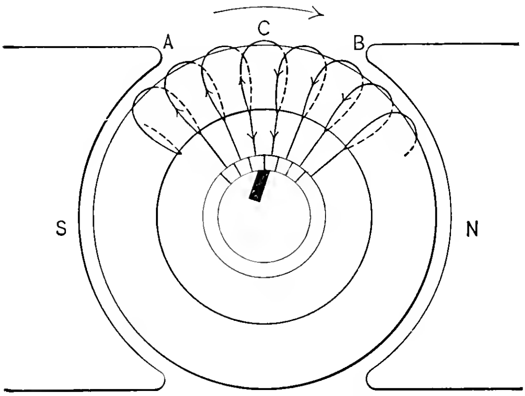


FIG. 211.—By inspection of this diagram, giving the direction of current flow for coils on both sides of the brush, it is evident that the current in any one coil must reverse its direction as it passes the brush.

185. Effect of Self-induction on Commutation.—The time in which this reversal of current has to take place is very short, so that *the rate of change of current* as the coil is commutated is very high, especially when the machine is carrying much load. As the coil possesses self-induction, this rapid reversal of current sets up an e.m.f. of self-induction which tends to maintain the current in its original direction. When this e.m.f. of self-induction is high it is almost impossible to obtain sparkless commutation, and because of this fact armature coils must be made with low self-induction. The low coefficient of self-induction is obtained by winding but few turns per coil and by placing the coil in an open or semi-closed slot. An armature wound in closed slots would not commutate well at all, because of the high self-induction of its coils.

186. Rate of Current Change During Commutation.—The time during which the coil is short-circuited is easily calculated. Suppose a commutator having 90 segments is making 1800 r.p.m. and that the brush has a width equal to two commutator bars. Any two adjacent bars have a coil connected between them, so that the coil is short-circuited during the time required for the mica insulation between the two bars to move the width of the brush contact.

In the above case, this is equal to $2/90 \times 60/1800 = 1/1350$ of a second. If the armature is carrying 10 amperes per path, the rate of change of the current during commutation is about 27,000 amperes per second. The e.m.f. of self-induction is equal to the coefficient of self-induction for the coil, L, multiplied by the rate of change of current, and as this latter term is so high, L must necessarily be kept low. For a given brush width and commutator speed, the rate of current change during commutation increases directly with the current the armature is carrying. This rate is very large on big machines, and therefore the coefficient of self-induction must be kept correspondingly low. It will be noticed that on large armatures the coils have very few turns; sometimes only one turn per coil is used.

187. Methods for Getting Sparkless Commutation.—As the coil moves under the brush, the *variation of the contact resistance* between the carbon brush and the two segments to which the coil is attached, tends to make the current in the coil reverse. In low-voltage, slow-speed machines this effect can be made great enough to produce sparkless commutation, so that it is not necessary to introduce into the short-circuited coil an e.m.f. to overcome that of self-induction. Commutation which depends only upon this resistance effect to eliminate sparking is sometimes called *resistance commutation.* If some means is employed to generate in the short-circuited coil an e.m.f. equal and opposite to that of self-induction, the machine is said to have *e.m.f. commutation.* Of course, even if a machine employs e.m.f. commutation, since carbon brushes are always used, the resistance effect is also present, helping the e.m.f. effect.

188. Resistance Commutation.—The idea of resistance commutation may be understood by studying the variation of the contact resistance as the commutator bars move under a brush. Consider only a few coils of the armature shown in Fig. 211; the coil undergoing commutation and a few on each side of it are shown in Fig. 212, where B is the coil about to be commutated.

Position (1) shows coil B before it begins to be commutated, when all the current, i, flowing through the left side of the armature has to go through coil B and lead c to reach the brush and so the external circuit. When the brush is in position 2, this current from the left side of the armature has *two paths* by which to reach the external circuit; through coil B and lead c as before, or else not through coil B at all but directly through the lead a. The division of the current between these two paths depends upon their relative resistances, and in the case of carbon brushes *practically all the resistance of either path is in the brush contact.* In position 2, three-quarters of the brush is represented as in contact with segment g, and one-quarter in contact with segment f. If brush-contact resistance is assumed as varying inversely as area of contact, we should expect the current flowing from segment g to the brush to be three times that flowing from segment f to the brush. All the current, i, from the right side of the armature must flow down lead d, so that the current from the left side must divide, one-half, i 2, passing directly to the brush through segment *f* and the other half $i/2$, continuing, as before, through coil *B* and lead *c* and flowing to the brush through segment *g*.

As the armature continues to move, it will be seen that the contact area between the brush and segment *f* increases and that between the brush and segment *g* decreases. In position 3 they are represented as equal, and it would be expected that all the current from the left side of the armature would enter the brush through segment *f*, and that from the right side through segment *g*. Accordingly, there would be no current through coil *B*.

In position 4, the contact area between the brush and segment *f* is represented as three times that between the brush and segment *g*. If, therefore, the current flowing to the brush from segment *f* is to be three times that from segment *g*, it is necessary that one-half the current, $i/2$, from the right side of the armature flow to the brush through lead *d* and segment *g* and the other half, $i/2$, flow through coil *B* and down lead *b*, uniting with the current, *i*, from the left side in segment *f*.

As the armature continues to move, the resistance of the path through segment *g* becomes higher as the area of brush contact on this segment becomes smaller, so that more and more of the current from the right side of the armature winding comes through leads *d* and *c*, through coil *B*, and then through *b* and *f* to the brush. Finally, in

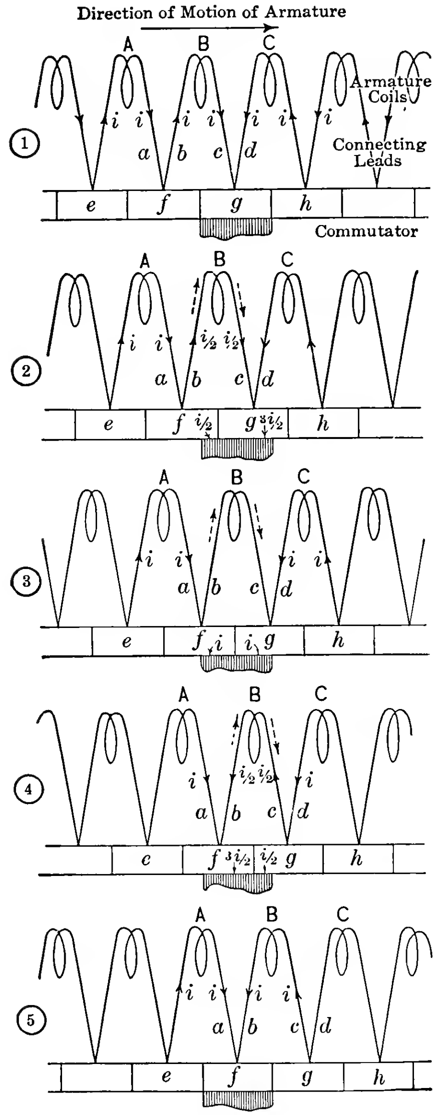


FIG. 212.—A detailed study of the conditions which must exist in any one coil, during the time it is short-circuited by the brush; the current must die down to zero and build up to an equal value in the opposite direction during the short-circuit period.

position 5, the brush and segment separate, and all of the current from the right side of the armature must pass through coil *B* to get to the brush.

Thus we see that the variation of brush-contact resistance first tends to stop the original current through B (positions 1 *to* 3) *and then tends to build it up in the opposite direction (positions* 3 *to* 5).

189. Current During Commutation.—If this resistance commutation is to work successfully, the current through *B* must have reversed and built up to the same value of current as that flowing in coil *C*, *before segment g leaves the brush.* If this condition is not satisfied, there will be more or less sparking. The variation of current in coil *B*, as it is being commutated, may be well shown by a curve, as in Fig. 213. If the current is 10 amperes in each side of the armature, and the time during which the coil is short-circuited is 0.001 second, and if the current in *B* completely reverses during this time, the current curve has the form shown by the full lines in Fig. 213.

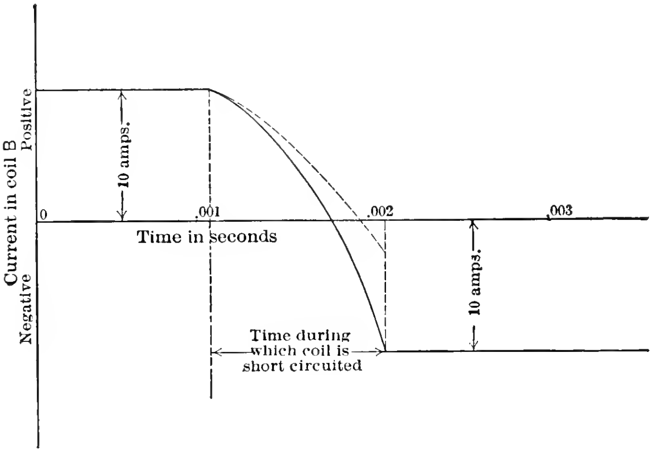


FIG. 213.—If, during the short-circuit period, here shown as 0.001 second, the current follows the full-line curve, the commutation will be satisfactory; no sparking at the brushes will occur. If the current follows the dotted line curve, sparking will occur at the trailing tip of the brush.

As was said before, an armature coil possesses some self-induction. During the first part of the commutation of coil *B*, when the current through it decreases, a counter e.m.f. must be generated because of its self-induction, which has the same direction as the decreasing current, since the c.e.m.f. tends to oppose the decrease. This c.e.m.f. is represented in Fig. 212 by the dotted arrows. When the current through *B* increases in the opposite direction, the c.e.m.f. of self-induction acts to oppose the increase of current. Its direction must then be opposite to the growing current, or the same as it was when the current was decreasing. In other words, *the direction of the c.e.m.f. of self-induction is always the same during commutation, provided the current always changes in the same direction.*

The effect of the c.e.m.f. of self-induction must then be to delay the reversal of the current in the coil being commutated. In position 3 of Fig. 212, for example, some current will still be flowing through coil B from left to right; it will not have reached zero as was supposed when discussing that figure.

Now suppose that the effect of varying the brush contact resistance is not great enough to reverse completely the current in B during 0.001 second. The current curve then might have the form shown by the dotted curve of Fig. 213. In this case, at the time when segment g leaves the brush, the current B is only 2 amperes (negative); it should be 10 amperes (negative). But if the current flowing in the right-hand part of the armature cannot reach the external circuit through segment g, *it must go through coil B,* and this means that at the instant the brush and segment g separate, the current in B must suddenly change from 2 amperes to 10 amperes. The rate of change of current in B is very large at this instant and so a large counter e.m.f. of self-induction is set up in B.

190. Cause of Sparking.—The current from C has then two possible paths; it may force its way through B against the high counter e.m.f. of B, or it may form an arc over the mica insulation and get to the brush without going through coil B. This condition is indicated in Fig. 214. The formation of this small arc depends upon the high counter e.m.f. of coil B and this in turn depends upon the rapid change in current through B when segment g leaves the brush. This excessively rapid change of current is necessary because during the time of short circuit the resistance variation has not been sufficient to reverse completely the current in B (dotted line, Fig. 213).

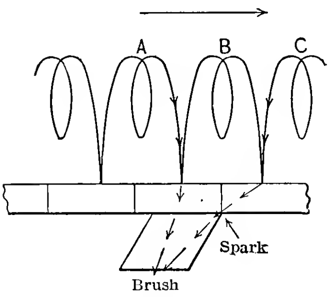


FIG. 214. — The sparking which occurs as indicated here, rapidly consumes the tip of the brush and also burns the commutator, causing a rough, blackened surface, which serves to augment the sparking and so make conditions worse.

The practical result of sparking at the commutator is to cause wearing of the receding edges of the commutator bars and the trailing brush tips. This is represented in exaggerated form in Fig. 215. It will be seen that this wear of the trailing brush tip materially decreases the area of brush contact and so tends to produce overheating, due to excessive current density, as well as due to the sparking itself; it also amounts practically to a slight shift of the brushes.

191. Condition for Sparkless Commutation.—If the current in B was completely reversed during the short-circuit interval (as in the full line of Fig. 213) there would be no change of current in B, as segment g left the brush, and hence no counter e.m.f. of self-induction to overcome. In this case there would be no tendency of the current from C to arc over the commutator, as segment g left the brush, and there would be no sparking. In fact, *commutation is always sparkless when the current in the coil undergoing commutation is completely reversed during the time the coil is short-circuited by the brush.*

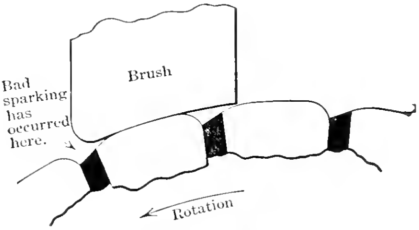


FIG. 215.—Bad sparking at the brush causes the commutator and brush to assume this shape. Really the surface of the commutator should be a smooth cylinder, and the brush should be in intimate contact with this cylinder over its whole cross-section.

192. E.M.F. Commutation.—E.m.f. commutation is used on nearly all modern machines. With high commutator speeds the resistance variation is never sufficient to reverse completely the current in the coil short-circuited. If, as was mentioned before, some means were employed to

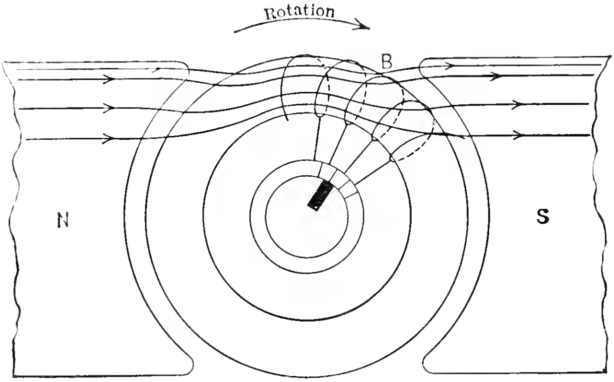


FIG. 216.—By advancing the brush the short-circuited coil may be caused to lie on the pole fringe.

generate, in the short-circuited coil, an e.m.f. equal and opposite to the the c.e.m.f. of self-induction, then resistance commutation could be depended on to accomplish the desired reversal of the current. This generated e.m.f., which we shall call the *commutation e.m.f.*, may be induced by the pole fringe or by separate poles intended especially for this purpose, called *commutating poles*. If the pole fringe is employed, *the brushes are advanced* on the commutator so that the short-circuited coil lies in the edge of the magnetic field under the leading pole tips, as shown in Fig. 216. (In case the machine is a motor and not a generator, this *brush shift must be backward,* not forward, as in Fig. 216).

In Fig. 212 commutation for a generator was considered, and it was shown that the direction of the c.e.m.f. of self-induction was opposite to the final direction of the current in the short-circuited coil *B*, that is, opposite to the direction of the e.m.f. generated by the coils to the right of coil *B* which are moving under the pole. So by moving the brushes forward a proper amount, as in Fig. 216, causing the short-circuited coil *B* to move in the pole fringe, we can generate a voltage in the coil during commutation, which, if equal and opposite to the c.e.m.f. of self-induction, should cause the current in the short-circuited coil to reverse according to the full line curve of Fig. 213.

193. Current Form in Short-circuited Coil.—The action of the commutation e.m.f., in assisting the current to reverse in the short-circuited coil, may well be considered from another viewpoint.

Suppose that in Fig. 216, with the brushes shifted as indicated, no current is drawn from the generator. With no current to be commutated, no c.e.m.f. of self-induction will exist in the short-circuited coil. Therefore, the commutation e.m.f., or voltage induced in coil *B* while moving within the pole fringe, will tend to set up a current, which, as indicated in Fig. 217, flows through the coil *B*, down lead *b*, through *f*, the brush, segment *g*, and up *c*. As the coil *B*, during the period of commutation, is continually moving into a stronger and stronger field, this current increases in magnitude as shown in Fig. 218 by the line *DE*. At the end of the short-circuit period, i.e., when the brush and bar *g* (Fig. 217) separate, this current is ruptured, resulting in a spark.

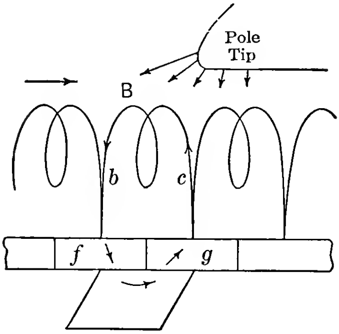


Fig. 217.—If we imagine an armature rotating, carrying no load current, with the brush so advanced that the short-circuited coil lies in the pole fringe, a current will flow through the coil, commutator bars, and brush, as shown here.

If current is now taken from the generator with its brushes shifted forward, as in Fig. 216, we may consider the actual current as the resultant of two components, the sum of which at any instant represents the actual current flowing at that time. One component will be the decaying current, shown in Fig. 213 and the other, the current produced in the short-circuited coil by the e.m.f. induced in it by the pole fringe, as was shown in Fig. 218. It is supposed that resistance commutation is not sufficient and that the current in *B* at the end of the short-circuit period is shown at *A*, Fig. 219, whereas it should be at *B*. The current produced in the short-circuited coil by the induced e.m.f. is shown by the line *DE*, and the total current in the short-circuited coil is the sum of the two component currents, *CA* and *DE*, shown by *CFB*. This is evidently the current required for sparkless commutation.

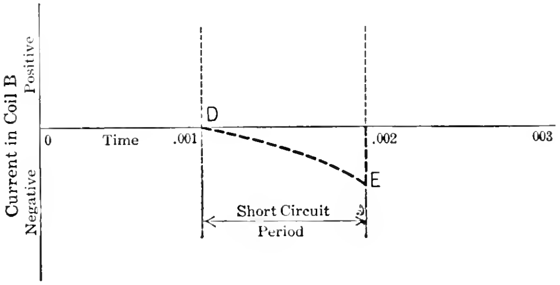


FIG. 218.—The form of the short-circuit current set up in the coil as indicated in Fig. 217, will be about as shown in the heavy dashed curve above.

If the load on the machine changes so that the current per path is 20 amperes instead of 10 amperes, the e.m.f. required for sparkless commutation is much greater than before. If the short-circuited coil still lies in the same field strength as before, the current in *B* during short circuit

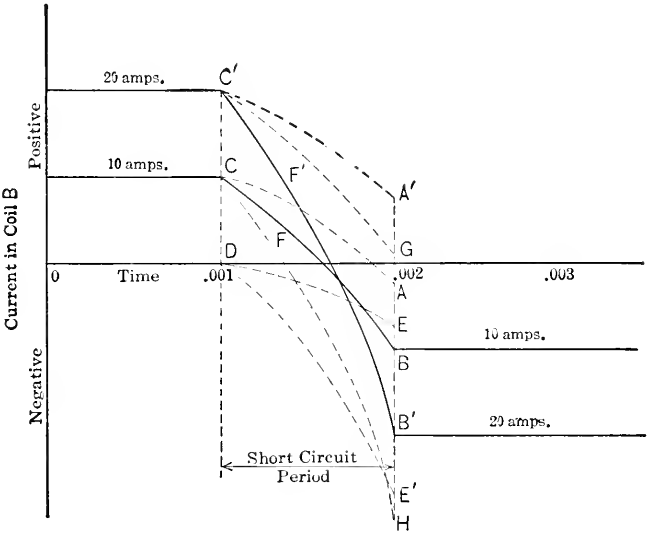


FIG. 219.—The position of the coil in the pole fringe, which gives successful commutation for one load, will not be suitable for another; the strength of field in which the short-circuited coil lies, should vary in proportion to the load carried by the machine.

follows the curve $C'G$. The resistance variation, as explained above, gives the current $C'A'$ and the addition of the short-circuit current, DE (the same as before because the e.m.f. producing it is the same) gives the current $C'G$. At the end of the short-circuit period the current in coil B should be at B' (negative), and it really is at G (positive).

194. Necessity of Shifting the Brushes with Load Variation.—If the brush is advanced more, so that coil B is in a denser field than before, a greater e.m.f. is induced in it while it is short-circuited and the short-circuit current becomes DE'. Now DE' added to $C'A'$ gives the current $C'F'B'$, which is just right for sparkless commutation.

If the load on the generator again changes, so that the current per path becomes 10 amperes as before, with the brushes shifted to give sparkless commutation for a current per path of 20 amperes, the current in B, during short circuit (in so far as resistance commutation is concerned) follows the curve CA. The addition of the current DE', produced by the the commutation e.m.f., gives the current CH, or a final value of about 30 amperes. But when the short-circuit period is over, the current in the coil must be 10 amperes; it is evident that this sudden change of current from 30 to 10 amperes will produce sparking at the commutator, so that the brushes must again be shifted backward.

When the conditions are such that the c.e.m.f. of self-induction is not opposed, or only partially neutralized, by a commutation e.m.f., the commutation is frequently referred to as "*under-commutation*"; such conditions are indicated in Fig. 219 when the current in the short-circuited coil follows curves CA or $C'G$. When the value of the commutation e.m.f. is greater than that required to neutralize the c.e.m.f. of self-induction, so that the current in the short-circuited coil "overshoots" the required final value, the conditions may be referred to as "*over-commutation.*" This condition was indicated in Fig. 219 by the curve CH.

In Fig. 220 are given three oscillograms of current in one coil of a generator, under full-load, light-load, and no-load conditions. The brushes were advanced sufficiently to produce sparkless commutation at full load, and left in this position for the other loads. It will be seen that the e.m.f. induced in the short-circuited coil is just right to completely reverse the current at full load but that at light load the current " overshoots " and at no load there is a large current (almost equal to full load current) in the coil while it is short-circuited by the brush. For light load and no load vicious sparking occurred at the brush contact.

It follows, then, that by means of *brush shifting,* sparkless commutation may be obtained; but the amount of shift required varies with the load, so that if brush shifting is used the operator has to change the position of the brushes (by moving the brush-holder yoke) as the load changes. The amount of brush shift necessary depends upon the design of the machine.

A modern well-designed machine operates fairly well with no brush shift at all. The brushes are placed somewhere between the proper no-load position and proper full-load position and are left there no matter what the load may be.

195. Use of Commutating Poles.—When the brushes of a machine are shifted, the short-circuited coil is moved to find its proper commutating flux in the pole fringe. When commutating poles are used, brush shifting is unnecessary, the proper commutating flux being brought to the short-circuited coil. Small poles, wound with a *series winding,* are placed midway between the main poles, and the flux produced by them acts in the same way as does the pole fringe just discussed. The brushes short-circuit the coil lying directly under the commutating pole and are left fixed in this position; the brushes on a commutating pole machine must never be shifted after correct adjustment has once been made.

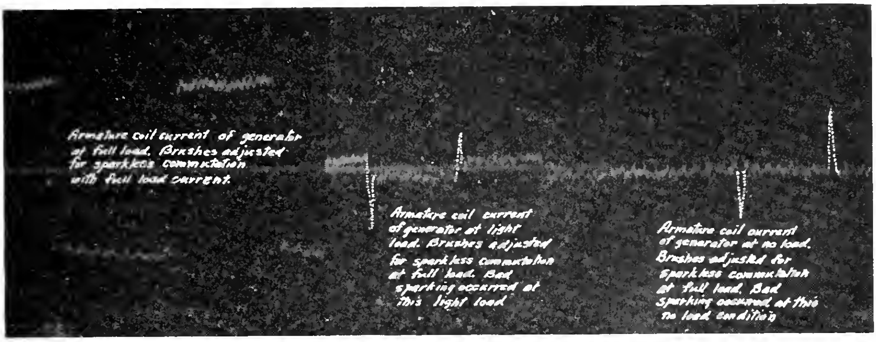


FIG. 220.—An oscillogram of the current in one coil of a bipolar generator; the brush position was advanced sufficiently to give sparkless commutation at full load and left in that position. It will be seen that the e.m.f. induced in the short-circuited coil gave a current just sufficient to commutate full load properly, but that this commutation current was too great for light load, and of course, correspondingly worse for no load.

To determine the proper polarity for a commutating-pole, it is only necessary to remember that in a generator without such poles, the brushes are shifted forward to place the short-circuited coil in the fringe of the flux from the next pole. As the commutating pole is to produce the same result, the polarity of a commutating pole must be the same as that of the next pole, in the direction the armature is moving. (As explained later, in a motor the brushes are shifted backwards as load comes on, therefore the polarity of a commutating pole is the same as that of the main pole just preceding it, in the direction of rotation.)

As these poles are equipped with a series winding (shown in Fig. 221) the *strength of field produced by them* (and hence the magnitude of the e.m.f.

induced in the short-circuited coil) is proportional to the load; the magnitude of the short-circuit current will then be proportional to the load, and, as was shown in the discussion of Fig. 219, this is the necessary condition for maintaining " black " commutation as the load varies. The use of commutating poles (sometimes called *interpoles*) has become almost universal on c.c. motors and generators, especially on *adjustable-speed motors* in which the speed variation is obtained by field weakening. Without the use of commutating poles, the adjustable-speed motors could not be built to operate satisfactorily.

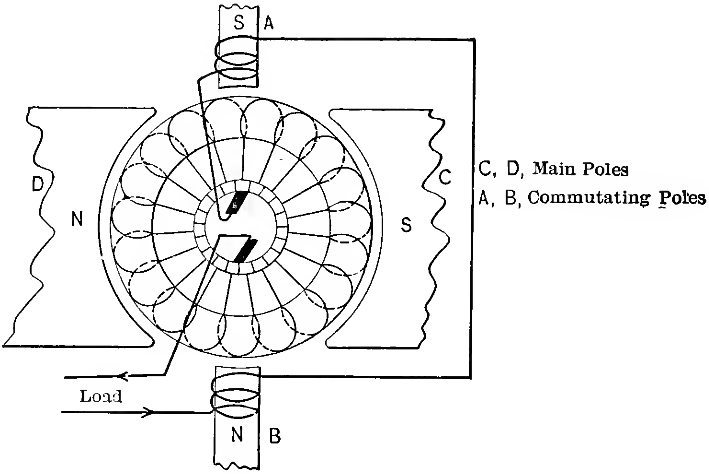


FIG. 221.—In practically all modern machines commutating poles are used; the windings of these poles, being in series with the load, automatically adjust the strength of the commutating field in accordance with the load.

The field frame of a machine equipped with commutating poles is shown in Fig. 144, page 153. The armature has been removed to permit a clear view of the field construction. The commutating poles are made very narrow; generally they reach over only two teeth (and the included slot) of the armature core.

When full-pitch armature windings are used, only one commutating pole per pair of main poles is required, the entire commutating e.m.f. per coil *being induced in one coil-side*. This is done, however, only in the case of small cheap machines; most machines have as many commutating poles as main poles.

196. Armature Reaction.—When the armature of a dynamo-electric machine is carrying current, the armature acts like an electromagnet. The current in the armature conductors sets up a m.m.f. independent of the main field windings *which alters both the distribution and the magnitude* *of the field which would be produced by the field windings alone;* this action of the m.m.f. set up by the armature windings is called *armature reaction.* It may be a distorting action only, in which case the magnetic field of the machine is not directly strengthened or weakened by the armature reaction, but is merely twisted out of its normal position. In other cases the armature m.m.f. may not only distort the main field but may either magnetize (strengthen) it or demagnetize (weaken) it.

When the main field is both twisted out of its normal position and either strengthened or weakened, the action of the armature m.m.f. may be resolved into two component actions, a *distorting* or *cross-magnetizing action* which causes the twisting of the main field, and a *magnetizing* or *demagnetizing action* as the case may be, which alters the magnitude of he main field.

197. Distortion of Field by Armature M.M.F.—In Fig. 222 is shown the field produced by the main field coils alone. This depends upon the excitation due to the field winding and upon the shape and materials of the magnetic circuit. If the air gap is of uniform length, the flux density in the air gap will also be uniform.

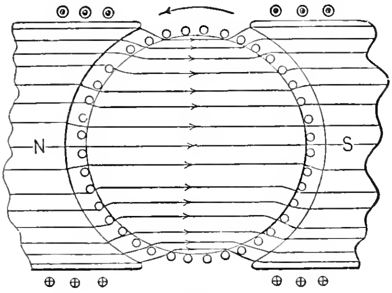


FIG. 222

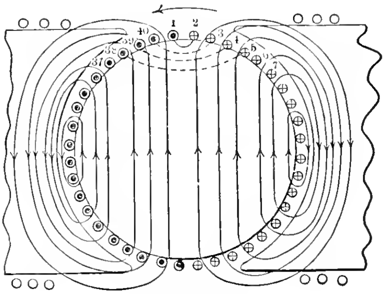


FIG. 223

FIG. 222.—The field windings of a bipolar machine tend to give a symmetrical field of practically uniform strength.

FIG. 223.—The armature windings, when carrying current, tend to give a fairly uniform field, perpendicular to the direction of the main field.

In Fig. 223 the main field excitation has been removed and current is being passed through the armature from an external source in the same direction as it would have if the generator were supplying current to a load of some kind, resulting in a magnetic field distribution approximately as shown. In this figure a plus sign (tail-end of an arrow) in the conductor cross-section, signifies that the current is flowing away from the observer, and those conductors with a dot (head-end of an arrow) are carrying current toward the observer.

That the armature winding does give such a field as is shown in Fig. 223 may be seen by supposing conductors 1 and 2 to form one turn of a solenoid; conductors 40 and 3 to form another turn, etc. Thus, each conductor may be imagined as connected with one on the opposite side of the armature, and the turns so formed all give an m.m.f. in the direction shown by the field in Fig. 223.

When the generator is supplying current, the armature and field m.m.fs. exist simultaneously, and the resultant field distribution will be the resultant of these two m.m.fs. We may, however, consider the resultant field distribution as approximately the combination of the two fields shown separately in Figs. 222 and 223, as indicated in Fig. 224. That the main field is twisted *in the direction of rotation of the armature,* as shown in Fig. 224, may be determined by noting in Figs. 222 and 223, that under the upper north and lower south pole tips, the main and armature fields are in opposite directions, so that we expect a weaker field in these regions than when the armature is not carrying current. Under the lower north and upper south pole tips, the two fields are acting in the same direction, so that a stronger field should exist under these pole tips than in Fig. 222.

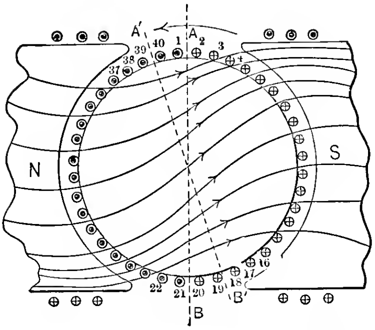


Fig. 224.—When both armature ampere-turns and field ampere-turns are acting together, the resultant field produced is an asymmetrical one; it seems to be twisted in the direction of rotation of the armature. This twisting, or skewing, of the field is due to armature reaction.

In case the machine is a motor, the armature currents in Fig. 223, for the direction of rotation assumed, would have to be in the opposite direction to that shown. As a result, the twist of the main field is in the direction opposite to that of rotation of the armature. (See Fig. 274.)

198. Distortion Proportional to Load.—As this twisting action is produced by the m.m.f of the armature windings, it must be proportional to the current flowing through the armature, i.e., to the load on the machine. At no load there is no current in the armature windings, and hence no armature reaction. The twisting effect is a maximum when the machine is carrying full load or overload.

199. Effect of Distortion on Commutating Plane.—The most important effect of this armature reaction is the shift it produces in the commutating plane. If no armature reaction were present, the coil which is short-circuited by the brushes should be in the plane *AB,* Fig. 224 (provided that the necessity of moving the short-circuited coil under the pole fringe for e.m.f. commutation is temporarily neglected). Now the coil in the plane *AB,* Fig. 224, lies in a magnetic field, due to the distortion produced by the armature reaction, and so has an e.m.f. induced in it. This e.m.f. is, moreover, in the *wrong direction* to produce sparkless commutation. The brushes must, therefore, be changed in position from the plane AB to the plane $A'B'$. If the necessity of moving the brushes under the pole fringe for e.m.f. commutation is considered, it will be seen that the plane $A'B'$ must be still further advanced, in fact, must lie under the pole tip, as in Fig. 225.

200. Effect of Brush Shift.—This change in brush position changes somewhat the current distribution in the armature conductors. Some conductors which, before the brush shift, were carrying negative current will now carry positive current, and vice versa. Figure 225 shows this new distribution of current after the brushes have been moved through the angle α; conductors 38, 39, 40, and 1, which, when the brushes were in plane AB, were carrying current towards us (·), are now carrying current away from us (+), and conductors 20, 19, 18, 17, which previously were carrying current away from us (+), now carry current towards us (·).

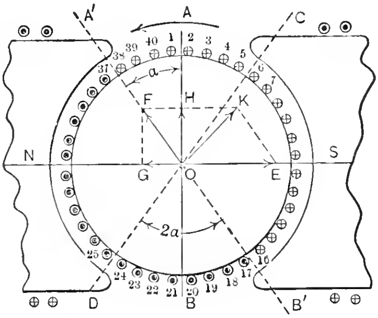


Fig. 225.—The armature ampere-turns, with brushes shifted forwards from the mid-position A–B, to position A'–B', for commutation purposes, may be divided into two groups, called the de-magnetizing turns and the cross-magnetizing turns.

201. Demagnetizing Turns and Cross-magnetizing Turns.—The armature conductors may now be considered in two groups. Considering conductors 38 and 24 as one turn, conductors 39 and 23 as another turn, etc., we find that the armature conductors in the angle $A'OC$, combined with those in the angle DOB', make up a set of turns whose m.m.f. is in direct opposition to that of the main field; these are called the *demagnetizing turns*. Those conductors included in the angle COB', combined with those in the angle $A'OD$, give a m.m.f. perpendicular to that of the main field; they constitute the *cross-magnetizing turns.*

The field m.m.f. is shown in Fig. 225 by OE, the armature m.m.f. by OF. The resultant m.m.f. (which actually produces the magnetic field through the armature) is obtained by adding OE and OF vectorially; it is shown by OK. The armature m.m.f. may be divided into its two components, OH, the cross-magnetizing force and, OG, the demagnetizing force. It is seen that this demagnetizing force is produced by all conductors in an angle equal to *twice* that through which the brushes have been shifted.

It will be evident from Fig. 225 that if the brushes of a generator were shifted backwards, the armature m.m.f. would tend to strengthen the field;

that is, the demagnetizing turns become magnetizing turns. This would, however, result in vicious sparking at the commutator, as the commutation e.m.f. generated in the short-circuited coil would be in the same direction as the c.e.m.f. of self-induction.

202. Reduction of Field Distortion.—The amount that the main field actually twists under the influence of the armature reaction depends upon the ratio of armature m.m.f. to field m.m.f., the degree of saturation of the pole tips, the length of the air gap, and other factors. If, without any distortion at all, the trailing pole tips are saturated, then the field flux cannot crowd any more into these pole tips even if the cross m.m.f. is very large. Laminated pole pieces are generally constructed with the laminations so shaped that only every other one extends into the pole tips, so that there are in the pole tip only half as many laminations as there are in the pole itself; such a construction is shown in Fig. 226. This construction tends to give saturated pole tips and consequently a field that is not easily distorted; such a field is said to be a "stiff" field.

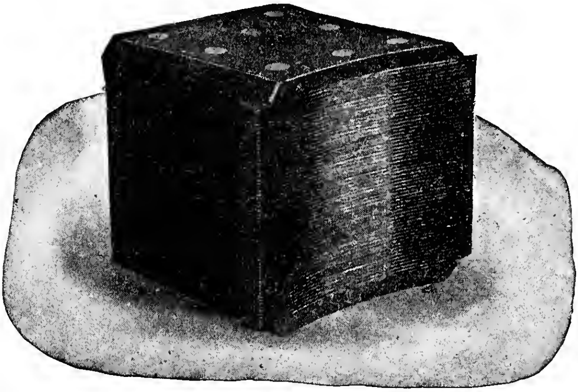


FIG. 226.—To limit the amount of twisting of the main field caused by armature reaction, the pole tips are sometimes made with half as many laminations as the main pole; this construction tends to stiffen the field, as the pole tips easily become saturated.

203. Flux Distribution Curves.—The distribution of the flux at the surface of the armature may conveniently be indicated by *flux distribution curves*, in which the surface of the armature is represented as a horizontal plane, as though the surface had been peeled off the armature and laid out flat. In Fig. 227, the flux distribution for a generator at no load is indicated; any abscissa, OX, represents the distance of a point on the armature (measured around the circumference of the armature) from a point midway between a north and a south pole, and the ordinate XY is proportional to the flux density in the air gap at the point considered. The flux leaves the armature under a south pole and enters it under a north pole. Figure 227 thus represents the same conditions as Fig. 222.

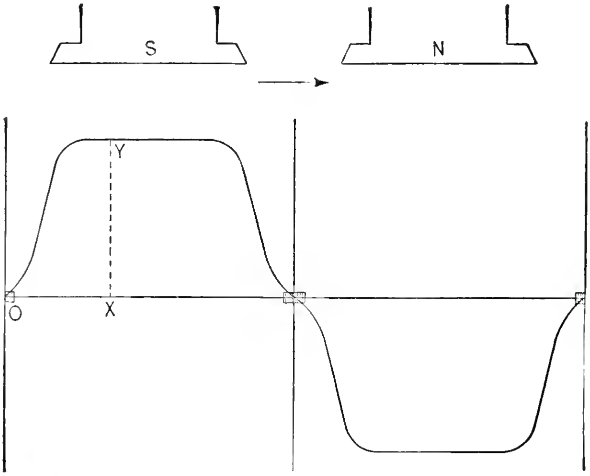


Fig. 227.—Indicating the field distribution due to the field ampere-turns acting alone.

The flux conditions existing when current flows only through the armature of a generator (no field current), with its brushes in the geometrical neutral plane AB of Fig. 224, are shown in Fig. 228. The armature conductors under the left-hand pole (south in Fig. 227) must all be carrying

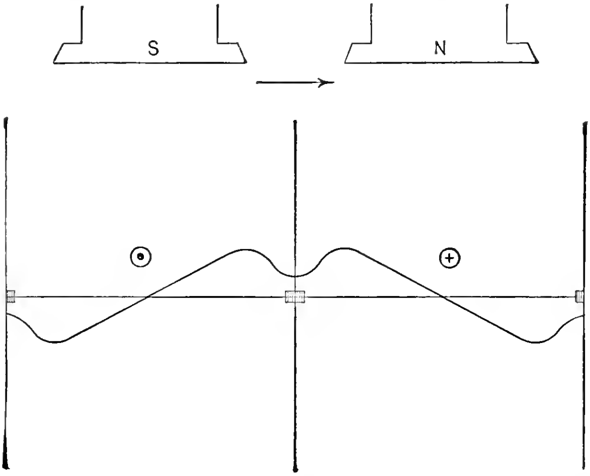


Fig. 228.—The field distribution which would be produced by the armature ampere-turns acting alone. The dip in the curve at the brush positions is caused by the comparatively high reluctance for the flux in this part of the magnetic circuit, there being a very long air gap opposite this part of the armature.

current towards us (Fleming's right-hand rule) and under the right-hand pole (north in Fig. 227) the conductors carry current away from us, producing the flux distribution as shown in Fig. 228. The hollows in the curve over the brushes are caused by the interpolar spaces, as may be seen from Fig. 223; although the flux due to armature m.m.f. is actually very small at these points, the armature m.m.f. is actually a maximum in these regions. The high reluctance of the magnetic circuit for the flux set up by the armature in these positions, accounts for the low values of flux density.

If, with the field excited, the armature carries current, the brushes still being left on the geometrical neutral, a flux distribution as shown in Fig. 229 results, being obtained by adding the curves of Figs. 227 and 228. It is to be remembered here that problems of this kind must generally be solved by super-imposing m.m.fs., getting the resultant m.m.f. and then the flux produced by this m.m.f. Fluxes can be superimposed, and the resultant thus obtained, only *when the relation between flux and m.m.f. is linear*, i.e., permeability is constant. It will be seen that the resultant flux obtained by adding the two component fluxes is distorted in the direction of rotation, that the neutral point has shifted from the position XX to the position YY. In order to generate a commutation e.m.f. of the proper direction, the brushes would have to be shifted still further in the direction of rotation, to a position ZZ, beyond YY.

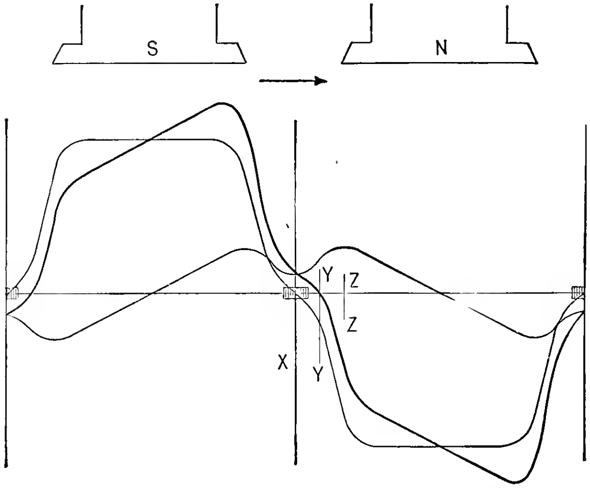


FIG. 229.—The flux distribution produced by the simultaneous action of field ampere-turns and armature ampere-turns. (This resultant curve has been obtained by merely adding the two component curves, a procedure permissible only when the permeability of the path is practically independent of the flux density.) The position of zero field is shifted ahead from X–X to Y–Y, by the effect of the armature reaction.

204. Experimental Determination of Flux Distribution Curves.— Flux distribution curves are readily determined experimentally either by means of two narrow auxiliary brushes, or by the oscillograph.

It will be noted that in Fig. 227, if any ordinate represents the flux density in the air gap, the voltage generated by a conductor on the armature at the point corresponding will also be proportional to the ordinate, provided the armature is rotating at constant speed. In one scheme for getting flux distribution, two narrow brushes, *bb*, insulated from each other and separated by a distance equal to one commutator bar, are mounted in suitable rigging, so that they may be placed in any position with respect to the main brushes, as indicated in Fig. 230. A low-reading voltmeter (with a small condenser across it to increase and steady the readings) is connected across the brushes; its reading will be proportional to the average flux density over the distance on the armature surface corresponding to the distance between the pilot brushes. By taking a series of voltmeter readings for successive positions of the pilot brushes, and plotting them against the distance of the pilot brushes from one main brush, a flux distribution curve is obtained.

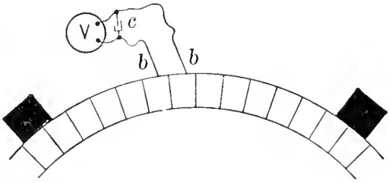


FIG. 230.—One scheme for getting the field distribution by a pair of pilot brushes, spaced from each other by the width of one commutator bar.

A set of flux distribution curves taken by means of two such pilot brushes is shown in Fig. 231. The machine tested was operated as a generator at no load and at full load, and also as a motor at full load. In each run the speed and the field current were the same, and the armature current the same in both load runs; the distortion of the field is evident.

Flux distribution curves can be determined by means of the oscillograph, by inserting a single fine wire into one of the slots and connecting it to suitable slip rings at either end of the armature. The voltage wave generated by this test wire will be a measure of the flux distribution of the machine.

It is to be noticed that this scheme, *using only one conductor* does give the flux distribution actually existing in the machine, whereas the method using the two pilot brushes does not. The brushes, *bb*, of Fig. 230, evidently connect *across one coil* of the armature; the coil has two sides, one under one pole and the other under an adjacent pole. If the coil has full pitch, the voltage on brushes *bb* will result from the average density of flux in two corresponding points under adjacent poles, and if the coil is not full pitch the curve of flux density obtained in the test is likely to be misleading. It represents the average flux density at two differently placed points, under adjacent poles.

205. Compensation of Armature Reaction.—The magnetizing effect of the armature coils may be neutralized by a compensating winding. This consists of a winding imbedded in slots in the pole faces, the winding having one-half as many turns as there are turns on the armature. These conductors in the pole faces are put in series with the armature, so that they carry the same current as the armature; but the connection is so made that every conductor in the compensating winding carries current in the opposite direction to an adjacent conductor on the armature. Figure 232 shows how these conductors are placed in the pole face, and the relative direction of the current in the armature and the compensating winding. The compensating winding is not used in the average machine because it makes a machine costly to build; it is only in high-speed, high-capacity, high-voltage generators that it is found necessary to supplement the commutating poles with the compensating winding. A modern

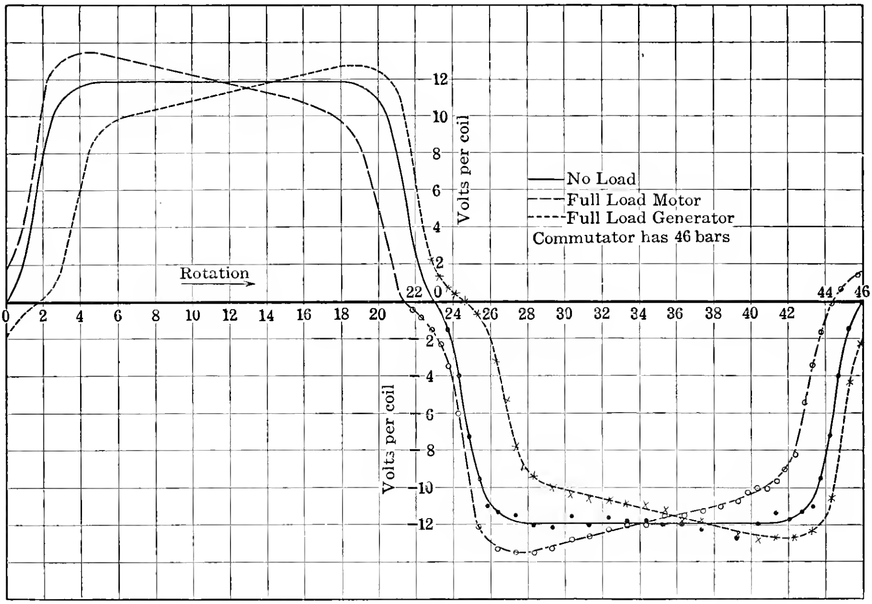


FIG. 231.—Actual field distribution of a bipolar machine obtained by the double pilot brush scheme of Fig. 230; the field had a uniform distribution when the machine was not loaded but when loaded the field was distorted, due to armature reaction. In the case of the generator the field is pulled ahead in the direction of rotation and in the case of the motor, against the direction of rotation. The average voltage of a coil is considerably higher when acting as a motor because of the reversed effect of the *IR* drop in the coil. Note the shift in commutating plane as machine changes from generator to motor.

500 kw. 1200-volt generator, equipped with a compensating winding, is shown in Fig. 193.

206. Effect of Armature Reaction on Commutating Poles.—The commutating poles must be designed with turns enough so that the armature cross-m.m.f. is neutralized *under the face of the commutating pole* and, in addition, the proper amount of flux required for sparkless commutation is forced into the armature core around the coil being commutated. The magnetic circuit of a two-pole generator equipped with commutating poles is shown in Fig. 233. It may be seen that the commutating poles do not prevent the flux from crowding into the tips of the main field poles. But this does comparatively little harm; the principal thing to obtain is the proper field for e.m.f. commutation at the two points *A* and *B*.

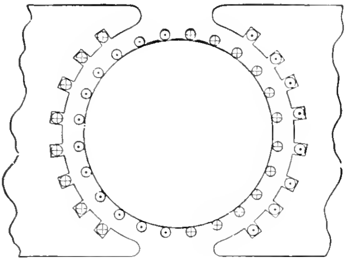


FIG. 232

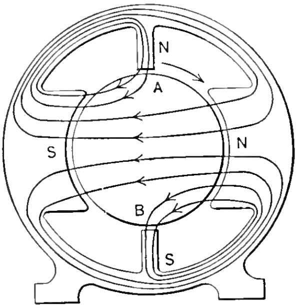


FIG. 233

FIG. 232.—To neutralize the effect of armature reaction a compensating winding, connected in series with the load, is sometimes wound in slots provided for it in the pole faces.

FIG. 233.—Approximate distribution of field flux in a bipolar, commutating pole machine. The flux density in the armature core, under the commutating poles, at full load is about one quarter that under the main poles.

207. Flux Distribution Curve with Commutating Poles.—A theoretical flux distribution curve for a generator equipped with commutating poles may be constructed in the same way as was done for Fig. 229. In Fig. 234, curves *F* and *A* represent the field and armature fluxes separately, and *C* the commutating pole flux which would exist at full load. When combined at full load these fluxes form the resultant flux curve, *R*. It will be noted that there still remains sufficient flux of the proper polarity at the geometrical neutral for e.m.f. commutation.

In Fig. 235 is given an oscillogram of the flux distribution curve of a small generator equipped with commutating poles, operated at no load; Fig. 236 represents the flux distribution in the same machine at half load and full load. It will be seen that the main field is twisted in the direction of rotation, the amount of twist being greater with heavier loads. The commutating poles prevent the building up of flux in the commutating plane due to armature reaction, and, in addition, give the small amount of flux required for commutation.

Some flux distribution curves for a commutating pole motor are given in Figs. 293 to 295.

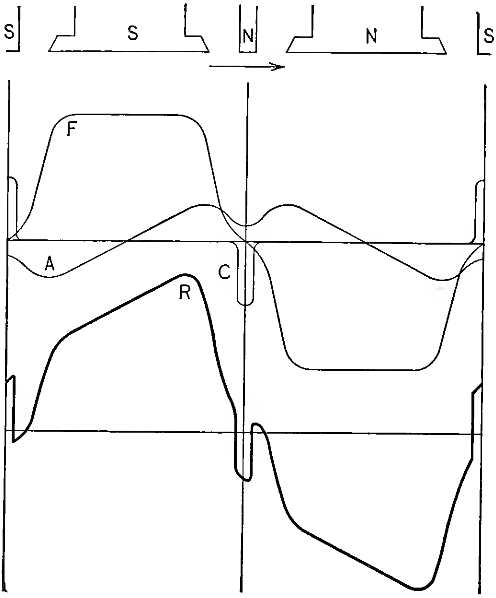


FIG. 234.—Field distribution in the air gap of a commutating-pole machine carrying load; the commutating pole, besides neutralizing the effect of the armature m.m.f., at the brush position, provides a proper flux density for black commutation. This curve is a theoretical one, the actual field distribution not showing the effect of the commutating poles as definitely as it is shown here.

208. Field Excitation.—The field current of any generator, whether c.c. or a.c., must be continuous; the poles must be continuously excited in the same direction. The field coils of a c.c. dynamo-electric machine may be supplied with current from the armature of the machine itself, in which case it is called a *self-excited machine*; or the power for the field coils may be furnished from some other electric circuit, in which case the machine is said to be *separately excited.* In general, we may say that all c.c. generators use self-excitation while practically all alternating current generators have to be separately excited.

209. Different Field Windings.—As was pointed out in the last chapter, the required number of ampere-turns may be supplied by using a large number of turns and a small current, or a few turns with a large current. The field winding may be connected in parallel with the armature of the

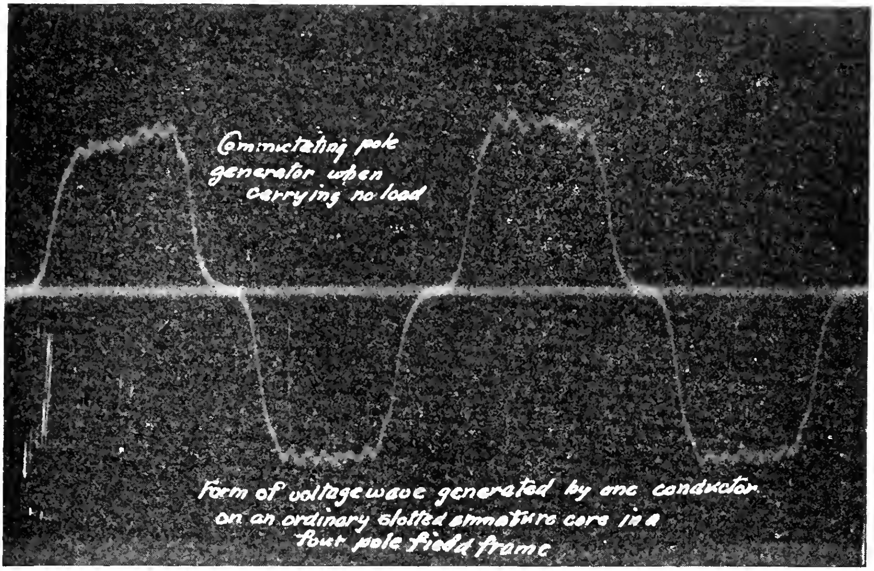


FIG. 235.—Flux distribution of a commutating pole generator when carrying no load, obtained by the oscillograph.

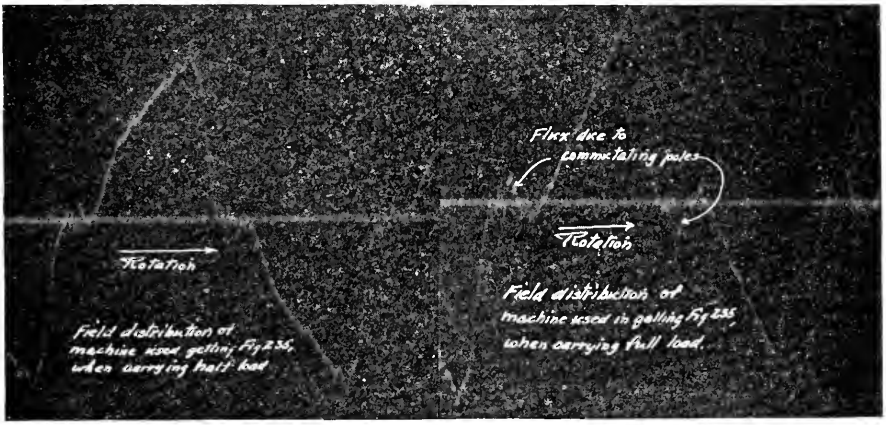


FIG. 236.—Flux distribution of the same machine as was used in obtaining Fig. 235, when the machine was carrying half load and full load, as noted on the two plates. The terminal voltage was the same for both curves and the scale of the two films was the same. It is seen therefore that the volts per bar (maximum) is much greater at full load than at half, or no load, although the terminal voltage and number of commutator bars is the same for both cases.

machine; the coils are then wound with many turns of comparatively fine wire, possibly as large as No. 12 on large machines, while small machines might be wound with No. 20 or smaller. When the field of a c.c. generator is so connected across the armature terminals, i.e., in shunt, or parallel, with the external circuit, the machine is said to have *shunt excitation,* and the winding is called a *shunt field.*

When but a few turns of heavy wire are put in the field coils, the field winding is connected *in series* with the external circuit and the field is called a *series field.*

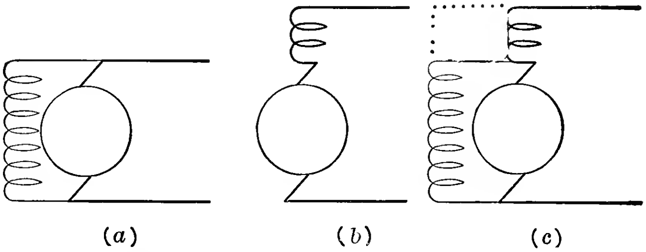


FIG. 237.—Various connection schemes for magnetizing the field of a self-excited generator.

In most c.c. generators, the field coil is wound in two parts. The larger part of the coil is made up of comparatively small wire and forms the shunt field, and the smaller part of the coil is wound of a few turns of large wire or copper ribbon and forms a series winding for the generator. Such a generator, having the two kinds of field coils, is said to be *compound wound.*

Diagrams of the three kinds of windings are shown in Fig. 237; (*a*) being the shunt, (*b*) the series, and (*c*) the compound winding. In the compound winding, the shunt field may be connected directly across the armature as in the full lines of Fig. 237 (*c*), or it may be connected across the armature and series field both, as shown in the dotted lines in the same figure. The first connection is called a *short shunt,* while the second (the one in dotted lines) is called a *long shunt.* A coil for a compound wound generator was shown in Fig. 174, page 184. It is seen that the coil is made in two parts. The outside coil of few turns is the series field; it will be noticed that the terminals which lead the current in and out of this coil are heavily constructed so as to carry a large current safely. The current through a series coil may be several hundred amperes on a large machine.

210. Field Rheostats.—In series with the shunt field of any generator is generally placed a variable resistance (see Fig. 238), made of some high-resistance material imbedded in enamel, porcelain, or other heat-resisting body. The amount of resistance can be varied by a movable shoe (carried on an arm that rotates) which makes contact with any one of many taps on the resistance. This adjustable resistance is called a *field rheostat.* A diagram of the connections of the movable contact and the taps on the resistance is shown in Fig. 239. Figs. 240 and 241 show the external appearance of the rheostat, the connections of which are given in Fig. 239. The cast-iron plate serves as a mechanical support for the enamel and resistance wire, and also serves to radiate the heat generated in the resistance wire.

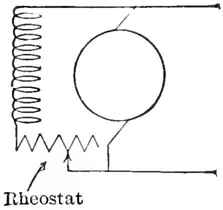


FIG. 238

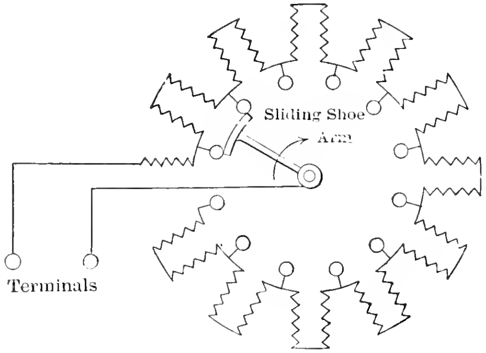


FIG. 239

FIG. 238.—A shunt-wound generator is always equipped with a variable resistance, called the field rheostat, in series with the field winding.

FIG. 239.—Connections of a field rheostat; the wire of the first steps of the rheostat can generally carry safely about twice as much current as can that used for the last steps.

The size of wire used in a field rheostat is "tapered"; the wire is much larger on one end than on the other. If the rheostat is connected in series with a shunt field (as in Fig. 238) and the amount of resistance in the rheostat is varied, evidently the current through the rheostat and field winding will vary. As the rheostat is "cut out" (i.e., as its resistance is lowered) the current in the circuit increases; for this reason the rheostat wire is made larger on the first steps than on the last steps. The amount of taper is generally 2: 1, i.e., the rheostat can safely carry twice as much current on one end as on the other.

211. Rating a Rheostat.—A certain field rheostat might be rated on its name plate: Resistance 35 ohms, maximum current 8 amperes, minimum current 4 amperes. This rating signifies that the total resistance of the rheostat is 35 ohms, that it will safely carry 4 amperes *through all of its resistance* and that the *first step* will safely carry 8 amperes. Such a rheostat would be suitable for a field having 28 ohms resistance, connected to

Fig. 240.—The cast-iron plate which forms the supporting structure for the rheostat has several ribs over that part which is close to the resistance wires; the ribs increase the heat dissipating ability of the plate.

a 220-volt source of supply. When the rheostat was "all in," the current would be somewhat less than 4 amperes, and when only the last step of the rheostat was connected in the circuit the current would be about 8 amperes.

212. Relation of Rheostat to Field Resistance.—If this rheostat were used to regulate the field of a small generator (say one having 200 ohms resistance in the shunt field) it would not have much effect. The change in field circuit resistance from the "all in" position of the rheostat to the "all out" position would be from 235 to 200 ohms and quite probably this amount of change would not be sufficient. The field rheostat to be used with any generator must be properly designed for the field circuit of that generator. A rheostat suitable for one field will not be at all suitable for another. In general, the rheostat should have about the same resistance as the field winding itself, and the current capacity of the rheostat in the first steps should be equal to the rated voltage of the machine divided by the resistance of the shunt-field winding.

213. Shunting the Series Field.—It is impossible to design the series field of a compound generator with exactly the right number of turns; these field coils are therefore always "over-designed." By this is meant that

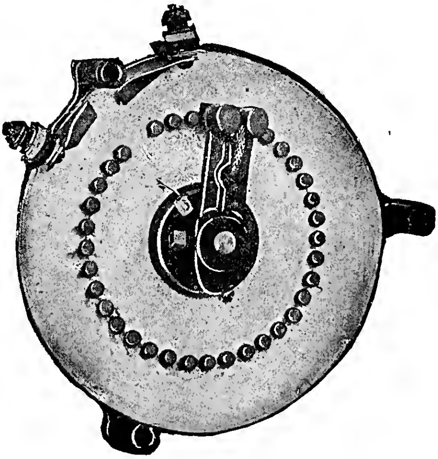


Fig. 241.—The resistance wires of the rheostat are completely enclosed in a vitreous enamel, as can be seen from this reverse view of the rheostat shown in Fig. 240; the contact buttons project through the enamel, are ground off smooth, and thus serve to permit connection of the wire with the sliding shoe.

the coil is wound with more turns than are really necessary to give the required number of series field ampere-turns, when all of the generator output current is flowing through them.

The action of this series field is weakened by putting *a shunt* across the terminals of the series field. This shunt, or *diverter,* generally made of German silver, ribbon-shaped conductor, serves as a by-pass for part of the load current. The division of the load current between the series field and the series-field shunt depends upon their relative resistances. By adjusting the resistance of the shunt, the number of ampere-turns in the series field (for any given load) may be changed as desired. After this shunt has once been adjusted it is not necessary to change it, and therefore it is not made adjustable; in this respect, it is different from the shunt-field rheostat.

214. Characteristic Curves of Generators.—If the load of a generator is increased (speed, etc., being kept constant), the terminal voltage of the generator decreases because of the effects of armature resistance and armature reaction. The curve showing how the terminal voltage changes with increase in load is called the *external characteristic* of the generator. If the terminal voltage is kept constant by increasing the field current (thus increasing the generated e.m.f.) as the load increases, and a curve is plotted to show how the field current varies with the load, the curve is called the *armature characteristic* (or field compounding curve) of the generator. Many other curves of similar nature may be constructed, and they are all grouped under the general name of *characteristic curves.* By inspection of these curves it may easily be seen how one quantity varies with respect to another, the rest of the variables involved in the operation of the machine being maintained constant.

The most important curves of a generator are the *external characteristic,* the *efficiency,* and the *magnetization curves.* The first two are plotted with terminal volts and efficiency respectively, as ordinates, and load current as abscissæ in both cases.

The efficiency curve of a generator will not be taken up here, as Chapter X will treat this subject in detail.

215. Magnetization Curve.—The magnetization curve is plotted between terminal volts and field current, the machine being operated at no load and normal speed while the data for the curve are being obtained, and the field being generally separately excited. When there is no load on a generator, the terminal volts and generated e.m.f. are the same. The generated e.m.f. is directly proportional to the flux through the armature, if the speed is held constant; the curve plotted between no-load terminal volts and field current shows, therefore, the relation between the field current and the flux which this field current produces; hence its name of magnetization curve.

If the reluctance of the complete magnetic circuit of the machine were constant, no matter how much the flux might be, the magnetization curve would be a straight line. The reluctance of the air gap (which, as shown before, constitutes the principal reluctance of the magnetic circuit) is independent of flux density; the reluctance of the iron part of the path, however, increases as the flux density increases, especially at the higher densities. Hence the magnetization curve of a machine is nearly a straight line, but tends to bend over slightly at the higher values of the field current. The amount of bending shows how nearly the iron part of the magnetic circuit is saturated. In a well-designed machine, normal voltage is obtained with that value of field current which gives a voltage somewhere near the *knee* of the magnetization curve, as it is called; this point is shown at *C* in Fig. 242. It will be seen that for an increase in field current over the normal field, *OB*, the increase in generated e.m.f. (hence in flux) becomes smaller and smaller. This condition indicates that the magnetic circuit is becoming saturated.

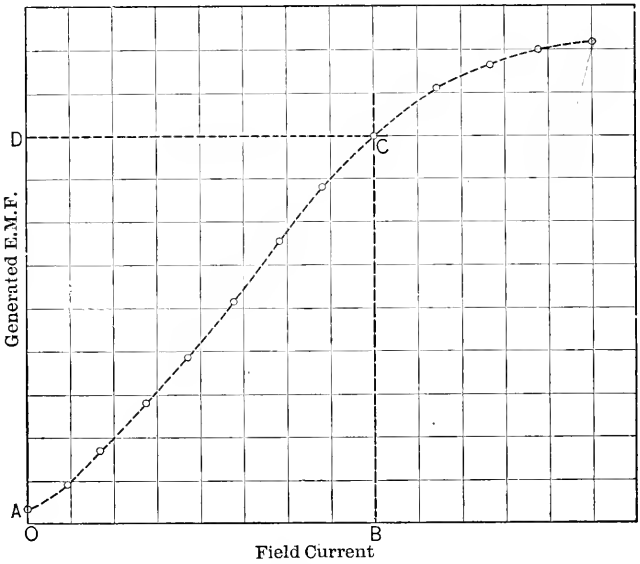


FIG. 242.—Magnetization curve of a generator; the normal operating density is just below the knee of the curve.

216. Residual Magnetism.—When the field current is zero, the generated e.m.f. is not zero but has some small value, perhaps 3 per cent of the normal voltage; this is due to the *residual magnetism* of the machine. After the field of a generator has once been magnetized, the iron pole pieces and yoke stay magnetized to a slight extent, and this magnetism which stays in the frame after the magnetizing current has been reduced to zero is called the residual magnetism of the machine. The amount of this magnetism depends upon the quality of iron used in the magnetic circuit; if all the iron of the magnetic path, poles, and yoke, as well as that of the armature core, was soft and well annealed, there would be practically no residual magnetism.

217. Separately-Excited Generator.—The terminal voltage of any machine can be obtained for any load, by subtracting the armature *IR* drop from the generated voltage at that same load. It has been shown that the armature of a generator exerts a demagnetizing effect on the main field; thus, even though the field current of a generator is maintained constant, as the machine is loaded the generated voltage falls, because of the field weakening due to the armature reaction.

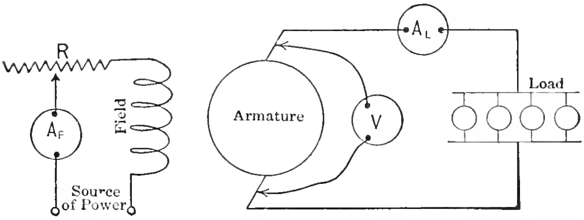


FIG. 243.—Connection scheme for getting the external characteristic, or armature characteristic, of a separately-excited generator.

Consider a generator with its field separately excited, as in Fig. 243, the field resistance, *R*, being adjusted to give the machine a field current *OA* in Fig. 244. From the magnetization curve, the machine will generate a no-load voltage equal to $AD = OB$. As load is added to the generator (its field current and speed being maintained constant), its generated voltage falls, on account of armature reaction; this relation between generated voltage and load current is represented by the curve *BC*.

If the armature *IR* drop corresponding to each load current is subtracted from the curve of generated voltage, the curve *BG* results; this represents the relation between terminal voltage and load current, and is the external characteristic of the separately-excited generator.

The terminal voltage of a separately-excited generator with constant field current, therefore, falls as its load is increased. If constant voltage with increasing load is desired, it is accomplished by adjustment of the field rheostat, permitting sufficient increase in the field current to bring up the generated voltage. If the generated voltage is maintained greater than the terminal voltage, by an amount equal to the armature *IR* drop, the terminal voltage remains constant.

218. Series Generator.—In the series generator, field and armature are in series and carry the same current, as represented in Fig. 245. The m.m.f. of the field is directly proportional to the load current, and so the generated voltage is nearly proportional to the load. Where a load current, as *OA* in Fig. 246, is flowing, we should expect the generated voltage to be equal to *AD*. The demagnetizing effect of the armature reaction reduces the generated voltage to the value *AC*; subtracting the armature and series field *IR* drops gives the terminal voltage corresponding to a load current of *OA*, as *AG*.

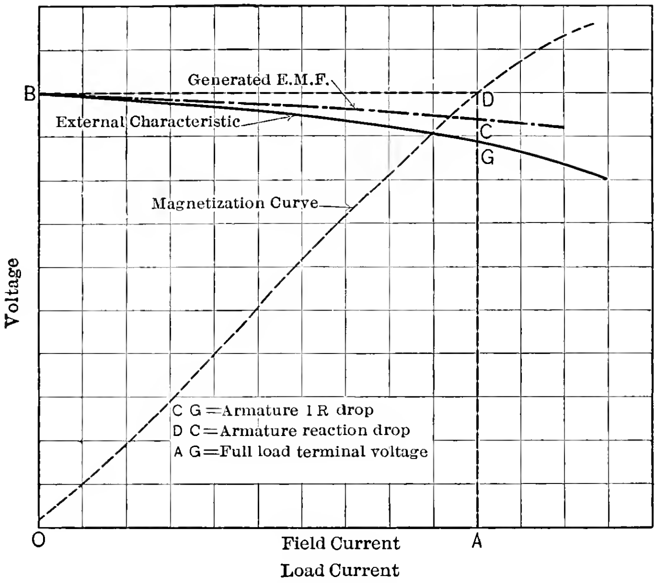


FIG. 244.—External characteristic, and curve of generated voltage, of a separately-excited generator.

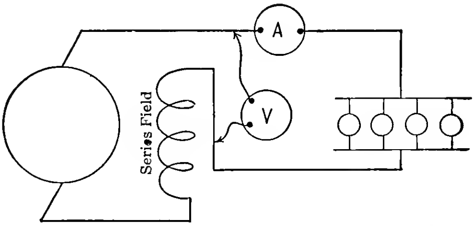


FIG. 245.—Connection scheme for getting the external characteristic of a series generator.

The demagnetizing effect of the armature really neutralizes a part of the field m.m.f. If the strength of this armature m.m.f. is taken as equal to *AB*, the *effective* current in the series field, in so far as its magnetizing effect is concerned, is really *OB* and not *OA*. The current *OB* gives a voltage in the magnetization curve of BF, and AC is equal to BF. Therefore, C is one point on the curve of generated e.m.f., and other points can be found in a similar manner. The armature demagnetizing effect (AB) is of course proportional to the load.

The series generator, having a different terminal voltage for each load, finds no application as such. Its only use at present is as a series booster (to be describe later) and in modified form for train and automobile lighting.

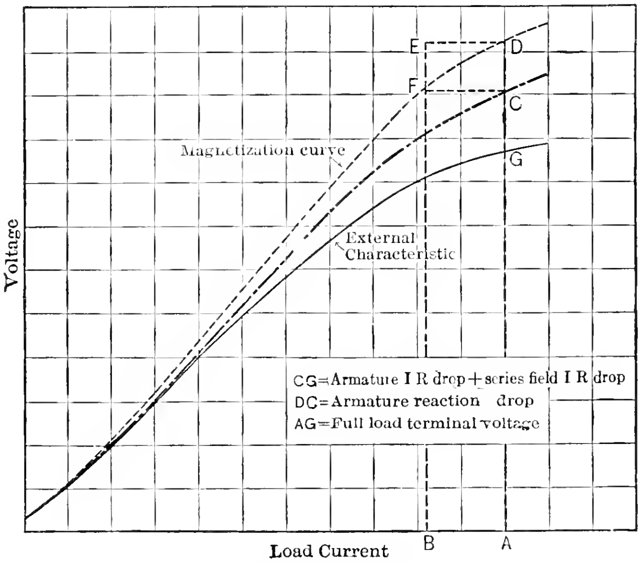


FIG. 246.—External characteristic, and curve of generated voltage, of a series generator.

219. Self-Excitation.—Shunt and compound generators are self-excited machines; both have shunt fields connected across the armature, which therefore supplies the shunt-field current. When such a machine is starting, there is no voltage developed in its armature, but as the speed increases, the residual magnetism (which plays a very important part in the operation of a self-excited generator) gives a small e.m.f. in the armature. *As the shunt field is connected across the armature, this small e.m.f. produces a small current through the shunt field of the machine* and even though this current is small, it increases somewhat the field strength of the machine. As a result, the e.m.f. generated in the armature increases and hence more current flows in the shunt-field circuit.

220. "Building Up."—This action and reaction between the armature and shunt field is called the "building up" of the generator; it continues until the generator is operating at normal voltage. It is evident that if there were no residual magnetism in the field this "building up" operation could not take place; it would be necessary to excite the fields from some outside source every time the generator was started, and this would complicate the operation of a generating station.

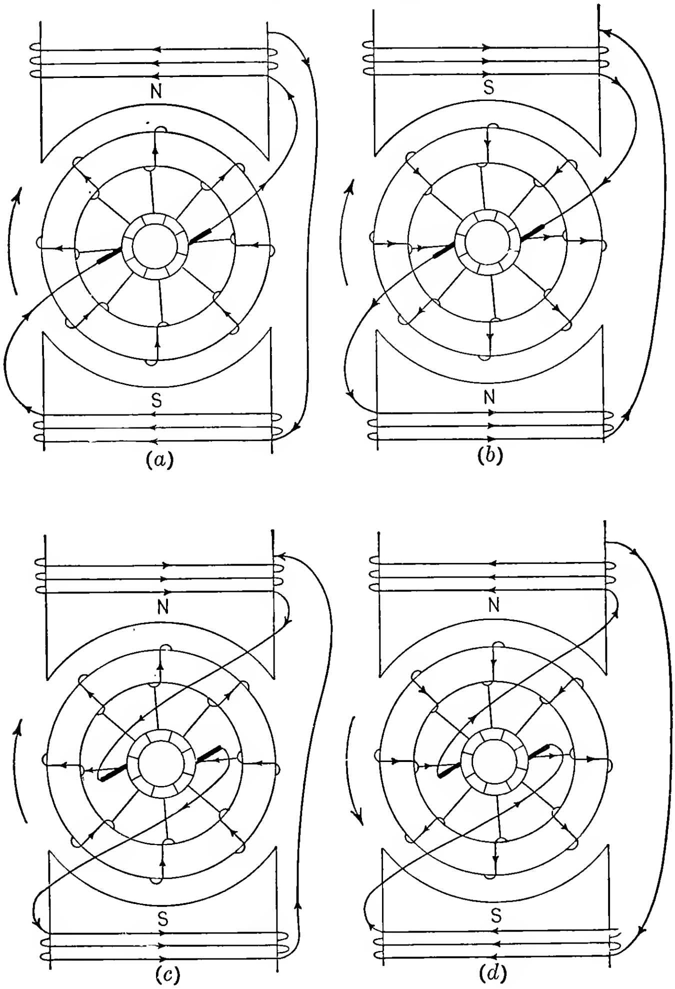


Fig. 247.—Conditions which may possibly be encountered when a shunt-wound generator is supposedly connected properly for "building up." The polarities indicated on the poles are those of the residual magnetism, not those set up by the current in the windings.

The successful building up of a given self-excited generator, and the polarity with which it builds up, depend on three conditions; the polarity and magnitude of the residual magnetism, the resistance of the field circuit and the order of the connections of the field winding to the armature terminals, and the direction of rotation.

In Fig. 247*a*, a shunt-wound generator is represented as rotating clockwise, its residual magnetism being such as to make its upper pole a north pole. As soon as the armature starts to rotate, current is induced as indicated, flowing out from the right-hand brush and through the field coils as shown; its direction is evidently such as to assist the residual magnetism, and the machine will tend to build up with polarity as indicated.

If the residual magnetism had been reversed, as in Fig. 247*b*, the upper pole being now south, the current induced in the armature would also be reversed, and the machine would tend to build up, *but with polarity opposite to that in Fig.* 247*a*.

It may happen that the direction of the residual magnetism, or the connection of the field to the armature, is such that what small current is sent through the field circuit by the voltage due to residual magnetism tends to decrease, instead of increase, the field magnetism. In this case the machine cannot build up as may be seen from Fig. 247*c*. By reversing the order of connection of the field to the armature, or, as it is generally stated, reversing the field, we come back to the connections of Fig. 247*a*, which are satisfactory. We may, however, also make the machine build up by reversing the direction of rotation from clockwise in Fig. 247*c*, to counter-clockwise, as in Fig. 247*d*.

If a shunt generator, by some chance, loses its residual magnetism (the jarring it receives during shipment might possibly effect such a result) it is necessary to connect its field circuit to some source of electric power, and so re-establish the residual magnetism.

If the residual magnetism of a generator should become reversed for any reason, it will build up with reversed polarity if the field is properly connected to the armature terminals. This condition of reversed polarity may be permissible in the laboratory; but if it happened on a railway generator, for example, the station meters would all deflect backward, and the polarity of trolley and ground would be reversed. In practice, it is therefore necessary to reverse the residual magnetism by supplying current in the right direction to the field coils from some outside source.

If a generator having residual magnetism refuses to build up, the difficulty probably lies with the connections of the field to the armature. If these are reversed the machine should build up. Certain other reasons why a generator will not build up are outlined below, as well as in Chapter XIII.

221. Why a Generator Builds Up.—The current which flows through a shunt field is

$$I_f = \frac{E}{R_f + R_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (102)$$

where E = impressed voltage;
R_f = resistance of field coils;
R_r = resistance of field rheostat.

Now suppose the magnetization curve of the machine is as given in Fig. 248. If a line AO is drawn through the origin, making an angle ϕ_1 with the horizontal, it is evident that, since

$$\tan \phi_1 = \frac{E}{I_f} = R_f + R_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (103)$$

it must represent some definite value of field resistance. (The same scale is supposedly used for both volts and amperes. If not, a proper change must be introduced in the value of tan ϕ_1.) Any other line, say KO, making a smaller angle, ϕ_2, represents some other smaller value of field resistance; if a field current OD is desired through this resistance, a voltage BD must be impressed upon it; for a current OH, the impressed voltage must be HF, etc.

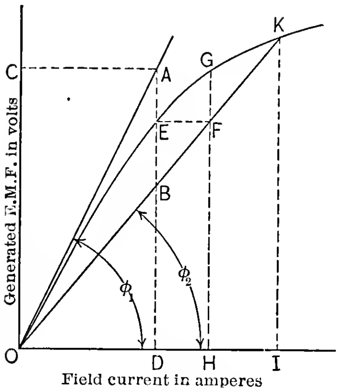


Fig. 248.—This diagram shows, graphically, the conditions required for a shunt-wound generator to excite itself; the tangent of the straight line, equal to the resistance of the field circuit (including the resistance of the rheostat) must be sufficiently low to make the straight line lie below the straight part of the magnetization curve.

With the resistance of the field circuit equal to tan ϕ_1, it is apparent that at no value of the field current (except possibly at very low values) does the armature generate enough voltage to force through the field circuit enough current to excite the field sufficiently to produce in the armature the voltage required. Hence, with this value for the field-circuit resistance, the generator could not build up.

But suppose that the field rheostat is cut down, so that the resistance of the field circuit becomes equal to tan ϕ_2. Now, when the generator has a field current equal to OD, it generates a voltage DE; but to force the current OD through the field circuit requires only the voltage DB. Hence, the voltage DE forces through the field circuit a current OH, which in turn makes the armature generate the voltage HG, which again increases the field current. This process continues until the point K is reached, where the generated voltage is just sufficient to force through the field circuit the current OI.

Therefore, in starting a shunt generator, it is necessary to properly adjust the resistance in the field rheostat. To obtain voltages higher than IK, more of the field rheostat must be cut out.

222. Shunt Generator.—The shunt-wound generator has an external characteristic that falls off more rapidly than does that of the separately-excited machine, because in the shunt generator there are *three effects* tending to make the terminal voltage fall as the load is increased. We have the armature reaction and the armature resistance drop, and, *in addition, the shunt-field current falls as the load is increased because of the lowered terminal voltage.* The shunt-field current is evidently proportional to the terminal voltage, as may be seen from Fig. 249, and so decreases

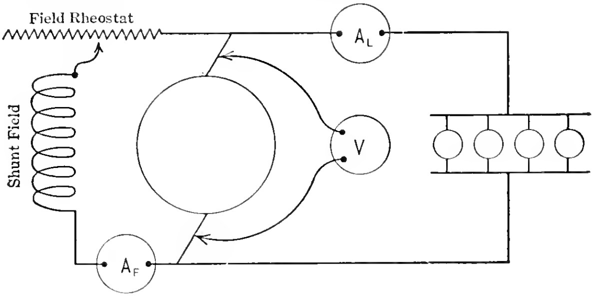


FIG. 249.—Connections for getting the external characteristic of a self-excited, shunt-wound, generator.

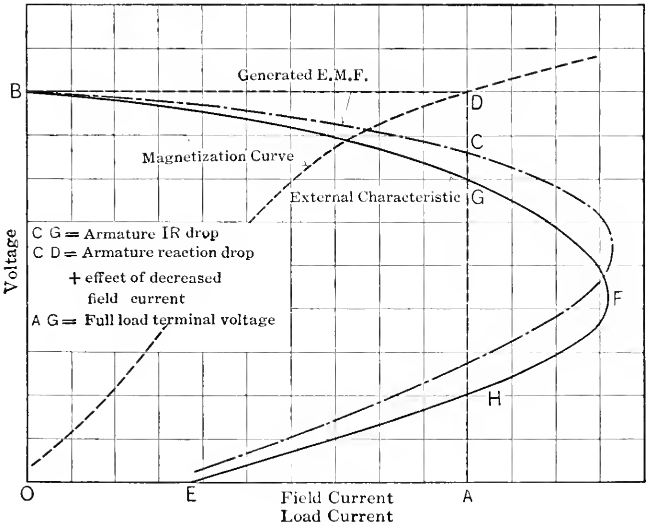


FIG. 250.—External characteristic, and curve of generated voltage, of a shunt-wound generator. Rated full-load current is shown by OA and rated full-load voltage by AG.

with load increase. The external characteristic of a shunt generator is shown in Fig. 250.

If the resistance of the external circuit of a shunt generator is continually decreased, the external characteristic doubles back because of the decreased field current. From B to F (Fig. 250) the machine is stable, and this is the working part of the external characteristic. The part of the curve from G to H is generally difficult to obtain, as the machine does not definitely maintain any special load on that part of the curve.

That the external characteristic of a shunt generator bends back on itself, may be seen if we consider the equations

$$E_t = E_g - I_a R_a, \quad \quad (104)$$

and

$$I_l = \frac{E_t}{R_e} \quad \quad (105)$$

where E_t = terminal voltage;
E_g = generated voltage;
I_a = armature current;
R_a = armature resistance;
I_l = load or external current;
R_e = resistance of load or external circuit.

Both R_e and E_t are decreasing quantities throughout the determination of the external characteristic; the operator causes R_e to decrease as he chooses, by adding load to the machine, but the rate of decrease of E_t is fixed by Eq. (104), the magnitude of armature reaction, and the shape of the magnetization curve. Whether the external current I_l, increases or decreases, depends upon the relative rates of decrease of R_e and E_t. At first, as R_e is decreased, E_t does not decrease relatively as much, and the load current increases. The relative rate of decrease of E_t, however, increases and when the machine reaches the straight portion of its magnetization curve, E_t decreases relatively faster than R_e, and I_l begins to decrease.

Even when the terminal voltage is zero (E, Fig. 250) there will be some current circulating through the armature. The field coils can produce no magnetic field because they have no current (the terminal voltage being zero), but there is some residual magnetism to give the armature e.m.f. which produces the short-circuit current OE. At this point the external circuit has zero resistance, and the machine is short-circuited.

The difference between the no-load terminal voltage and full-load terminal voltage (field excitation and speed constant) expressed in percentage of the full-load voltage, is called the *regulation* of a generator. Thus, a machine having no-load voltage of 121 and full-load voltage of 110 would have a regulation of 10 per cent.

The above analysis is for machines having no commutating poles. Machines equipped with commutating poles, even if the brushes are set directly under the center of these poles, show a certain compounding action which makes the external characteristic droop less rapidly than that shown in Fig. 250.

Because of its poor regulation, the shunt generator is used but very little in practice. It is well adapted for charging storage batteries, because of its drooping characteristic; as the batteries become charged their voltage rises, and the charging current is thereby reduced. Such installations are rare, however.

223. Armature Characteristic of a Shunt Generator.—If constant terminal voltage is desired from a shunt generator, with increasing load, it is necessary to increase the shunt-field current by adjustment of the shunt-field rheostat. This manipulation compensates for the armature *IR* drop and the effect of armature reaction, and results in an increasing field current with increase of load; the curve expressing their relation is called the armature characteristic. Such a curve is shown in Fig. 251; it is usually concave upward, because the increase in flux, per unit increase of the shunt-field current, decreases as the magnetic circuit approaches saturation, as shown by the magnetization curve.

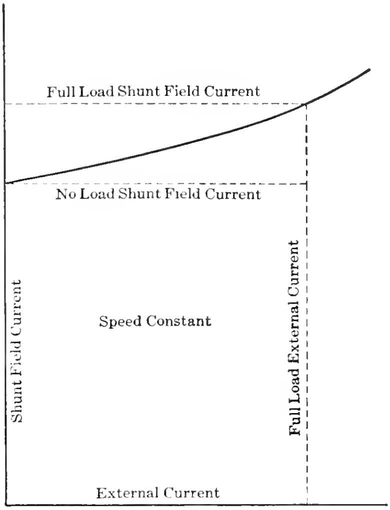


FIG. 251.—Armature characteristic, or field compounding curve, of a shunt-wound generator.

The increase in the field ampere-turns required to compensate for the armature *IR* drop and armature reaction, may be determined from this curve, being equal to the increase in shunt-field current from no load to full load, multiplied by the number of shunt-field turns; the latter is, of course, constant.

224. Compound Generator.—A very simple way of causing the necessary increase in the field ampere-turns, to compensate for armature *IR* drop and armature reaction, is to employ the external current (which causes the terminal voltage to fall), by passing it through the series-field winding, which consists of several turns of large wire and is therefore of low resistance. A machine in which this has been done is called a compound generator; its connections are indicated in Fig. 252. The shunt field provides the correct no-load flux of the machine, while the series field, carrying all, or a fixed percentage of the external current, increases the flux by a predetermined amount.

If the series field provides more flux than is required to compensate for all the losses of e.m.f., then the terminal e.m.f. will rise with increase of load and the generator is said to be *over-compounded.* If the series field provides just enough increase in flux to make the terminal voltage at full load the same as at no load, the machine is said to be *flat-compounded.* If there are not enough turns in the series field to keep the terminal voltage from dropping as the load increases, the machine is said to be *under-compounded.*

In an over-compounded generator, the shunt-field current increases somewhat with increase of load, because of the increase in terminal voltage brought about by the series-field winding.

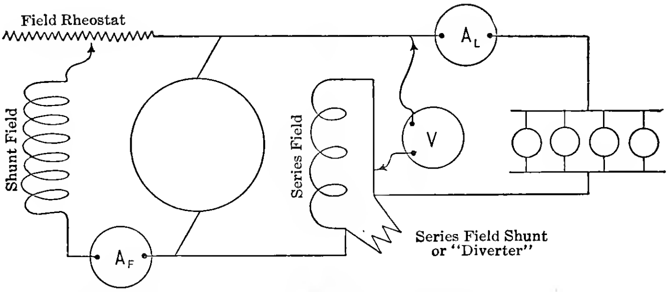


Fig. 252.—Connections for getting the external characteristic of a compound-wound generator.

In Fig. 253, the external, or compound, characteristic of an over-compounded generator is shown by the curve *BG.* The curve of generated voltage, *BC* is seen to rise as the load increases.

225. Calculation of Series-field Turns.—The number of series-field coil turns required to convert a given shunt generator into a compound generator may be determined from the armature characteristic of the shunt generator.

Consider that the armature characteristic of Fig. 251 is that for a 7.5 kw. shunt generator the voltage of which was maintained constant at 110 volts from no load to full load. The shunt-field current at no load was 4 amperes, and with a full-load current of $\left(\frac{7500}{110}=\right)$68.2 amperes, it was 4.7 amperes. If there are 1500 turns per pole in the shunt-field winding, the increase in field ampere-turns was $(4.7-4.0)\times1500=1050$. Neglecting the *IR* drop in the series field, the minimum number of series turns necessary is therefore $\frac{1050}{68.2}=15.4$ per pole. More turns per pole than this would be added, say 25, and a German silver shunt or diverter, adjusted across the series field. Its resistance would be such as to allow $\frac{1050}{25}$, or 42.0 amperes to pass through the field and divert 26.2 amperes. Considering that there will be a small *IR* drop in the series field to be compensated, the resistance of the diverter would have to be slightly higher than this value so as to make the series-field current somewhat greater than 42 amperes.

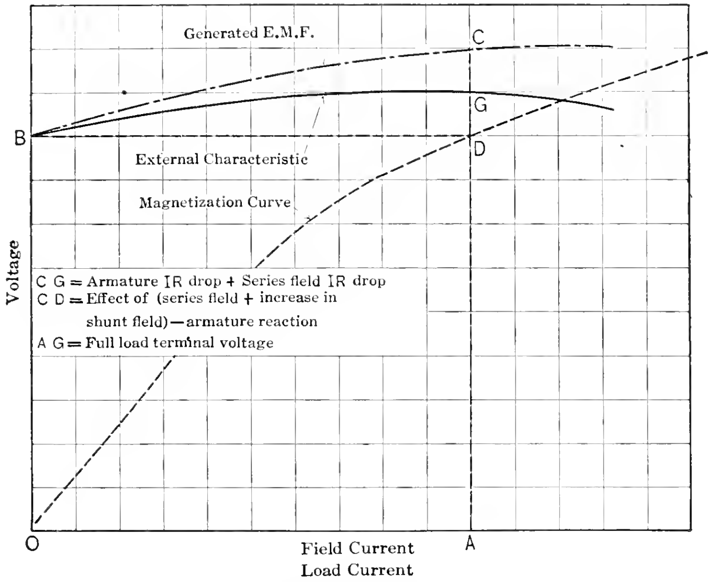


FIG. 253.—External characteristic, and curve of generated voltage, of a compound-wound generator.

If the armature characteristic of Fig. 251 had been determined with the voltage increasing uniformly with load increase, from 110 volts at no load to 125 volts with a full-load current of 68.2 amperes, the calculation would be altered by the fact that when the machine is converted into an over-compounded generator, the shunt-field current will automatically increase with load.

Suppose the no-load shunt-field current in this case to have increased from 4.0 to 5.35 amperes, increasing the shunt-field ampere-turns by $1.35 \times 1500 = 2025$. As a compound generator, the no-load shunt-field current will be 4.0 amperes, and the shunt-field circuit resistance $\frac{110}{4} = 27.5$ ohms. At full load (again neglecting the *IR* drop in the series field) the voltage impressed on the field circuit will have risen to 125 volts, so that the

shunt-field current becomes $\frac{125}{27.5}=4.55$ amperes, increasing the ampere-turns provided by the shunt field when operating as a compound generator, from 4×1500 to 4.55×1500 or an increase of $0.55\times1500=825$ per pole.

The series field to be added in this case need therefore furnish only $2025-825=1200$ ampere-turns per pole, which can be done with a minimum of $\frac{1200}{68.2}=17.6$ turns per pole.

226. Use of the Compound Generator.—Practically all machines used in lighting or railway service are over-compounded. In such work the characteristic to be obtained is *a constant voltage at the load,* not at the generator. The life and efficiency of an incandescent lamp are both greatly affected if the voltage of the line to which it is connected goes either above or below the voltage at which the lamp is rated. Now, if the terminal voltage of the generator should remain constant, the voltage of points on the distributing system must fall as the load is increased because of increased *IR* drop in the wires of the distributing system.

In Fig. 254 this point is illustrated. A group of lamps is connected to a center of distribution *B*; the generator is located at *A*. If the voltage at *A* remains constant as the load increases, it is evident that the voltage at *B* must fall because it is always equal to the voltage at *A* minus the drop in the line *AB*. But if the increase in voltage at *A*, from no load to full load, is just equal to the *IR* drop in *AB* when full-load current is flowing, then the voltage at *B* will be the same at full load as it is at no load. At intermediate loads, the voltage at *B* will be somewhat above normal; the external characteristic of a generator is always more or less curved (because of the variable reluctance of the magnetic circuit) so that if the machine is properly compounded at full load it is always somewhat over-compounded at half load. This effect is not great enough to cause any trouble on the system.

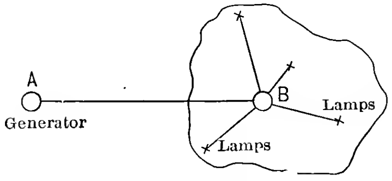


FIG. 254.—In electrical distribution systems it is customary to keep the voltage constant at some distant feeding point, such as point *B* of the diagram; this requires an increase in the terminal voltage of the generator proportional to the load current, to offset the *IR* drop in the feeder between *A* and *B*.

The amount of over-compounding used in practice depends upon the service for which the machine is intended. For isolated plants in single office buildings, it may be as low as 3 per cent or as high as 10 per cent, while with railway generators it will generally be 10 per cent, the voltage rising in the latter case from 550 volts at no load to 600 volts at full load. Under-compounded generators are never used in practice.

227. The Three-wire Generator.—A great many electric light installations are equipped with a *three-wire distribution system.* In such a system three wires are used for carrying the current from the generator to the lamps. A diagram of the scheme of connections for such an installation is given in Fig. 255; the voltages given here are those generally used for incandescent lamp circuits.

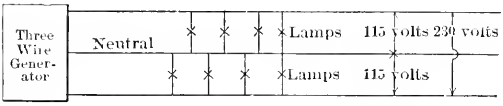
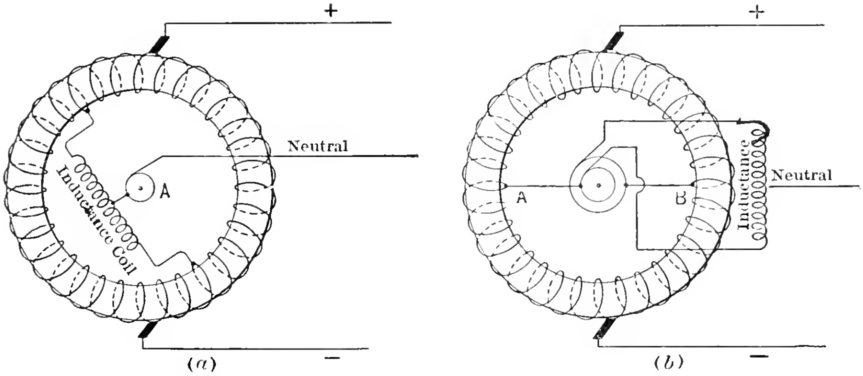


Fig. 255.—A three-wire network connected to a three-wire generator; the neutral wire generally carries a much smaller current than either of the two outside wires.

The generator which supplies such a three-wire system must evidently have three wires connected to it; the extra wire is called the *neutral.* Between each outside wire and the neutral, the lamps are connected, so that there is on the lamps an e.m.f. which is just one-half that between the outside wires.

The ordinary generator has only two sets of brushes; the special feature in the three-wire generator is the arrangement for connecting the third wire (the neutral) to the armature windings. This is always done by connecting an inductance coil between two diametrical taps on the armature winding and connecting the neutral wire to the center of this coil.



(a) (b)

Fig. 256.—Two schemes for connecting the neutral wire to the armature winding of a generator.

The inductance coil consists merely of several turns of insulated wire wound on a laminated iron core. Sometimes this coil is built right into the armature spider; in other cases it is not in the armature at all but located behind the switchboard or other convenient place.

Figure 256 illustrates these two schemes. In (*a*) the coil is mounted in the armature spider and its center point is connected to a slip ring, *A*. The neutral wire connects to the brush bearing on this ring. (In Fig. 256 the commutator has been omitted and the two ordinary brushes are shown as making contact on the periphery of the armature. This is done merely to keep the diagram clear.) In the scheme shown at (*b*), the two diametrical taps, *A* and *B*, connect to two slip rings, and so through brushes and leads to the outside inductance.

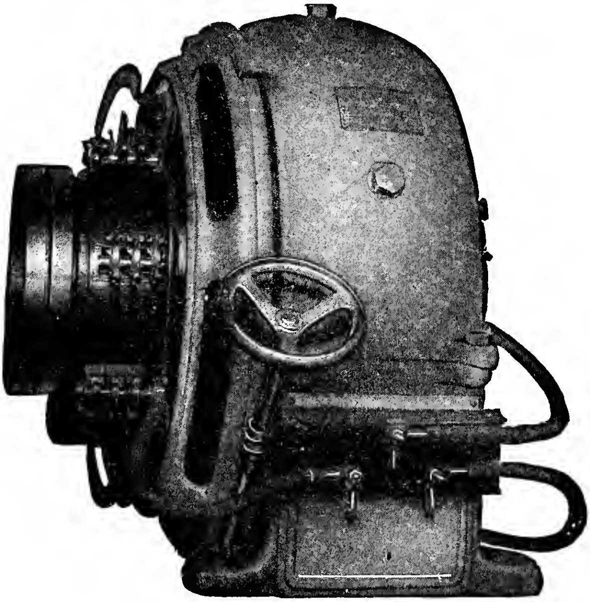


FIG. 257.—A modern three-wire generator; in this machine, scheme (*b*) of Fig. 256, is to be used; the two slip rings being easily seen in the cut, to the left of the commutator.

A three-wire generator with slip rings for connection to an outside inductance, is shown in Fig. 257.

skip to § 243

228. Boosters.—A booster is a generator the armature of which is connected in series with a circuit, its generated voltage being added to or subtracted from that of the circuit to increase or decrease the voltage of the line; the former is the more general usage. Boosters may be driven by any type of prime mover; a motor taking its power from the line in which the booster is connected is generally most convenient.

Boosters are much used for the control of the charge and discharge of storage batteries used in connection with generators in isolated plants.

In such installations the batteries are operated in parallel with the generators, taking the peak loads, but generally charging during the day. When the load is light, as during the night, the generators are shut down, and the battery takes the entire load. The boosters used may have their fields wound either shunt, series, compound or differential; in the last case the series field opposes the shunt field.

Series boosters are much used for compensating for the *IR* drop in long, heavily loaded feeders. In Fig. 258 a station is shown supplying a number of short feeders leading to distribution centers nearby, together with one long feeder. If it is required that the voltage at the end of this long feeder be maintained constant, this may be conveniently done by means of a series booster connected as in Fig. 259.

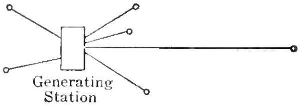


Fig. 258.—When a long feeder is supplied from the same bus-bars as are the short ones it becomes necessary to use a booster to offset the high *IR* drop in the long feeder.

The field current of the booster is evidently the load current of the feeder. Since the *IR* drop in the feeder varies directly as the load current, by designing the booster to operate on the straight portion of its magnetization curve, thereby making its generated voltage proportional to its field current, the *IR* drop in the feeder may be exactly compensated.

Obviously, the polarity of the booster, i.e., whether it adds its voltage to that of the feeder, or subtracts it therefrom, depends upon the field connections. By reversing the field of a booster which is raising the voltage of a feeder, the polarity of its armature will be reversed and it will subtract its voltage from, or *crush*, the voltage of the feeder.

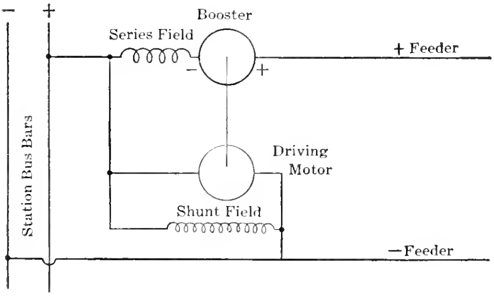


Fig. 259.—Connections of a booster and its driving motor.

In Fig. 259, the booster is shown as driven by a shunt motor, connected on the station side of the booster; if the voltage on the station side of the booster is substantially constant, the booster will be driven at practically constant speed.

Another common use of the series booster is as a *negative booster*. In railway work, where the current is brought back to the power house by means of the track and the surrounding earth, there is danger of the current leaking from the track and following gas or water mains, as illustrated in Fig. 260. This leakage current causes electrolytic action, which results in corrosion of the pipes. In order to prevent leakage of current to water and gas mains, insulated *negative feeders* are placed in parallel with the track, to assist in bringing the return current back to the power house. Such negative feeders, alone, naturally reduce the resistance of the return circuit, but their equivalent resistance may be further reduced by placing a series booster in series with them, as shown in Fig. 261. With a properly designed booster so connected, most of the return current will flow back to the station over the feeder.

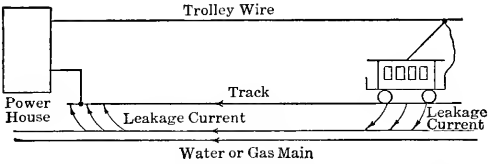


FIG. 260.—In some electrical networks, especially trolley systems using rail return, stray currents may cause serious damage to water and gas mains by electrolytic action.

229. Constant-current, Variable-speed Generators.—For generators designed to charge batteries and furnish power for lamps on automobiles or trains, where the speed may vary widely, a special form of field excitation must be employed; one commonly used is that shown in Fig. 262, this being known as a " third-brush " generator. The field is excited by the voltage between brushes *C* and *B*, much less than 180° apart. Owing to armature reaction, the field flux is twisted in the direction of rotation, as shown, and of course the amount of twist depends directly upon the magnitude of current flowing in the armature. With increasing twist, less and less flux is available for producing e.m.f. in the armature conductors between brushes *C* and *B*, and this results in a decreasing field current (if speed stays constant.)

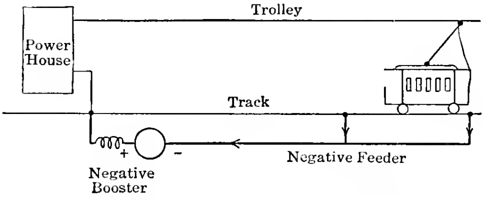


FIG. 261.—The use of a negative feeder and booster as here shown, will generally get rid of trouble from electrolysis.

The peculiar action of this third-brush excitation results in a nearly constant current delivered to the load circuit of the generator, even though the speed varies through wide limits. The action of the coil short-circuited by brush *B* further complicates the action of the machine, actually resulting in a current curve (for fixed load-circuit resistance) which increases at first with increase of speed and then decreases as the speed is raised still further.

There are other types of generators designed to deliver constant current as speed is varied, but the third-brush scheme is probably used more than any other, especially for automobile generators.

230. Factors Limiting the Capacity of a Dynamo-electric Machine.—The capacity of a generator or motor is limited by either one or the other of two effects, namely, *heating* or *commutation.* As the load on a machine increases, more current must flow through its armature (and series field if the machine has one), and the heat produced in these circuits is proportional to the square of the current.

The other limit to the capacity of a machine is fixed by the sparking at the commutator. As was explained in the discussion of commutation, the current in any coil has to reverse during the short interval of time during which it is short-circuited by the brush. Now, the liability to spark at the commutator was shown to depend upon the *rate of change of current* necessary during the commutating period. As the time during which commutation has to take place is constant (for a given speed), the rate of change of current depends directly upon the load the machine is carrying, and so the liability to spark depends directly upon the load. It has been shown, however, how the commutating pole, producing in the short-circuited coil an e.m.f. *proportional to the load,* overcomes this difficulty, hence, in properly designed commutating-pole machines, the limit of capacity is set by heating and not by commutator sparking.

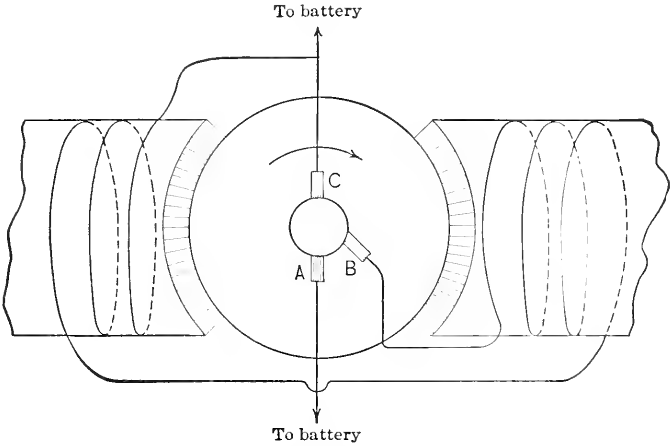


FIG. 262.—A special type of self-regulating generator which will deliver to a storage battery a practically constant current as the speed varies within very wide limits; it is largely used in automobile equipments.

231. Features Limiting Temperature Rise.—The temperature to which the windings of a motor or generator will rise depends upon two factors, the rate at which heat is *produced,* and the rate at which it can be *radiated,* or sent off into the surrounding medium. The rate at which heat is radiated from any body (e.g., an armature) depends upon the difference in temperature between it and the surrounding air and upon the amount of air that is carried over the radiating surface. This second condition is really involved in the first, because if but little air is supplied it soon gets hot and so reduces the temperature difference, whereas if much air is supplied, as in forced ventilation, the air is carried away from the radiating surface before it has time to get hot.

232. Effect of Ventilation upon Capacity.—In Fig. 263 are shown two sets of curves, those for heat radiation and that for heat production. The curve *OAB* gives the rate of heat production plotted against the armature current as abscissæ. The three lines *OC*, *OD*, and *OE*, show the rate of radiation for good, medium and poor ventilation; the abscissæ for these curves are the difference between the armature temperature and room temperature. (It is supposed that the air for ventilation is taken directly from the room and is therefore at the same temperature as the room.)

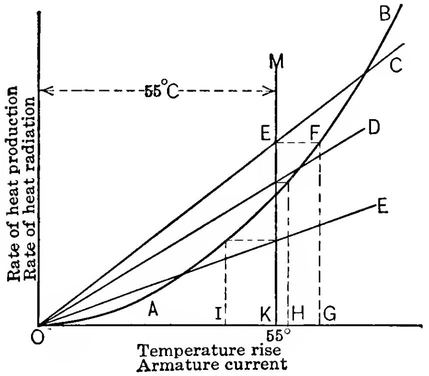


FIG. 263.—Curves showing the factors which determine the temperature rise of a machine.

The highest temperature rise of the hottest part in ordinary machines has been fixed by the *A.I.E.E.* at 55° C., above a room temperature of 40° C., hence the line *KM* is erected (in Fig. 263) at this value. With good ventilation, the rate of radiation at this temperature is equal to the rate of heat production by the current *OG*, and so we say the safe current, in so far as heating is concerned, is *OG*. If the ventilation is medium, the safe current is *OH*; and if the ventilation is poor, the safe current is *OI*. This diagram shows the enormous advantage of forced ventilation gained by equipping an enclosed machine with a ventilating fan and proper air ducts. By this means it is possible to increase its safe current capacity by 200 per cent or 300 per cent. A certain railway motor for example might be rated as 75 h.p. with ordinary ventilation, and by suitable ventilation *its rating might be increased to perhaps* 200 *h.p.* Practically all enclosed machines are now designed with the idea of getting the best ventilation possible.

233. Special Insulation Increases Capacity.—There is, of course, one other method by which a machine of given size may have its rating much increased, and that is by using only insulating materials which will stand higher temperatures. The limit of 55° C. above room temperature is proper for such insulating materials as cotton, shellac, oiled cambric, etc. If a machine is built with nothing but mica and asbestos for insulation, it may be run safely up to temperatures as high as 125° C. Asbestos has been used to some extent, but it is difficult to work this material into uniform sheets and coverings and, also, it absorbs moisture readily. If some good heat-resisting insulator could be discovered, the possible output of electric machinery could be much increased.

There is one bad feature connected with this high temperature of operation, however; the resistance of the windings increases quite rapidly with the temperature, and so results in an increased I^2R loss and therefore in a lower efficiency.

234. Operation of C.C. Generators in Parallel.—A c.c. generating station has ordinarily several generators, all of which, or only one of which, may be operated to supply the station output.

There are two reasons why several small-sized generators are used to supply the station output, instead of one large one; these reasons are reliability and efficiency. Suppose the load on a station is represented in the form of a curve, load in kilowatts being plotted against time. An average curve would be as shown in Fig. 264, the supposed load being made up of railway traffic, lighting, and power for factories. The peak load occurs at 8 o'clock in the evening, and is about 5000 kw. As the peak lasts only a short time, one generator of 4000 kw. rating would fit the needs of the station in so far as quantity of power was concerned. Such a generator would be run overload for a short time, but as large generators usually are able to carry 25 per cent overload for a short time, no harm would result. But there would be two bad features about such a station: First, if an accident should happen to the machine, the station would be without power of any sort until the machine was repaired; and second, during most of the day the machine would be operating with a load of less than 50 per cent of its full-load rating and consequently the efficiency would be rather low. (See Chapter X.)

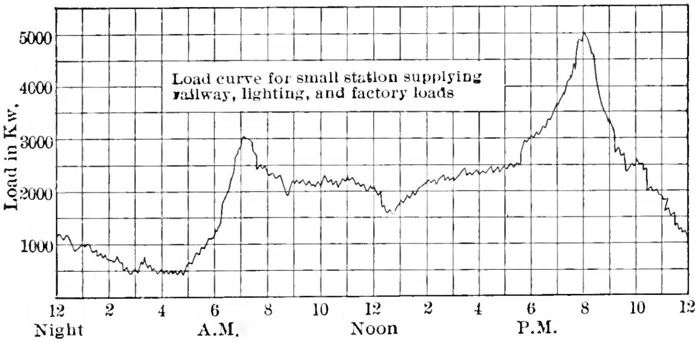


FIG. 264.—Typical load curve for a small station.

The proper installation to supply a load curve as given above, would consist of two 2500 kw. machines and one 1000 kw. machine. Then, from 12 o'clock until 6 o'clock A.M., the 1000 kw. machine would be running and the other two would be shut down. During most of the day, one 2500 kw. machine would be used, and during the two hours of peak load the two 2500 kw. machines would be used; or one 2500 kw. and one 1000 kw. machine would do for the very short time the overload occurs.

In such a station it is to be noticed, first, that *whatever generators are running are operated at practically full load, under which condition the efficiency is a maximum* and second, *any two generators of the three which are installed have sufficient capacity to carry the station load.* Therefore, continuity of service is guaranteed and the station operates efficiently; of course the first cost of the station would be considerably higher for the three-generator equipment than for the one-generator equipment.

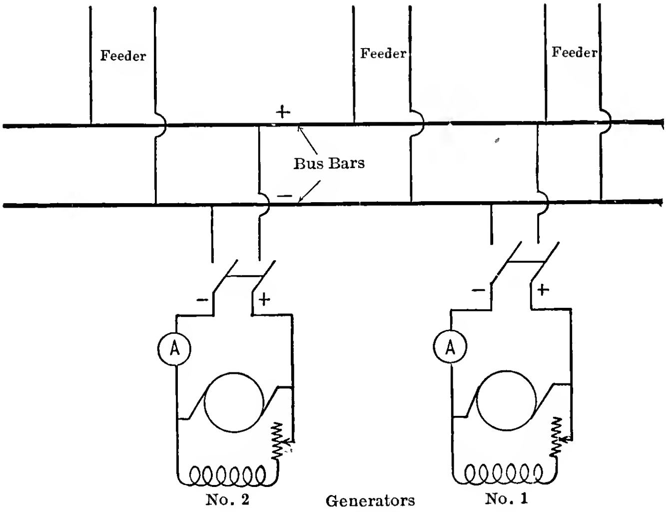


Fig. 265.—Connection of shunt machines for parallel operation on common bus-bars.

235. Parallel Connection of Generators.—In a c.c. station, only one set of bus-bars is used, no matter how many generators there may be in the station. *Bus-bars* is the name given to the heavy copper bars which run the length of the switchboard (behind it) and to which all generators and feeders are connected. The positive bus connects to the positive terminals of all machines in operation, and all negative terminals are connected to the negative bus. Such a connection is called a *parallel connection* of generators; all generators send their power into the one set of bus-bars, and the load, supplied by various feeders, is taken from them, as shown in Fig. 265.

236. Load Division with Shunt Generators Operated in Parallel.—The question naturally arises as to how these generators will divide the total load. Will each take half, or what will the division be, and how is it determined? We have said that motors and generators are reversible in their action, and it may be that under special conditions generator No. 2 may not be a generator at all but may be running as a motor, generator No. 1 supplying the necessary power to do this, besides supplying all of the power required by the load.

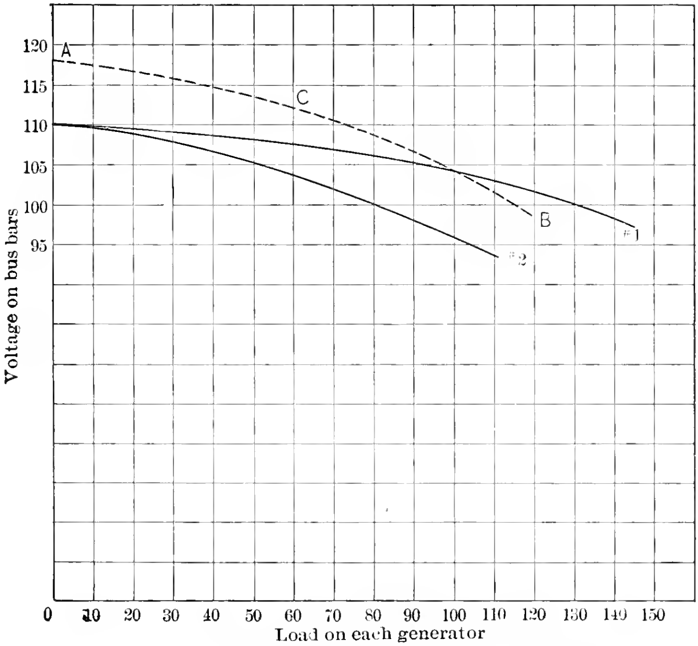


FIG. 266.—If the external characteristics of the two shunt generators operating in parallel are not the same, the two machines will not maintain an equal division of load, as this is varied.

In the case of shunt-wound generators, as shown in Fig. 265, the division of load is easily regulated and may be shifted from one machine to the other by simply manipulating the field rheostats. If the external characteristics of the two machines are known, the division of load may be determined at once, because of the fact that *the terminal voltage of both machines must be the same* as they both connect to the same bus-bars.

Figure 266 shows the external characteristics of the two machines operating at constant rated speed; it is supposed that at no load the two machines generate the same e.m.f. This is brought about by variation of one or the other of the field rheostats. Machine No. 2 is supposed to have an external characteristic that drops with load increase more than that of No. 1. When the load is 140 amperes, evidently No. 1 must be supplying 90 amperes and No. 2 supplying 50 amperes, and the line e.m.f. (i.e., the terminal voltage of the two machines) is 105 volts.

If the load current is increased until the bus-bar voltage falls to 100 volts, machine No. 1 must be carrying 130 amperes and No. 2 must be carrying 80 amperes; the load is therefore 210 amperes.

Equalization of Load.—Suppose it is now desired to equalize this load between the two machines, always being driven at their rated speeds, so that each machine carries 100 amperes. This may be done either by increasing the field current of No. 2 (cutting out some of its field rheostat) or by decreasing the voltage of No. 1 (by increasing its field-rheostat resistance). Suppose that the voltage of No. 2 is increased by increasing its field current, until its external characteristic crosses that of No. 1 at 100 amperes; the external characteristic of No. 2 is raised throughout the whole range of the curve and is shown by the dotted line *ACB*, in Fig. 266. When the load is 200 amperes, each machine will take one-half of the load, but at lighter loads No. 2 will now take more current than No. 1; when the load is 120 amperes, for example, No. 2 takes 80 amperes and No. 1 only 40 amperes.

237. Generators of Different Capacities.—If the two external characteristics coincided with each other throughout their length and the load was once equally divided, it would be equally divided for all loads. It may be, however, that the two machines are not of the same capacity; No. 1 may have a full-load capacity of 100 amperes and No. 2 of only 50 amperes, in which case we should want the division of load to be proportional to the capacities. In this case the two characteristics *must be similar in shape, and the no-load and full-load voltages of the two machines must be the same;* if No. 1 gives 110 volts at no load and 105 volts when carrying 100 amperes, No. 2 should give 110 volts at no load and 105 volts when carrying 50 amperes. With such external characteristics, No. 2 would always carry one-half as much current as No. 1

238. Connecting and Disconnecting a Generator from the Bus-bars.—When it is necessary to connect an additional generator to the bus-bars, on account of load increase, the machine is first brought up to speed; its terminal voltage is then made equal to the bus-bar voltage and the generator switch is closed; the machine will take no load on closing the switch, but its load may be brought to any desired value by decreasing the resistance of its field circuit. If it is desired to disconnect one generator from the bus-bars, its load is first reduced to zero by increasing the resistance of its field rheostat until its *generated e.m.f.* is the same as the bus-bar e.m.f.; when the generated and terminal e.m.fs. of a machine are the same, there can be no current flowing through its armature. When the load is zero, the generator switch is opened, and then the generator may be stopped.

239. Load Division with Compound Generators.—When over-compounded generators are operated in parallel, it is necessary to install an extra bus-bar, called the *equalizer bus-bar.* The diagram of two compound generators, connected for parallel operation, is shown in Fig. 267.

Without this equalizer bus, the parallel operation of two compound generators is unstable; they will not divide the load equally, but, on the contrary, one machine will stop furnishing load altogether and will operate as a motor from the other generator.

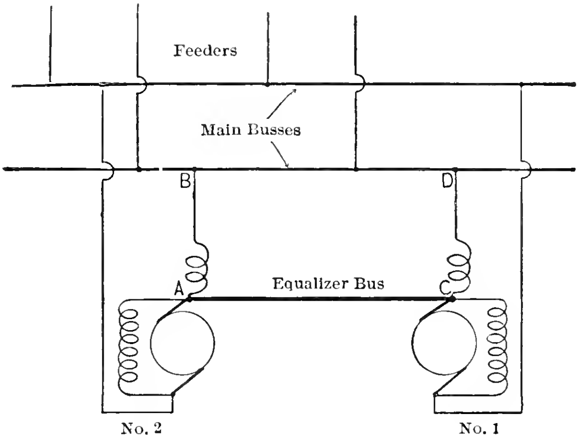


Fig. 267.—When compound-wound generators are operated in parallel it is necessary to provide an extra bus-bar, called the "equalizer bus," connected as shown here.

The external characteristics of the two machines, operating at constant rated speed, are given in Fig. 268 at AB and AC. Suppose the load is 130 amperes and that machine No. 1 is supplying 50 amperes, while No. 2 furnishes 80 amperes, the bus-bar voltage being 115 volts. Now, something may happen to change the load on No. 1 to 63 amperes, and so its voltage tends to rise to the point F on the line AB. As the whole load is 130 amperes, machine No. 2 will now carry only 67 amperes, and so we should expect machine No. 2 to operate at the point G of its external characteristic. But the terminal voltage of No. 1 would be higher than that of No. 2, and as they are connected to the same bus-bars this is really an impossible condition. Machine No. 1, however, which tends to be at an higher terminal voltage than No. 2, will force current through the armature of No. 2, besides furnishing the current to the feeders. The final result is that No. 1 operates at some extreme point on its external characteristic, furnishing all of the 130 amperes for the load and in addition, enough to run machine No. 2 as a motor.

If the action had started the other way (i.e., machine No. 1 had, for some reason, decreased its share of the load from 50 amperes to 40 amperes) then the decrease in the load of No. 1 would have continued until No. 2 was furnishing all of the load current and enough to run No. 1 as a motor. The operation of two such machines is therefore unstable; any action which starts a re-distribution of the load continues until one machine is carrying all of the load and the other has become a motor.

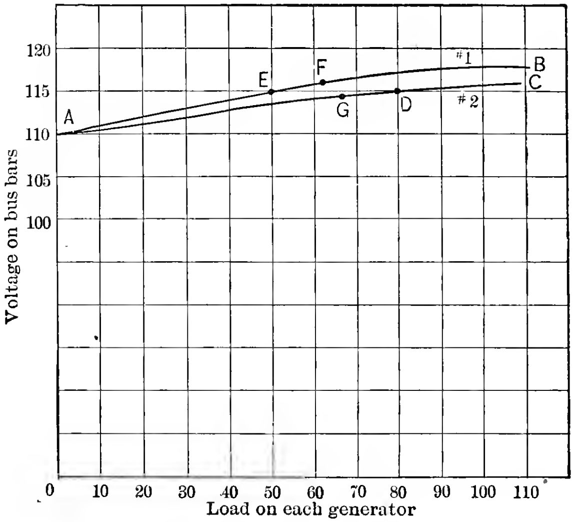


FIG. 268.—Without an equalizer bus over-compounded generators in parallel are unstable; if one starts to increase its share of the load this action continues until it has taken all of the load and is, in addition, running the other as a motor.

The necessity of having the equalizer bar becomes more apparent if we consider that, without such a connection, machine No. 2 is operating as a motor, taking its power from machine No. 1, as in Fig. 269. The armature and series-field current of No. 1 (generator) flows from the negative to the positive bus, while its shunt-field current flows from its positive to its negative brush, as indicated by the arrows. The machine being a compound generator, the series-field m.m.f. augments that of the shunt field. Machine No. 2 being a motor, its armature and series-field current, as well as its shunt-field current, flow from the positive to the negative bus. In passing from generator to motor, the shunt-field current of No. 2 has remained in the same direction, but the direction of its series-field current has reversed; its series field now tends to oppose the shunt field. The greater the current taken by No. 2 becomes, the stronger its series field and the weaker its resultant field, until its net field is zero;

at this time machine No. 2 is practically a short circuit for the generator, No. 1.

240. Action of the Equalizer Bus.—It will be noticed that this instability is due to the action of the series field; that machine which momentarily increases its load at the same time increases its terminal voltage, and the terminal voltage of the other machine must fall as part of its load is taken away by the first machine. Now, the function of the equalizer bus is to get rid of this instability, and *it does this by maintaining the IR drop over the two series fields equal at all loads.* As the resistance of the equalizer is practically zero, as is that of the main bus-bars, it is evident that the drop in voltage from A to B must be equal to that from C to D (Fig. 267). It will be noticed that the resistance of the cables connecting the machines to the switchboard must be treated as part of the series-field circuit. Evidently, the drop in voltage from A to B (or C to D) is equal to the drop in the series field proper, plus the drop in the connecting cables.

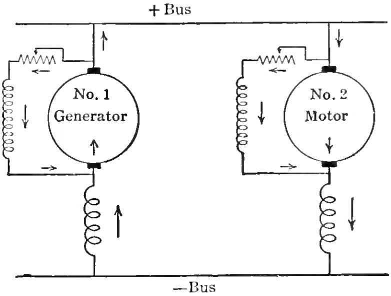


FIG. 269.—This diagram, showing the direction of the various currents of two compound generators, helps to explain their instability.

Suppose No. 1 tries to increase its share of the load; the IR drop from C to D must increase, and *therefore so must the IR drop from A to B increase.* But the IR drop from A to B can only increase if the current through the series field of machine No. 2 increases; and if this field current increases the voltage of No. 2 must increase also. Hence, any increase in voltage of No. 1 is accompanied by an increase in the voltage of No. 2 and therefore No. 2 will continue to furnish its share of the load.

It was noticed that, without the equalizer bus, when No. 1 increased its load (and thereby its voltage) the voltage of No. 2 *decreased,* and this was the cause of the unstable operation; when the equalizer bus is used, however, if No. 1 increases its load the voltage of No. 2 increases also, thereby causing No. 2 to continue to carry its share of the load.

Another way of looking at the same problem is to notice that the two series fields are put in parallel by the use of the equalizer bus; thus, any increase in one field current must be accompanied by a corresponding increase in the other.

If two compound generators of equal capacity are to operate in parallel and divide the load equally, they must have identical external characteristics and *the resistances of their series-field circuits must be equal.* We say series-field circuit instead of series field, because generally the series field is paralleled by a shunt, and it is the resistance of the double circuit, series field with shunt in parallel, which must have the same value for both machines.

241. Compound Generators of Different Capacities.—If two compound generators of unequal capacity are to operate well in parallel, dividing the load according to their capacities, the two machines must be adjusted to give the same no-load and full-load voltages, and *the resistances of the two series-field circuits must be inversely proportional to their capacities.* This last condition evidently is fulfilled if the full load IR drop is the same for each series-field circuit.

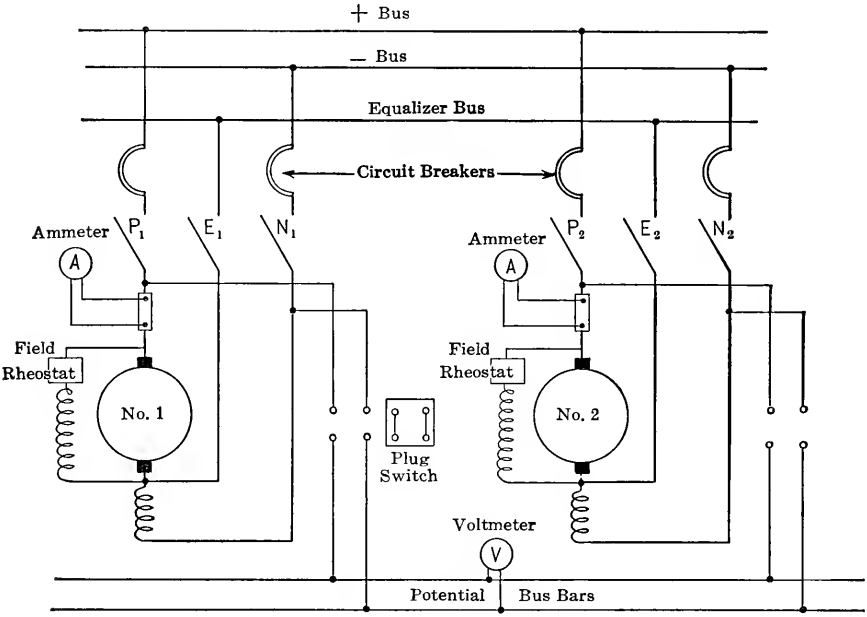


FIG. 270.—Proper layout of machines and apparatus for compound generators in parallel operation.

242. Switching of Compound Generators.—In Fig. 270, detail connections for two compound generators, intended for parallel operation, are given. The connections for two shunt generators would be the same, except for the series fields and equalizer connections.

When it is necessary to connect an additional generator (say No. 1) to the bus-bars, it is first brought up to rated speed, and the main circuit-breakers closed. Switches E_1 and N_1 are then closed, passing some of the load current from the other generators through the series field of the incoming machine. The voltage of No. 1 is then adjusted to be the same as that of the bus-bars, and switch P_1 closed.

It is not necessary that switches E_1 and N_1 be closed before the voltage of No. 1 is adjusted; if the voltage is first adjusted, it will be found that further adjustment will be necessary after switches E_1 and N_1 are closed owing to the current set up in the series field of No. 1. Whatever order of switching is followed, however, it is necessary that the switch P_1 be the last to be closed, the voltage of the machine having been properly adjusted to be equal to that of the bus-bars. The machine is then made to assume its proper share of the load, by strengthening its shunt field.

To disconnect a generator from the bus-bars, its load is first reduced to zero by increasing the resistance of its shunt-field circuit, when its generated voltage will be equal to the bus-bar voltage. Switch P_1 is *first* opened, after which the other switches and the circuit-breakers may be opened in any order, and the machine shut down.

It is evident, when a generator is to be connected in parallel with others already operating, that it must come in with proper polarity; i.e., its positive and negative terminals must be connected to the positive and negative bus-bars, respectively. The voltmeter connections of Fig. 270, being arranged symmetrically, take care of the fulfillment of this condition. If a generator, by any chance, had had its residual reversed and had built up with reversed polarity, the operator would recognize the fact at once; the voltmeter, which he would require in adjusting the voltage of the machine, *would be reading backward.*

To meet the difficulty arising when a machine intended for parallel operation builds up with reversed polarity, the obvious thing to do is to properly restore its residual magnetism. With compound generators, this will ordinarily be done if, with considerable load on the system, switches E_1 and N_1 (Fig. 270) are closed, exciting the series field. Should this fail, the shunt field may be separately excited. An easy way of doing this is by lifting all the positive or negative brushes and closing all switches and breakers of the machine for an instant. The brushes of the machine must not, however, be dropped until at least switch P_1 is again opened.

PROBLEMS

1. Assuming that each coil generates an e.m.f. wave of the form of a sine wave, find the variation in line e.m.f., in percentage of the maximum line voltage, of

(*a*) A two-coil winding with a two-part commutator;

(*b*) A four-coil winding with a four-part commutator;

(*c*) An eight-coil winding with an eight-part commutator.

2. A bipolar generator has 36 coils of 12 turns each. The active length per turn is 14 inches. The pole face covers 65 per cent of the armature periphery, and the flux density under the pole is 8500 gausses. Peripheral speed is 3600 feet per minute. What is the generated e.m.f.?

3. If the total length per turn in the above problem is 26 inches and the wire is No. 12, what is the safe current capacity of the armature, and what is the full load terminal voltage? Allow 2 volts for brush contact drop.

4. An armature winding requires 800 feet of No. 10 wire. There are six paths in parallel, with the winding scheme used. What is the resistance of the armature at 20° C.? At 75° C.?

5. An armature winding having 288 conductors has a two-path winding. The field has four poles, and the flux per pole is 750,000 lines. The armature turns 1800 r.p.m. What is the generated voltage?

6. An eight-pole generator has a multiple-circuit winding. There are 264 coils of 4 turns per coil, the conductor having 30,000 circular mils cross-section. The total length per turn is 44 inches, of which 18 inches is active in cutting flux. The armature is 40 inches in diameter and turns 250 r.p.m. The flux density directly under the pole, where 65 per cent of the conductors lie, is 9500 gausses, and in the pole fringe, where 10 per cent of the conductors lie, is 4400 gausses. Find the generated e.m.f., the armature resistance at 90° C., and the terminal voltage at full load, 90° C., allowing 2.5 volts for total brush drop.

7. A small bipolar machine has 240 armature conductors and 650,000 lines of flux per pole. The wire used in the armature winding is No. 14, and the machine turns 2400 r.p.m. How much current can it safely carry? What is its generated voltage? What is the terminal voltage at 80° C., the total length of wire per turn being 16 inches and the brush contact drop being 2.6 volts?

8. The commutator of the machine, in Problem 5 above, has 72 bars, and the brush thickness is 0.5 inch. If the full-load line current is assumed as 50 amperes and the commutator diameter is 5 inches, what is the average rate of change of current during commutation?

9. What is the rate of current change during commutation in a coil of the machine given in Problem 6 when carrying full load current, the commutator being 24 inches diameter and the brushes being $\frac{5}{8}$ inch thick.

10. If the rate of current change in a full-pitch coil during commutation is 25,000 amperes per second, and the self-induction of the coil is 0.0008 henry, what is the proper flux density under the commutating pole, the length of the pole being 8 inches, the peripheral speed of the armature being 3500 feet per minute, and the number of turns per coil being six? There are as many commutating poles as there are main poles. Neglect the effect of armature reaction in the calculation.

11. The number of series ampere-turns per pole required at full load for a certain four-pole compound generator is 1250. There are $4\frac{1}{2}$ turns per pole, of 100,000 circular mils cross-section, the average length of a turn being 34 inches. If the rating of the machine is 110 volts, 750 amperes, what must be the resistance of the series-field diverter, if the temperature of the field coils at full load is 95° C.?

12. To generate 20 volts, a certain 110-volt shunt generator requires (at rated speed) a field current of 0.172 ampere. If the shunt-field circuit has a resistance of 135 ohms, will the machine build up? If the speed were increased 15 per cent above normal, what would be the answer?

13. The shunt-field current of a certain 110-volt, 25-kw. shunt generator has to be increased from 3.6 amperes to 4.1 amperes, to give flat compounding. There are 1300 turns on each field pole. If the machine is to be equipped with series field for flat compounding across the armature, how many turns per pole are required?

14. The armature copper of a certain machine rises 10° C. with a current of 75 amperes. Assuming a safe temperature rise of 55° C., how much current can the machine carry? (Neglect the effect of core loss in this problem.)

15. Two equally compounded generators of 50-kw. and 150-kw. rating are to be operated in parallel. The series-field circuit resistance of the 50-kw. machine is 0.002 ohm. What must be the resistance of the series-field circuit of the other? If the actual resistance of the field winding is 0.0015 ohm, what must be the resistance of its diverter?

16. The type and speed of a 400-kw., 550-volt machine are such that a temperature rise of 90° C. in the armature would dissipate one watt per square centimeter. The available radiating surface is 62,000 sq. cm.; the armature resistance is 0.0135 ohm, and the iron loss is 9000 watts. What will be the rise in armature temperature at full load?

17. A 100-kw., 250-volt, eight-pole generator has an armature periphery of 360 cm. The width of each pole face is 35 cm., and there are 161 slots with two conductors in each, all conductors connected as a two-path winding. How many cross-magnetizing and demagnetizing ampere turns are there per pair of poles, if the brushes are advanced to be opposite the pole tips? How many, if the same conductors are re-connected as an eight-path winding, the current density in the conductors to be the same as before?

18. A 6-pole, 150-kw., 250-volt generator has an armature surface and ventilation such that it radiates 0.05 watt per square inch per degree C. rise. The armature resistance is 0.0095 ohm, the hysteresis loss in the teeth is 610 watts, and in the core proper 1240 watts; the eddy-current loss in the teeth is 50 watts and in the core proper 90 watts. The radiating surface is 2600 square inches. What is the temperature rise? What is the safe current through the armature, if a rise of 55° C. is allowed.

19. A rectangular field coil, 9 inches by 15 inches, carries a current of 10 amperes and has a resistance of 3.9 ohms. The heat dissipated is 0.003 watt per square centimeter per degree C. rise. How long must the coil be, if a temperature rise of 50° C. is allowed?

CHAPTER IX

THE CONTINUOUS-CURRENT MOTOR

243. Relation between Generator and Motor.—As has been said before, the c.c. generator and the c.c. motor are nearly identical in construction. The construction which is best for one is generally best for the other; a machine which runs well as a generator will generally operate satisfactorily as a motor.

244. Difference in Operation.—Although the construction of the two is practically the same, the operation of one machine differs quite materially from that of the other. The generator is always rotated by some prime mover, at a speed as nearly constant as possible. The motor, on the other hand, has no prime mover, but is supplied with electrical power, instead of mechanical power, as is the case with the generator. The speed of an electric motor is generally not constant as the load is varied; in some cases this speed variation may be small, while in others the highest speed of operation may be several times as great as the lowest speed.

The function of a generator is to *generate an e.m.f.* by moving conductors through a magnetic field, while the prime function of a motor is to *produce a turning effort*, or *torque.*

245. Torque Acting in a Generator.—When a machine is operating as a generator, and its armature is carrying current, it too, develops a torque (because the armature consists of conductors carrying current, in a magnetic field). This torque *opposes* the motion of the armature and must be overcome by the prime mover, in order to keep the armature of the generator operating at constant speed. The current in the armature flows *with* the e.m.f. generated in the moving armature. We may say, therefore, that so far as mechanical power is concerned the generator is *absorbing* energy, and so far as electrical power is concerned the machine is *giving out* energy.

246. Torque Acting in a Motor.—In the case of a motor, the armature turns in the same direction as that in which it is urged by the force acting on the armature conductors; the mechanical power must therefore be positive, or *output.* When the armature revolves it must generate an e.m.f. (conductors moving in a magnetic field generate an e.m.f.), and the current in the motor armature flows in a direction opposite to this e.m.f.; the e.m.f. generated in a motor armature is therefore called a *counter* *e.m.f.* As the current flows against the armature e.m.f., the electrical power of a motor must be negative, i.e., *input.*

247. Torque of a Motor.—A motor develops torque, because on the periphery of its armature are placed conductors, through which current flows, and those conductors lying under the pole faces are in a magnetic field. These conductors are then acted upon by a force which tends to move them in a direction perpendicular to the magnetic field and to their length; such a force must then act as a tangential force on the periphery of the armature.

The fundamental formula which we shall use in calculating torque (Eq. 6, Chap. III) gives the relation between the length of the conductor, the strength of the field in which the conductor is lying, the current in the conductor, and the force on the conductor. *If a conductor l cms. in length lies in a uniform magnetic field of a density of B lines per square centimeter (direction of conductor being perpendicular to field) and carries a current of I amperes, then the conductor is acted upon by a force which tends to move it in a direction perpendicular to its length and to the direction of the magnetic field; the magnitude of this force, in dynes, is given by the equation*

$$f = BlI/10. \qquad (107)$$

If we wish to express B in lines per square inch, l in inches, I in amperes, and f in pounds, since there are 2.205 pounds and 981,000 dynes in a kilogram, we shall have,

$$f = 2.54l \times \frac{B}{2.54^2} \times \frac{I}{10} \times \frac{2.205}{981{,}000} = 0.885BIl \times 10^{-7}. \qquad (108)$$

Suppose that a conductor 10 inches long lies in a field of 60,000 lines per square inch and carries a current of 100 amperes. The force on the conductor in pounds is evidently

$$f = 60{,}000 \times 100 \times 10 \times 0.885 \times 10^{-7} = 5.31 \text{ pounds.}$$

If we desire the force in dynes, we have

$$f = \left(\frac{60000}{2.54^2}\right) \times (10 \times 2.54) \times \frac{100}{10} = 2{,}360{,}000 \text{ dynes.}$$

248. Direction of Torque the Same for all the Armature Conductors.—In Fig. 271 is sketched the section of a four-pole motor. The conductors marked (+) are carrying current away from the observer, and those marked (.) are carrying current toward the observer. The conductors under pole A carry current away from observer and the flux is directed downward; therefore they will all exert a force toward the left, as indicated by the arrow under them. Those under pole C carry current in the same direction as those under A, but the direction of the field is upward;

therefore the force is toward the right, as shown by the arrow under pole *C*. The conductors under the two poles *B* and *D* give forces up and down respectively, as shown by arrows, and it is now seen that all conductors tend to turn the armature in the same direction, i.e., counter-clockwise.

It may also be seen from Fig. 271 that the torque of an electric motor is a constant or steady one, not pulsating as in the case of reciprocating steam engines, gasoline and oil engines. This may readily be appreciated when one compares the acceleration of an electric trolley car with the acceleration of a train connected to a steam-driven locomotive.

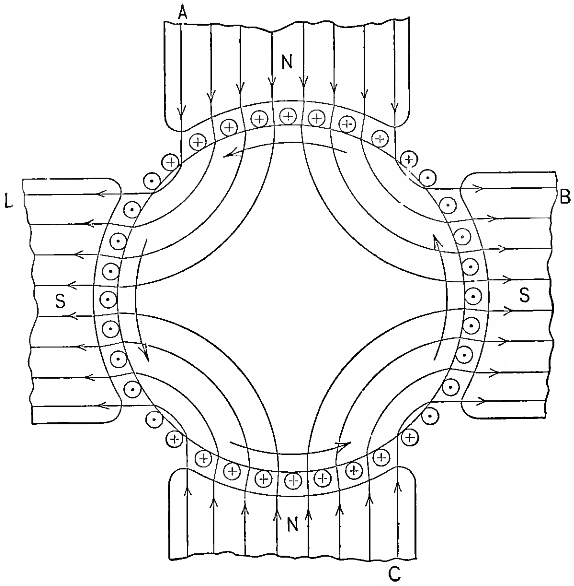


Fig. 271.—All of the conductors on the periphery of a motor give torque in the same direction; this is due to the fact that, by the scheme of connecting the conductors together, the currents in conductors under poles of opposite polarity flow in opposite directions.

249. Torque Calculation.—To calculate the magnitude of this torque, or turning effort of a motor, we must know the number of conductors lying in the magnetic field, the active length of each conductor (i.e., length of conductor under the pole face), strength of field in the air gap where the conductors are situated, and the current carried by the conductors. By the use of equation 107 we can calculate the force in dynes on one conductor, and this value, multiplied by the number of conductors situated in the magnetic field, gives the total force acting on the periphery of the armature.

It is to be noticed that those conductors lying in the interpolar space produce no turning effort; they are *inactive* conductors. When calculating the e.m.f. of a generator it was pointed out that these same conductors were inactive in the production of an e.m.f. in the armature; in fact, *that portion of the armature winding which generates an e.m.f. when the machine is operated as a generator serves to produce torque when the machine is operating as a motor.* In calculating the e.m.f. of a generator it was found advisable to divide the active conductors into two parts, those lying directly under the pole face, where the field has its maximum intensity, and those lying in the pole fringe where the field is weaker and non-uniform. The same division of conductors is advisable when calculating the torque of a motor.

Suppose we wish to calculate the peripheral force on the armature shown in Fig. 271. The length of the pole face we take as 15 cm.; the field we consider uniform and of a density equal to 8000 lines per square centimeter. There are 200 conductors on the armature, of which 60 per cent lie under a pole face, and the current in each conductor is 20 amperes. The force (in dynes) per conductor is given by

$$\begin{aligned} f = BlI/10 &= 8{,}000 \times 15 \times 20/10 \\ &= 240{,}000 \text{ dynes} \\ &= 0.245 \text{ kg.} \end{aligned}$$

There are 200×60 per cent = 120 active conductors.

The peripheral force on the armature is therefore $0.245 \times 120 = 29.4$ kg., and if the radius of the armature is 10 cm., the torque developed is 2.94 kilogram-meters.

250. Calculation of H.P. of a Motor.—If the peripheral speed of the armature is known, the output of the motor in ft.-lbs. per minute, or horsepower, is readily determined. If a body is exerting a force f, and moving in the direction in which this force is acting with a velocity v, then the rate of doing work is equal to fv. Hence, if we multiply the peripheral pull on the armature by the velocity of the armature periphery, the product obtained is equal to the amount of power which the motor is giving.

Let us consider a lap-wound armature 4 ft. in diameter, having 12 poles. The winding consists of 240 coils of 4 turns each and the length of the pole face is 10 inches. Sixty per cent of the conductors lie under the pole face where the flux density is 60,000 lines per square inch, and 15 per cent lie in the pole fringe where the average density is 35,000 lines per square inch. What horse-power is the motor developing if the current flowing into the armature is 480 amperes and the machine is rotating 200 r.p.m.?

There are two things to find—the peripheral pull in pounds and the peripheral velocity in feet per minute. The product of these two quantities divided by 33,000 (the number of foot-pounds per minute in 1 horsepower) will give the horse-power of the motor.

As the armature is lap wound and the machine has 12 poles, the winding must have 12 paths. Therefore the *current per path* $= 480 \div 12 = 40$ amperes, and this is the current in each conductor. The active length of each conductor is 10 inches. The total number of conductors $= 240 \times 4 \times 2 = 1920$. Of these, 60 per cent (i.e., 1152), lie in a field of 60,000 lines per square inch and 15 per cent (i.e., 288) lie in a field of 35,000 lines per square inch.

The force in pounds is therefore equal to

$$0.885\times10^{-7}\times40\{(1152\times10\times60{,}000)+(288\times10\times35{,}000)\}=2970;$$

The peripheral speed in feet per minute $= 4\times\pi\times200=2515;$

The horsepower developed therefore $=\dfrac{2515\times2970}{33000}=226$

The general expression for the horsepower developed by a motor may be written

$$\text{H.P.}=\frac{2\pi FLN}{33000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (109)$$

where F = force exerted at the periphery of the armature (or pulley);
L = radius of the armature (or pulley);
N = r.p.m.

The derivation of this equation is obvious when we consider that the product $2\pi LN$ is the peripheral velocity of the armature (or pulley). The product FL is the torque exerted by the motor; representing this by T, we may rewrite Eq. (109) and obtain the useful equation.

$$\text{H.P.}=\frac{TN}{5250} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (110)$$

251. Necessity of Commutator in a Motor.—We saw in the case of the generator that the function of the commutator was to so rectify the alternating e.m.fs. induced in the several coils, that the e.m.f. impressed on the external circuit was uni-directional. It will be seen from Fig. 271 that if in a motor, the torque exerted by the conductors on the armature is to be always in the same direction, the current in any one conductor must be reversed as the conductor passes from one pole to another. A c.c. motor is supplied with uni-directional current, and the commutator must therefore change this uni-directional current so as to make it an alternating one in each conductor.

252. Commutation in a Motor.—In Fig. 272, coil X of a motor armature winding is being commutated. In order that the forces exerted by the conductors under the north pole may be to the right, current in the conductors on the periphery of the armature must flow towards the observer (Fleming's left-hand rule); and under the south pole, it must flow away from the observer. The brush of the motor shown is therefore a negative one.

Coil X is shown in such position that the current in it is beginning to decrease; from what was said on the subject of commutation in the previous chapter, it is evident that the c.e.m.f. of self-induction, induced in the coil as its current reverses, will be in the same direction as the decaying current, as indicated by the dashed arrows. A commutation e.m.f., to be opposite to this c.e.m.f. of self-induction, must, by Fleming's right-hand rule, be induced under the south pole, so that, to get the proper condition for sparkless commutation, the brush must be moved backward to such a position that the coil being commutated is moving under the fringe of flux from the south pole. The direction of e.m.f. induced in the coils of the armature by their motion in the magnetic field is indicated by the dotted arrows.

Commutation flux, in a motor without commutating poles, must therefore be that of the pole which a coil is leaving. Applying this to the motor with commutating poles, it follows that the polarity of a commutating pole must be the same as that of the main pole just preceding it in the direction of rotation. Figures 233 and 234 can be made to apply to the commutating-pole motor by reversing the direction of rotation from that shown there.

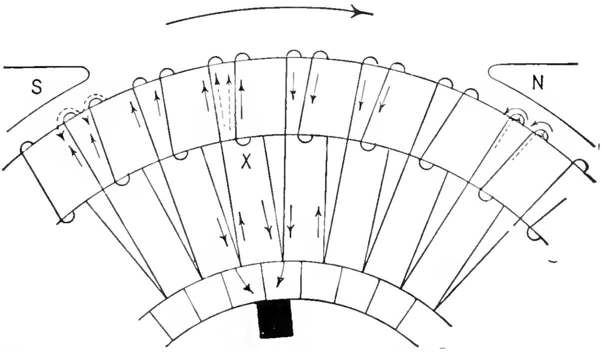


Fig. 272.—While the coil of a motor armature is short-circuited by a brush its current must reverse. The e.m.f. of self-induction of the coil tends to prevent the reversal, acting in the direction of the dashed arrow in coil X. The direction of the generated e.m.f. in the coils under the poles is shown by the dotted arrows. To overcome the e.m.f. of self-induction the brush must be moved backwards to bring the short-circuited coil in the fringe of the south pole.

253. Effect of Armature Reaction in a Motor.—In a generator, the armature current *flows in the direction of the induced e.m.f.*, and in the motor, the armature current *flows against the induced e.m.f.* (called the counter e.m.f.). We should, therefore, expect the armature m.m.f. of the motor and generator to be in opposite directions, and the main field in the motor to be twisted *against the direction of rotation.*

This may be seen from Fig. 273, which represents conditions in a motor, just as Fig. 225 represented them for a generator. As in the latter figure, the conductors in the angle $A'OC$, combined with those in the angle DOB', make up the demagnetizing turns. The conductors included in the angle COB', combined with those in the angle $A'OD$, constitute the cross-magnetizing turns. The main field m.m.f. is shown vectorially by OE, the armature m.m.f. by OF, and the resultant m.m.f. by OK; the armature m.m.f. is resolved into its components, the demagnetizing force, OG, and the cross-magnetizing force OH.

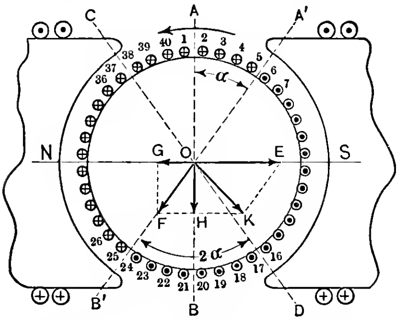


FIG. 273.—The direction of current in the armature conductors and the backward shift of the brushes, through the angle α, results in the armature m.m.f. as shown at OF; this, combined with the main field m.m.f., OE, gives as resultant OK, and this will therefore be nearly the direction of the actual magnetic field in the armature.

Armature reaction in a motor, therefore, produces the same effects as in the generator; the main field is both weakened and distorted, the direction of distortion, however, being opposite to the direction of rotation in the motor, whereas it was found to be in the direction of rotation in the generator. This is indicated in Fig. 274, the direction of the twisted field for generator action being indicated in dotted lines and that for motor action in full lines.

254. The Armature of a Motor as a Rotating Body.—To rotate a body requires the application of a driving torque, or turning effort. A resisting torque, due to the friction of the bearings, will always be present in addition, and will be augmented by the resisting torque due to the load if the rotating body is doing work.

Now, if the driving and resisting torques are exactly equal, the angular speed of the rotating body will remain exactly constant; but if one becomes greater than the other, a change in speed must take place. Such a change in speed will continue as long as the inequality between driving and resisting torques exists. Thus, if a rotating body with constant driving torque has its resisting torque increased so as to be greater than its driving torque, the speed will decrease steadily until the body stops. To start it again, the driving torque must be made greater than the resisting torque; the body will then continue to accelerate.

Consider that a line of shafting, driven by a steam engine, is to be started. Steam is turned on, and if the steam admitted is sufficient to give the engine enough torque, the shafting starts to rotate; the driving torque applied to the shafting is thus somewhat greater than the resisting torque due to friction of the bearings. If the torque of the steam engine remained constant after the shafting had started to rotate, and if the bearing friction remained constant, the speed of the shafting would gradually increase indefinitely; but as friction in practice generally increases with the speed, the speed finally reaches some definite value; at this point, the resisting torque is just equal to the driving torque. If the speed is to be further increased, or load put on the shafting, the driving torque must be increased by admitting more steam into the engine.

The armature of a motor, being a rotating body, is subject to the same laws. The driving torque of a motor is generated by the forces exerted between the armature current-carrying conductors and the field; its resisting torque will be due to such rotational losses as occur, together with that due to the load.

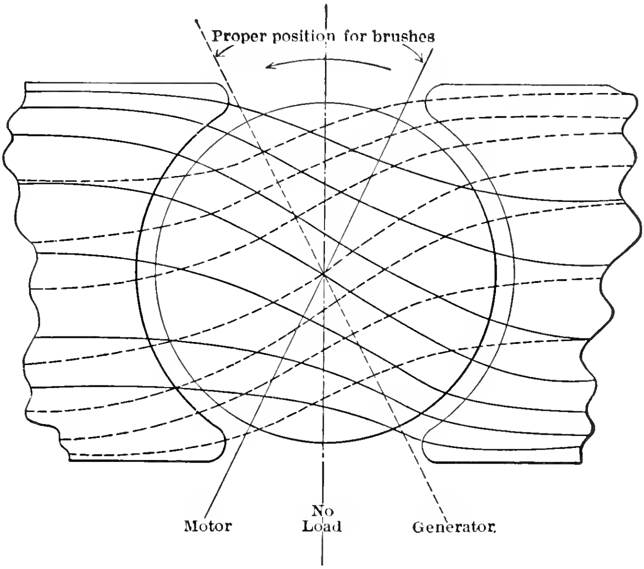


FIG. 274.—Showing the twist of the field of a motor to be in a direction opposite to that produced by the armature reaction of a generator.

It will be shown that the electric motor is a self-regulating machine. Whenever the resisting torque applied to the motor is changed, a change in the speed will, in general, occur, and this change in speed will automatically bring about reactions within the motor armature which will cause the driving torque to again become equal to the resisting torque.

In considering how a given type of motor will act under load, the student is advised always to begin his reasoning with the ideas on rotating bodies presented above, and then apply whatever conditions are imposed upon the motor.

255. Expression for the Torque of a Motor.—In Eq. (107) it was shown that the force on any motor armature conductor is dependent only upon the flux density in which the conductor is placed and the current flowing through it; *the force is therefore independent of the speed with which the conductor is moving.* The torque exerted by a motor armature, being the summation of the forces of all the armature conductors, multiplied by their distance from the center of the armature, is therefore dependent only on the flux entering the armature per pole and the armature current.

We may write, then,

$$T=K'\phi I_a, \quad \ldots \ldots \ldots \quad (111)$$

where T = the torque of the motor;
ϕ = the flux per pole entering the armature;
I_a = the armature current;
and K' = a constant involving the number of poles, the number of armature conductors, the number of parallel paths in the armature winding and such factors as are necessary to express the torque in desired units, as pound-feet, kilogram-meters, etc.

256. Counter E.M.F. of a Motor.—We have said that, in the case of the motor, the armature turns in the direction in which it is urged by the force acting on the armature conductors. With the field of a motor set up in a certain direction, the direction of the force acting on the armature conductors is dependent on the direction of the armature current, and this must be that in which the voltage of the line, to which the motor is connected, is acting; in a motor the impressed voltage and armature current are in the same direction.

The motor armature conductors, moving in a magnetic field, generate a counter e.m.f., which, being opposite to the direction of current flow, is also opposite to that of the voltage impressed on the armature.

The generated e.m.f. of a generator was found in Eq. (101), to be

$$E_g=\frac{p\phi NZ}{m\times 60\times 10^8}$$

in which Z = total number of conductors on the armature;
p = number of field poles;
m = number of parallel paths in armature winding;
ϕ = total flux per pole, entering the armature;
N = revolutions per minute of armature.

This equation, for a given machine, reduces to

$$E_g = K\phi N, \quad \dots\dots\dots\dots \quad (112)$$

in which $K = \frac{pZ}{m \times 60 \times 10^8}$, all of which are constants.

This must also be the expression for the c.e.m.f. of a motor, since it is the generated e.m.f. of the armature. We have then

$$e = K\phi N, \quad \dots\dots\dots\dots \quad (113)$$

where e = the counter e.m.f. of the motor.

257. Types of Continuous-current Motors.—The classification of motors is generally made according to the kind of field windings they have. The three types are the *shunt, series*, and *compound.* In the shunt-wound machine, the field consists of many turns of fine wire and is connected in parallel with the motor armature. The series motor has a field winding consisting of a few turns of heavy wire connected in series with the armature. The compound motor has two sets of field coils; one of many turns of fine wire in parallel with the armature, and another of a few turns of heavy wire in series with the armature.

The series winding of a compound motor may be so connected that it *assists* the shunt winding in magnetizing the field, or it may be so connected that the m.m.fs. of the two windings *oppose* one another. The first is called a *cumulative-compound* motor and the second a *differential-compound*, or, as it is generally called, simply a *differential* motor; the latter type is of so little use and of so little practical importance that it is seldom met in practice. The term, compound motor, is therefore used to designate that type in which the series and shunt fields assist one another.

The characteristics of the three types of motors are given here, and will be explained more fully in the later paragraphs.

The shunt motor has a fair starting torque and nearly constant speed for all loads. It is used where the load requires practically constant speed and the starting torque demanded is not excessive.

The series motor operates through a wide range of speed as the load changes, and at very light loads the motor is likely to "run away," i.e., reach dangerous speeds. It gives very great starting torque and is therefore used where a heavy starting torque is demanded and the motor may be positively connected to its load. The principal application of this motor is in railway service.

The compound motor has a starting torque greater than that of the shunt motor, but less than that of the series motor. It has, however, a definite upper speed limit, and even if all of its load is removed it will not run at dangerous speeds. Its principal application is in elevator service, machine-tool drive, etc., where a fixed speed limit is necessary and considerable starting torque is required. The number of series turns used on the field coils varies somewhat, according to the service required of the motor; but, in general, we may say that, at full load, the series ampere-turns are from 10 per cent to 50 per cent of the shunt ampere-turns. The decrease in speed from no load to full load may vary from 12 per cent to 50 per cent in different motors.

258. Shunt Motor. Current-torque Curve.—The relation between the torque developed by a motor and the current flowing through its armature winding is shown by a current-torque curve. This curve has different forms in motors with different styles of field windings.

The simplest case is that of the shunt motor. The field current of this machine is independent of the current flowing through the armature, because its field windings are connected directly across the supply line, the voltage of which is assumed constant. Now, if the field current of a motor is constant, the strength of its magnetic field is practically constant. Hence, from Eq. (111), it is seen that the current-torque curve of a shunt motor must be a straight line, the torque being directly proportional to the armature current. This is shown in Fig. 275 in which the result of a laboratory test is given. The curves are practically straight lines, curve *A* being for a weak field (all of the field rheostat was in series with the field windings) and curve *B* for a strong field (field rheostat " all-out ").

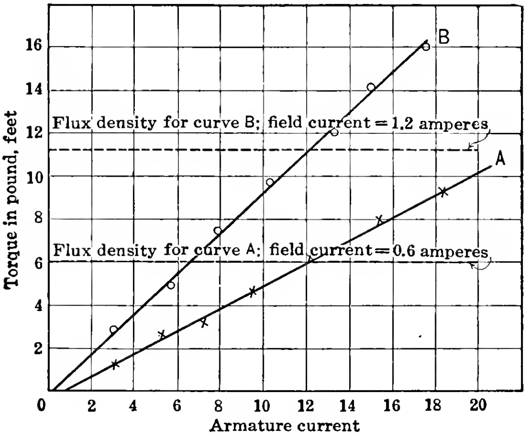


Fig. 275.—Variation of torque of a shunt motor as the armature current is varied; it is shown for two values of field density. Although one field current is twice as large as the other, the field density, and hence torque, is not twice as great, because with 1.2 amperes in the field coils the magnetic circuit is approaching saturation.

The fact that the flux density did not increase proportionately with the field current indicates that the iron of the field circuit is approaching saturation.

259. Shunt Motor. Action under Load.—Since the counter e.m.f. and the resistance reaction of the armature both act against the impressed e.m.f. of the motor, we may put

$$E=e+I_aR_a, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (114)$$

where E = the voltage of the line to which the motor armature is connected;
e = c.e.m.f. of the motor;
I_a = the armature current;
R_a = the resistance of the armature.

From the last equation we get

$$I_a=\frac{E-e}{R_a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (115)$$

If the load on a shunt motor is increased, its driving torque (i.e., the torque developed by it) must increase; and, since the field flux of a shunt motor is approximately constant, it follows from Eq. (111) that the armature current must increase.

The only variable in the right-hand member of the expression for armature current, Eq. (115), is the counter e.m.f.; therefore, if the armature current is to increase, e must decrease, and vice versa. From the equation, $e=K\phi N$, with the field flux constant, a decrease in e requires a decrease in speed.

Consider that a shunt motor is carrying a load torque T_1, with an armature current I_1, and running at a constant speed N_1, as represented in Fig. 276; the driving torque of the motor is hence also T_1. At time t_1, the load torque is suddenly increased from T_1 to a value T_2. The load, or resisting torque, being thus greater than the driving torque, it follows, from the analysis given above, that the speed of the motor must at once begin to fall, along some such line as AB. Since the field flux will remain practically constant, the c.e.m.f. will also fall, nearly in proportion to the speed, allowing the armature current to increase along some such line as CD; with increasing armature current and field flux practically constant, the driving torque increases along the line EF. This action continues until, at time t_2, the driving torque is again equal to the load torque, and the speed continues at the constant value N_2.

At time t_3, the load torque drops to a value T_3, and at once the speed starts to increase, the driving torque being greater than the load torque at the instant after the load decreased. The rising speed increases the c.e.m.f. proportionally, which in turn reduces the armature current, so that the driving torque becomes less. At time t_4, the driving torque has decreased sufficiently to be equal to the new load torque, and equilibrium is again reached.

260. Shunt Motor. Speed-load Curve.—From what has just been said, it is evident that the shunt motor must have a definite no-load speed.

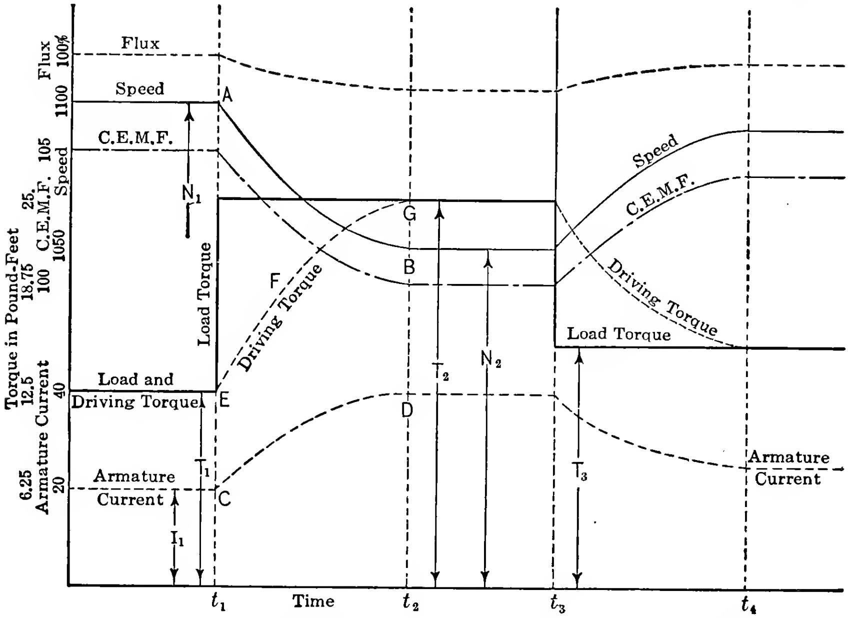


FIG. 276.—These curves show the cycle of events when the load on a shunt motor is first increased and then decreased. With increased armature current the flux diminishes somewhat, due to increased demagnetization by the armature m.m.f.

A relatively small armature current is necessary to run the motor without any load; its no-load c.e.m.f., therefore, approaches within about 1 per cent of the value of the impressed voltage; obviously, the c.e.m.f. cannot become equal to the impressed voltage if the machine is to operate as a motor.

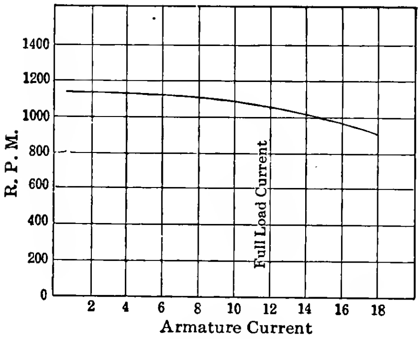


FIG. 277.—Speed-load curve for a typical shunt motor.

As the load increases, the speed drops somewhat, decreasing the c.e.m.f. and so permitting more armature current to flow; with the average shunt motor, the drop in speed from no load to full load may be between 5 per cent and 10 per cent of the no-load speed. The amount of decrease in speed depends upon the armature resistance, as may be seen from Eq. (114); the greater the resistance the greater the decrease in speed.

The speed-load curve of a typical shunt motor is shown in Fig. 277.

261. Calculation of No-load Speed.—The no-load speed of a compound or a shunt motor is easily calculated. At no load, there is practically no IR_a drop, so that the counter e.m.f. must be equal to the impressed e.m.f. The no-load speed may therefore be found by calculating what speed the motor would have to run (as a generator) to generate in its armature an e.m.f. just equal to that of the supply line.

262. Armature Reaction and Resistance Act to Neutralize One Another.—Although the shunt motor is generally considered as having constant flux for all loads, there is some reduction in its field strength, due to armature reaction.

The effect of field weakening, brought about by armature reaction, tends, in general, to cause an increase in the speed. The question of field weakening will be considered in greater detail later on, but its general effect may be seen from the equation for the c.e.m.f., $e=K\phi N$. If the field is weakened, to generate a given counter e.m.f., the motor speed must increase, inasmuch as armature current is fixed by the torque that the motor must exert.

Although this speed increase does not usually occur in commercial motors, the effect of field weakening by armature reaction may be made sufficient to overcome the effect of armature resistance drop, in which case the speed-load curve will be nearly flat, giving no speed decrease with increase of load. If the brushes are shifted too far back, the weakening of the field caused by the armature reaction may be sufficient to make the motor *speed up* with increase of load. With this condition there will generally be some sparking at the brushes.

If the brushes are shifted forward, the armature reaction tends to magnetize the main field; the speed-load curve of the shunt motor with brushes having a forward shift is nearly the same as that of a compound motor. This condition is never met in practice as the brushes of a motor spark viciously if shifted forward to any extent.

263. Series Motor. Current-torque Curve.—The series motor gives a current-torque curve differing from that of the shunt motor, because the field strength of a series motor depends upon the current flowing through its armature, the field and armature being connected directly in series. The equation for torque involves the product of the field strength and armature current; when the field strength is directly proportional to the current through the field coils, the *torque must vary as the square of the current.*

This is the case with the series motor at light loads. At values of armature current near full load (and for all currents of higher value), the field is approaching saturation, and therefore the field strength is not proportional to the current through the windings. At very high values of current, the strength of the field is practically independent of the current, and thus the current-torque curve tends to become a straight line, similar to that of a shunt motor. In Fig. 278 the results of a test are shown. The magnetization curve of the machine is given for reference, and it is seen that as long as the magnetization curve is a straight line the torque-current curve is parabolic in form, but that for currents which begin to saturate the field (shown to be about 14 amperes in Fig. 278), the curve becomes less steep and approaches a straight line in form.

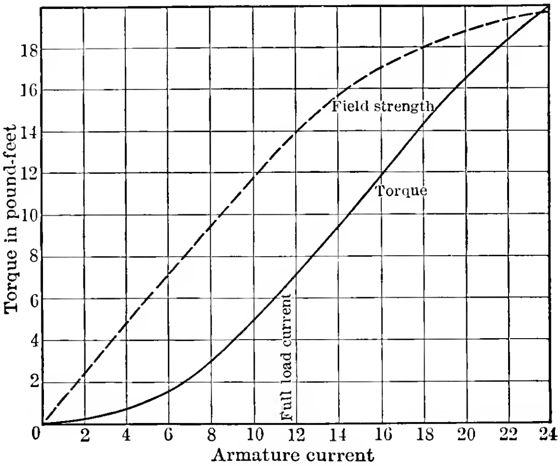


FIG. 278.—Current-torque curve for a series motor; it is parabolic in form until the current is sufficiently large to produce saturation in the magnetic circuit.

264. Series Motor. Speed-load Curve.—The fact that the field strength of a series motor depends upon the current flowing through its armature, also gives it a speed-load curve different from that of the shunt motor.

Since the current taken by a series motor flows through both armature and series field, we may write

$$E = e + I_a(R_a + R_{se}), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (116)$$

where R_{se} = the resistance of the series field, and the other terms have the same significance as in Eq. (114).

The resistance of the series field is always low, less usually than that of the armature; the value of the c.e.m.f. will therefore approach within perhaps 15 per cent of that of the impressed voltage.

If the load on a series motor is increased, the speed of the motor must start to fall; this allows the c.e.m.f. to decrease, with an attendant increase in both armature current and field flux. The generated torque thus increases, owing to an increase in both armature current and flux; the field flux cannot, however, increase unless the armature current becomes greater; therefore, from Eq. (116), there must be a decrease in c.e.m.f., or rate of cutting of the field flux, to which the c.e.m.f. is proportional. We may say; then, that in the series motor, with increased load the speed must fall a good deal, to permit the necessary decrease in c.e.m.f., in spite of the increase in flux. In the shunt motor, as the flux remained constant, the speed had to fall comparatively little to allow the c.e.m.f. to decrease the proper amount.

The series motor thus decreases its speed very much as its load is increased, as may be seen from Fig. 279. As full load is approached, the field generally begins to be saturated, so that the field flux does not increase as much for a given increase in armature current as at light loads. As a result, the speed does not have to fall as much as at light loads, and the speed-load curve tends to straighten out at about full load.

As the load on the series motor is decreased, the speed increases rapidly and at very light loads the motor runs at speeds far above the safe speed. For small values of generated torque, only small values of armature current are required, resulting in low values of flux. If the armature current is small, the value of the c.e.m.f. will be nearly that of the impressed voltage; since $e=K\phi N$, a high value of speed is required with a weak field. If all load were taken off a series motor, the speed would rise to such a value that excessively large centrifugal forces would be brought into play, with the result that the winding would be thrown off the armature core and the commutator would probably be thrown to pieces.

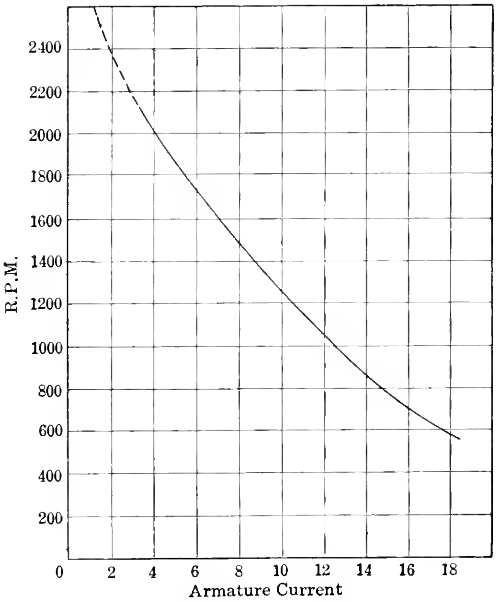


FIG. 279.—Speed-load curve of a series motor; compare this with the curve of Fig. 277, which is for a shunt motor having the same full-load speed.

This peculiarity of the series motor limits its use to those classes of service in which the motor may be directly (or by gears) connected to its load, so that under no condition would it be running without enough load to hold its speed down to a safe value. A series motor should never be belted to its load, because if the belt should run off the motor pulley, the motor would immediately start to "race," and in a few seconds would be damaged. In electric railway installations, the motor is *geared* to the car axle, so that there is never any possibility of its running at excessive speeds.

265. Compound Motor. Current-torque Curve.—The compound motor gives a current-torque curve the shape of which is intermediate between those of the shunt and series types. The curves for such a motor are given in Fig. 280; the field strength at zero armature current is given by the shunt coils only, but as the armature current increases the field strength increases somewhat, on account of the m.m.f. of the series coils.

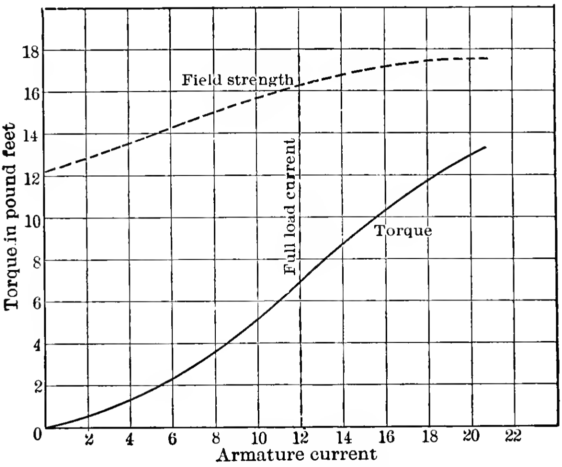


FIG. 280.—Current-torque curve of a compound motor; the dashed curve shows how the field strength increases with armature current, due to the action of the series field winding, which carries the armature current.

266. Compound Motor. Speed-load Curve.—The curve of the compound motor has generally much the same shape as that for the shunt motor, but the speed decrease with increase of load is much more marked, as may be seen from Fig. 281. The relative strengths of the series and shunt fields of a compound motor are fixed by the service for which the motor is intended. In most applications, a series field, such that at full load about 40 per cent of the total field m.m.f. is furnished by it, is sufficient. As the series field is made relatively stronger and stronger with respect to the shunt field, the compound motor approaches the characteristics of the series motor. Rolling-mill motors are sometimes so built that the series field furnishes as high as about 80 per cent of the full-load field m.m.f. Such motors will have practically the characteristics of a series motor, the shunt field being added to prevent them from running away in case they are accidentally disconnected from the rolls.

267. Comparison of Motor Characteristics.—In Figs. 282 and 283 are shown the respective current-torque and speed-load curves for shunt, series, and compound motors of the same full-load output and speed; the full-load torque of each motor must therefore be the same.

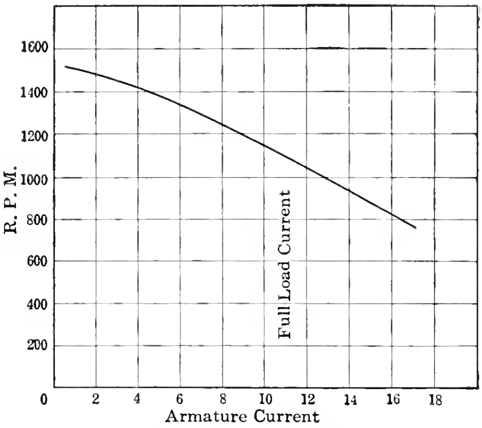


FIG. 281.—A typical speed-load curve for a compound motor.

We have seen that the torque exerted by a motor is dependent on the value of its armature current and field flux, and independent of speed. The current-torque curves of Fig. 282 are therefore characteristic of the respective motors, whether the machines are running or not. The torque exerted by a motor when not running is called its *starting torque*, and it will be seen from the current-torque curves of Fig. 282 that with 200 per cent full-load current (which might be safely put through a motor for the short time necessary for starting) the series motor gives much larger starting torque than either the shunt or compound motor, and that the compound motor will exert more starting torque than the shunt motor.

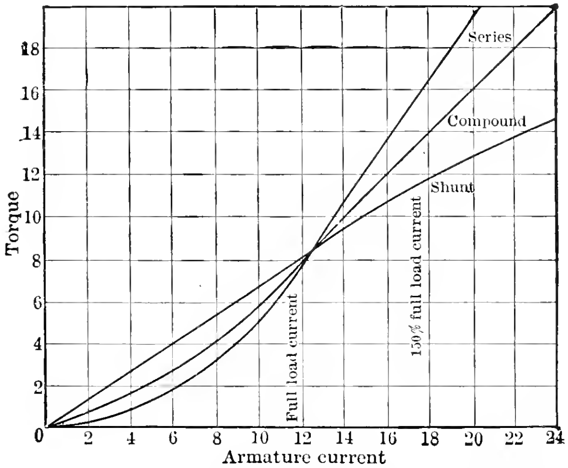


FIG. 282.—Current-torque curves for shunt, series, and compound motors having the same torque and current at full load.

Reference to the speed-load curves of Fig. 283 will show the wide range of speed of the series motor in comparison with the fairly constant speed of the shunt motor. For the series motor considered, a speed of about 2150 r.p.m. is considered safe; higher speeds are dangerous. The safe speed for any motor is a matter of design and construction, depending upon the means employed to care for the centrifugal forces developed in the armature and commutator.

The motors chosen for comparison in Figs. 282 and 283 were all of the same rating, giving equal horse-power output at the same full-load speed, and therefore developing the same full-load torque. Let us also consider a set of three motors, shunt, series, and compound, all of the same horse-power output but having the same speed *at light loads,* as represented in Fig. 284.

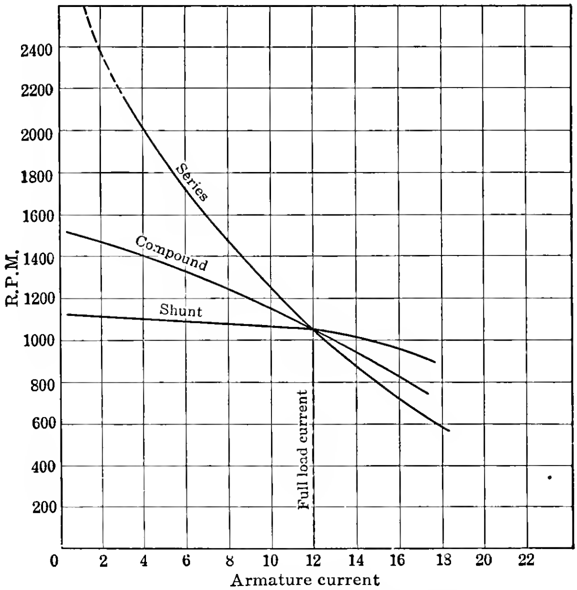


FIG. 283.—Comparative speed-load curves for shunt, series, and compound motors having the same speed at full load.

From Eq. (110), since the three motors have the same output, their full-load torques will be inversely as their full-load speeds. It is obvious, then, from these curves, that the series motor considered is able to start and turn over a very much larger load with a given armature current than the shunt motor with which it is compared, and that it is able to exert the same torque as the shunt motor with much less current at all but very light loads.

Considering the equation for the torque of a motor ($T=K\phi I_a$) in terms of flux and armature current, in connection with the group of motors of Figs. 282 and 283, since they all develop the same full-load torque with the same value of armature current, it follows that they must all have the same full-load flux; the dimensions of their armatures and field structures will therefore all be about the same.

However, if the series motor of Fig. 284 exerts four times the torque at full load, with the same armature winding, as the shunt motor of the same figure, it must have four times as much flux threading its field frame and armature. It would probably have twice as much flux, and twice as many armature conductors. Obviously, either of these conditions requires that the series motor be of much larger dimensions than the shunt motor to which it is compared.

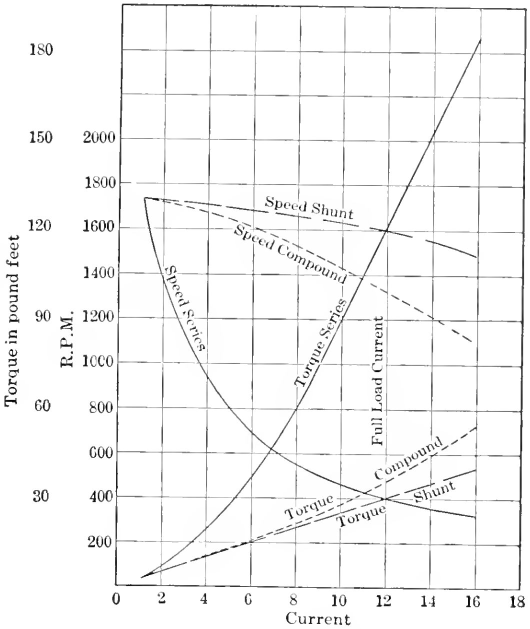


Fig. 284.—Comparative speed-load curves for shunt, series, and compound motors having the same speed at light load.

268. Applications of Motors.—The principal factor which determines the selection of one type of motor or another for a certain class of work is the variation of its speed as the load on the motor is varied.

The shunt motor is used for such service, where practically constant speed is required regardless of load variation, such as driving machine tools, blowers, fans, line shafting, etc. A special form of the shunt motor, known as the *adjustable-speed* motor, will be considered further on. In this type of shunt motor, the speed can be fixed by the operator at any value between a minimum and maximum value, but when once so set will remain substantially constant for all loads. Adjustable-speed motors are used largely in individual drives for machine tools.

Compound motors are used for driving machines that are subject to sudden applications of heavy loads, as in punches, shears, etc. Their use in connection with flywheels will be taken up later. Compound motors are also useful in cases where large starting torque is required, but where it is desired to have less speed variation under load than a series motor would give.

Series motors are particularly adapted for railway and hoisting service, because of their rapid drop in speed as the torque demanded by the load is increased. Railway practice requires a reasonably high speed on level track, and the development of large torque for starting and operation on grades, with as small a current as practicable. That the series motor exactly meets these requirements may be seen by comparing the operation of the shunt and series motors of Figs. 282 and 283, when used to propel a street car. Both would start their loads equally well, and climb a grade requiring full-load current in about the same time. However, on level track, where the load is light, a car equipped with shunt motors would operate at a very low speed compared with one equipped with series motors.

If motors with characteristics as shown in Fig. 284 are used, the speed of the cars on level track would be the same. When called upon to climb a grade, the shunt motor would exert the required torque by excessive increase of its armature current and would continue at practically the same speed as on level track. The series motor, however, would promptly slow down and exert the required torque with very much less current than that which the shunt motor would draw; it would, of course, take longer in climbing the grade. The fact that the series motor can exert a much larger torque with a given overload armature current is of great importance, inasmuch as it affects the size of the feeders and generating station.

The characteristic curves of a typical series railway motor are shown in Fig. 285.

269. Effect of Change of Line Voltage on Speed.—If the voltage impressed on a motor varies, the speed of the motor may be expected to vary correspondingly; from Eq. (114), by substituting for e its value as given in Eq. (113).

$$E=K\phi N+IR_a,$$

or

$$N=\frac{E-IR_a}{K\phi}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (117)$$

This shows how the speed of a shunt motor may be expected to vary as either E or ϕ is changed.

By similarly using Eq. (116), we get for the speed of a series motor

$$N=\frac{E-I(R_a+R_{se})}{K\phi}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (118)$$

As the field circuit of a shunt motor is connected directly across the supply line, any change in line voltage must affect the flux of the motor, ϕ. Now, from Eq. (117) a reduction in impressed voltage causes a decrease in speed, whereas a reduction in ϕ tends to cause the motor speed to increase. If the field of the motor is not operated near saturation, the change in speed for a small reduction in line voltage is not very marked. If E increases, ϕ increases in nearly the same ratio, so that the speed does not increase very much.

In the case of a series motor, there is no shunt field and thus for a *given current in the armature circuit,* the speed varies in nearly the same ratio as the impressed voltage.

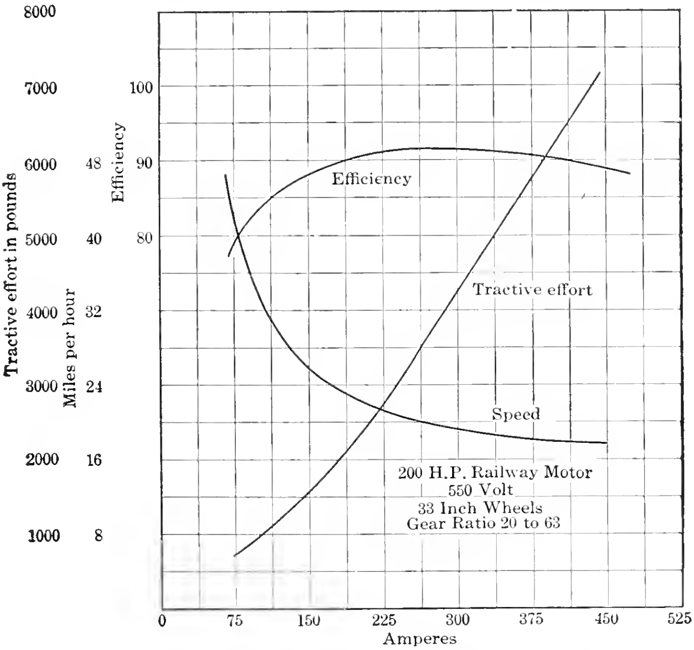


FIG. 285.—Characteristic curves for a typical railway motor. The motor is geared to the 33-inch car wheel in the ratio of 20 to 63, giving to the car, speeds and tractive efforts as indicated.

The compound motor, having both shunt and series coils, has a change of speed, with change in line voltage, greater than that of the shunt motor but less than that of the series motor.

270. Effect of Field Heating on Speed.—When a shunt motor is first started up after having been idle for several hours, the temperature of its field winding will be the same as that of the room. After it has run for several hours, however, the temperature of the field will rise and its resistance increase, reducing the field current.

A 50° rise in temperature will cause an increase in resistance of about

20 per cent and a decrease in field current of about 17 per cent. The resulting decrease in flux depends upon the degree of saturation at which the field operates, but there will usually be sufficient decrease in flux to bring about an increase in speed of about 10 per cent.

It is to be noticed that this action might be compensated by a field resistance which could be cut out as the field heats up. Commercially, however, shunt motors are not usually provided with field rheostats, the effect of field heating being of no importance.

In the compound and series motors, the effect of field heating has less effect. In the series motor it acts to cause a slight reduction in the speed for a given load.

271. Motors with Commutating Poles.—The problem of commutation is just as difficult to solve for the motor as for the generator. If a motor is not equipped with commutating poles, the brushes must be shifted just as much in the motor as in the generator, if sparkless commutation is to be obtained.

Commutating poles, which are very important for the successful operation of a generator, are even more important in motor operation, because the load variations are more sudden and violent. Practically all modern motors are fitted with commutating poles. The brushes of such motors are carefully adjusted before the machine is sent out from the factory, and are generally held in place by a set screw or clamp; *the brushes on a commutating-pole motor must not be shifted from this position*, as correctly determined in the factory.

272. Effect of Various Armature Conductors.—The poles and armature of a bipolar, commutating-pole motor are shown in Fig. 286, and the direction of the c-e.m.f. induced in each conductor is indicated. If the brushes are in the neutral position (shown by line AB in Fig. 286) there is no demagnetizing effect produced by the armature reaction, and in each half of the armature winding there is a certain number of active conductors generating the necessary counter e.m.f.; these conductors are indicated by X and Y in Fig. 286.

The conductors marked M and N add nothing to the counter e.m.f. of the armature, because in each half of the winding the e.m.fs. generated in these conductors neutralize each other. Considering the right-hand half of the armature in Fig. 286, it is seen that under the S commutating pole there are two inductors with a "negative" e.m.f., and under the N commutating pole there are two inductors generating a "positive" e.m.f.; hence in the right-hand side of the armature the effective counter e.m.f. must all be generated by the inductors marked X. The same reasoning shows that the only inductors generating the counter e.m.f. in the other path of the armature are those marked Y.

In spite of the balancing of the c.e.m.f. of conductors M and N, tending to show that the commutating poles have no effect on the speed load curve of a motor, there is an appreciable effect on the speed of a motor produced by the flux of the commutating poles where it enters or leaves the main poles. This effect depends upon the degree of saturation of the magnetic circuit, and will not be further analyzed here. Experimental verification of the effect is suggested in Experiment XI, Chapter XIV.

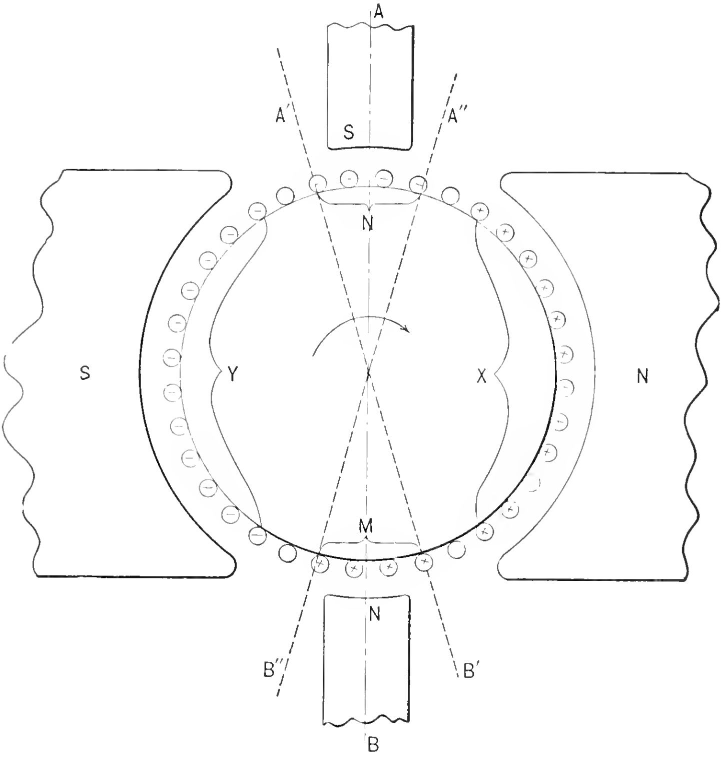


Fig. 286.—Practically all modern motors are equipped with commutating poles to bring about sparkless commutation. This diagram is to show that the e.m.f. generated under the commutating poles contributes nothing to the c.e.m.f. of the armature unless the brushes are shifted from the center line *A–B*.

273. Effect of Forward Brush Shift.—If the brushes are shifted *forward* to the plane $A'' B''$, the motor will slow down with an increase of load. This is due to three effects. The IR_a drop increases with the load, and so slows the motor down as shown before; the armature reaction in a motor tends to *magnetize the main field* when the brushes have a forward shift, and so increases the counter e.m.f., thus causing a speed decrease; when the brushes are in the plane $A''B''$ it is seen that the inductors marked M and N help in generating a counter e.m.f. in the armature winding.

Those inductors marked M evidently add their e.m.f. to that generated in the inductors X, and those marked N add their e.m.f. to that generated by the inductors Y.

The e.m.f. generated in the inductors M and N is proportional to the load, because the strength of field produced by the commutating poles is proportional to the load. The e.m.f. generated in the inductors X and Y is independent of the load (as long as the speed remains constant), but the counter e.m.f. of the armature winding as a whole tends to increase with a load increase because of this effect of the inductors M and N. In fact, the effect of the conductors M and N is just the same as though there were no commutating poles and the main field increased in strength with an increase in load. Hence, these conductors and the armature reaction (with a forward brush shift) tend to make the motor act like a compound-wound motor which, as we know, has a speed-load curve which drops quite rapidly as the load is increased.

A certain commutating-pole motor ran at a speed of 1000 r.p.m. with no load; with the brushes in the plane AB (Fig. 286), the full-load speed was 900 r.p.m. and with the brushes in the position A'' B'' the full-load speed was 670 r.p.m. This increase in speed decrease (670 r.p.m. as compared to 900 r.p.m.) was caused by the combined effect of the armature reaction and the commutating poles. When the motor was carrying full load with the brushes in the position $A''B''$, they sparked badly.

274. Effect of Backward Brush Shift.—When the brushes are moved *backward* as shown at $A'B'$ (Fig. 286), the speed-load curve tends to become more nearly flat. The effects of the inductors M and N and of the armature reaction *both decrease* the counter e.m.f. of the armature as the load increases, and thus tend to make the speed of the motor *increase* with a load increase. The effect of the IR_a drop, however, again tends to make the speed decrease as the load is increased, so that the shape of the speed-load curve is determined by the relative magnitudes of these two effects.

If the armature resistance is high and the armature reaction and commutating poles relatively weak, the motor speed will fall off as the load is increased. In a motor having a smaller armature resistance, the speed will actually rise with an increase of load; one motor tested increased its speed from 1000 r.p.m. at no load to 1080 at three-fourths load, and before full load was reached the motor "ran away," i.e., the speed suddenly increased to such a high value (thereby drawing so much current from the line) that the protecting devices in the armature circuit opened the supply line.

Of course, the brushes were sparking to some extent when in the position $A'B'$, but not as badly as when in the position $A''B''$. In general, it may be said that, with a backward shift of the brushes, the operation of a commutating-pole motor is unstable; under certain conditions the speed of the motor may oscillate violently even when the load on the motor is constant. The motor mentioned above, with its main field weakened somewhat from its normal strength, when the brushes were in the position $A'B'$, " hunted " between speeds of 800 r.p.m. and 1200 r.p.m. until, finally, the protecting fuses blew and opened the circuit.

We may conclude, therefore, that on a commutating-pole motor the brushes must be accurately set in position; *a shift of even one-half the width of one commutator bar* from the proper position will often produce sparking and unsteady running.

275. Necessity of a Motor Starting Rheostat.—When a motor armature is stationary, there can be no e.m.f. generated in its windings because there are no conductors cutting lines of force. If, then, the stationary armature of a shunt motor is connected directly to the supply line, the current which will flow in the armature circuit may be calculated from the equation

$$E=e+IR_a, \quad \text{and as} \quad e=0,$$

$$E=IR_a \quad \text{or} \quad I=\frac{E}{R_a}. \qquad \ldots\ldots \qquad (119)$$

Now, the current calculated from this equation will be *ten or twenty times the full-load current,* and will be disastrous in its results.

Consider a 110-volt motor, the full-load current of which is 40 amperes. The armature resistance of such a motor would be about 0.2 ohm. If the stationary armature were connected directly to the 110-volt line, the current through the armature would be $110 \div 0.2 = 550$ amperes, whereas the full-load current is only 40 amperes. The current of 550 amperes would burn the brushes, commutator, and winding, and would also blow the fuses and circuit breakers in the supply line.

After the motor is running, there is not an excessive current flowing through the armature because the current is limited by the counter e.m.f. But while the armature is accelerating, this counter e.m.f. is small, and some other means must be employed to limit the starting current. This is the function of the *motor-starting rheostat,* or, as it is frequently called, the *starting box.*

A starting rheostat consists of a variable resistance, placed in series with the armature, and which may be gradually cut out as the motor speeds up and which can be cut out altogether when the motor has reached nearly normal speed. The total resistance of the starting box must be of such a value that, when it is connected in series with the armature, directly across the motor supply line, the current which flows through the circuit *will not be greater than about* 150 *per cent of the full-load current* for the motor. The starting box is sometimes designed so that it limits the current to a value not greater than the full-load current of the motor.

276. Example of a Proper Starting Rheostat.—Suppose that a starting box is desired for the 110-volt, 40-ampere motor mentioned in the previous paragraph. If the current in the armature circuit is to be 60 amperes at the start, i.e., 150 per cent of the full-load current, the total resistance of the armature circuit (armature resistance and starting box) must be 110/60 $=1.83$ ohms. As the armature resistance is 0.2 ohm, the total resistance of the starting box must be $1.83-0.2=1.63$ ohms. The wire of which the starting box is made must be of sufficient size to carry safely 60 amperes during the short time required for the acceleration of the motor armature.

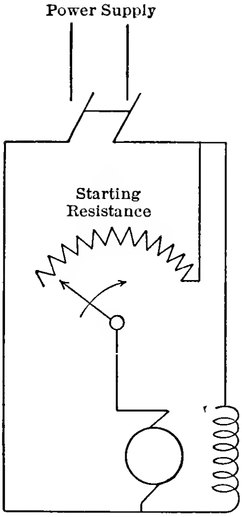


FIG. 287.—When starting a motor a variable resistance, called a starting resistance, or starting box, is inserted in series with the armature circuit.

The connection of the armature to the line through this variable resistance is shown in Fig. 287. The field is connected directly across the line, and is at full strength during starting. The wires of which the starting-box resistance is composed are imbedded in sand, or wound on porcelain tubes, in the best types of starting boxes; they are then enclosed in fireproof material, so that if the operator keeps the starting resistance in the circuit for a longer time than that for which it was designed, thus overheating and possibly melting the wire, no fire risk occurs.

The resistance of 1.63 ohms would have taps brought out at a number of points, so that it could be cut out by means of a handle operating a sliding contact, as the motor speeds up.

Let us consider what happens as the motor is started up against a constant load torque equal to the full-load torque of the motor. The operator moves the handle, closing the circuit through the armature and all of the starting resistance. With 1.63 ohms in series with the armature, the armature current jumps to 60 amperes, as is shown in Fig. 288, at time t_1.

With constant field current and flux, the torque exerted by the motor is directly proportional to the armature current; with 60 amperes armature current the motor will exert 150 per cent rated torque or a value 50 per cent greater than the load torque assumed. At once the motor begins to accelerate and generates a c.e.m.f.

From the equation

$$E=e+IR_a,$$

as soon as e starts to increase, with the rising armature speed, the armature current must begin to decrease along some such curve as AB. Increase in motor speed therefore causes a decrease in armature current and in generated torque.

Now, we have already seen that a motor will increase its speed only if its generated torque is greater than the load torque; the motor will therefore increase its speed and decrease its armature current until the latter has fallen to 40 amperes; with this current the motor is developing full-load torque which is equal to the value of the load torque assumed.

If we know the full-load speed of the motor considered, we can calculate the value to which the motor speed will rise at time t'_1 with all of the 1.63 ohms in series with its armature. With constant field flux, the speed, from the equation for c.e.m.f. ($e = K\phi N$), will be directly proportioned to the c.e.m.f.

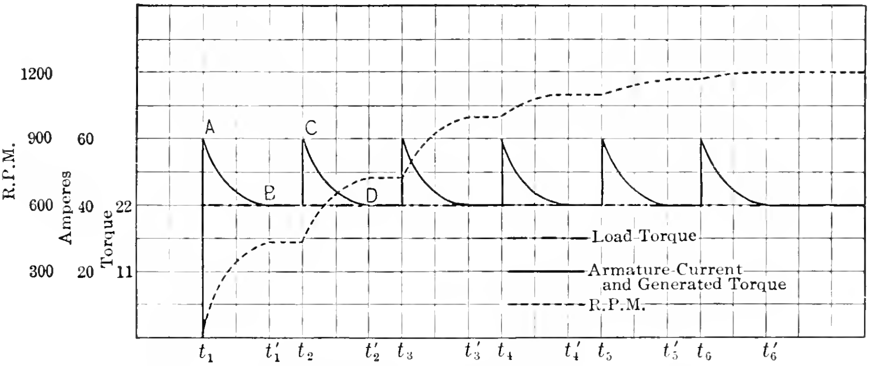


FIG. 288.—This curve shows how the armature current of a motor, its torque, and speed, vary as the starting resistance is cut out, step by step.

Let us assume that the full-load speed of the motor is 1200 r.p.m. At full load, with no added resistance in series with its armature, its c.e.m.f. will be $110 - 40 \times 0.2 = 102$ volts.

At time t'_1, with 40 amperes flowing and 1.63 ohms in series with the armature, the c.e.m.f. will be $110 - 40\ (1.63 + 0.2) = 110 - 73.2 = 36.8$ volts.

We have then the following proportion:

$$\frac{\text{Speed at time } t'_1}{\text{Speed at full load}} = \frac{\text{c.e.m.f. at time } t'_1}{\text{c.e.m.f. at full load}},$$

or

$$\frac{N'_1}{1200} = \frac{36.8}{102},$$

from which $N'_1 = \frac{1200}{102} \times 36.8 = 433$ r.p.m.

To further increase the speed of the motor, its generated torque must again be increased. This can be done by cutting out a part of the 1.63 ohms in series with the armature, by moving the handle of the starting rheostat farther to the right. This will naturally cause a momentary increase in the armature current, and if this increase is again to be limited to 60 amperes, the amount of resistance left in series with the armature may be calculated from the equation

$$E=e+IR_a+IR_2,$$

where R_2 is the resistance to be left in series with the armature.

Since the value of the c.e.m.f. at time t_2 is 36.8 volts, we have

$$R_2=\frac{110-36.8-60\times0.2}{60}=\frac{61.2}{60}=1.02 \text{ ohms.}$$

Cutting out $1.63-1.02=0.61$ ohm will therefore cause the armature current to jump to 60 amperes at time t_2. At once the speed will increase, and the armature current will decrease, along some such curve as *CD*, until it is again 40 amperes. At time t'_2, when everything is steady again, the c.e.m.f. will be

$$110-40(1.02+0.2)=110-48.8=61.2 \text{ volts,}$$

and the speed

$$N'_2=\frac{1200}{102}\times61.2=720 \text{ r.p.m.}$$

The resistance to be left in at time t_3, to allow the current to rise again momentarily to 60 amperes, is then

$$R_3=\frac{110-61.2-60\times0.2}{60}=\frac{36.8}{60}=0.61 \text{ ohm.}$$

When the current has again fallen to 40 amperes at time t'_3, the value of the c.e.m.f. will be

$$110-40(0.61+0.2)=110-25.2=84.8 \text{ volts,}$$

and the speed

$$N'_3=\frac{1200}{102}\times84.8=998 \text{ r.p.m.}$$

The resistance of 1.63 ohms would therefore have taps at about five points, so that it could be gradually cut out as the motor speeded up. The steps are not even; the above rheostat is divided into steps of 1.63 ohms, then 1.02 ohms, 0.61 ohm, 0.22 ohm, and 0.08 ohm.

277. Special Features of a Starting Rheostat.—A starting box of good design and manufacture is shown in Fig. 289. The lever, or arm by means of which the resistance is cut out as the motor speeds up, is so connected to a spring that it constantly tends to fly back to the " off " position, thus opening the armature circuit.

There are two conditions under which the armature circuit should be opened, and the starting box, shown in Fig. 289, is designed to take care of these automatically. These two conditions are, excessive current through the motor armature, and failure of line voltage. If too much load is put on the motor, a dangerously large current may flow through the armature circuit; under such a condition the armature should be opened, thus stopping the motor and warning the operator of the overload. The "*over-load release*" of the starting box, shown in Fig. 289, actuates a stop which releases a small circuit-breaker located on the face of the starting box, thus opening the circuit when the safe current is exceeded.

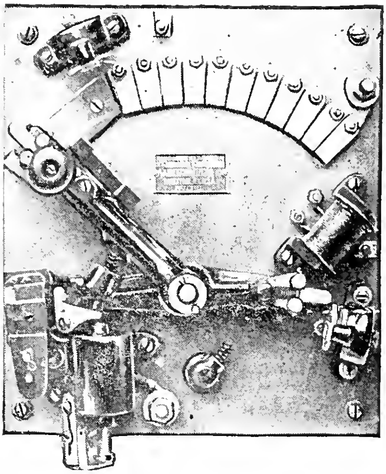


Fig. 289.—A good type of starting box fitted with over-load circuit-breaker, no-voltage release, etc. The contact plates are so assembled that they may be easily replaced when they become so worn as to be unserviceable.

278. Necessity of "No-voltage" Release.—The "no-voltage release" permits the rheostat arm to fly back to the " off " position if the line voltage drops below a certain value. The object of this release is always to open the armature circuit when the supply line becomes " dead." When this occurs (as, for example, when the station circuit-breaker on a feeder blows) the motor immediately slows down and stops. If the line is again made alive (circuit-breaker re-set), as the motor armature is directly connected across the line, with no resistance in series, an excessive current will flow through the stationary armature and may injure it. Of course, the over-load release would open the circuit under such conditions, but these over-load releases are designed only to break currents of about the same magnitude as the full-load rating of the motor. As was shown in section 275, the current which flows through a stationary armature when it is connected directly to a normal voltage line is many times the full-load current; and if the small circuit-breaker on the front of the starting box were depended upon to break such large currents, it would soon be damaged.

279. Connections for Starting Rheostats.—The solenoid which operates the no-voltage release is connected (in series with a suitable resistance) directly across the supply line, and the solenoid which operates the overload release is connected in series with the motor. The connections of such a rheostat to the motor and supply line are shown in Fig. 290. In starting a motor, the arm must not be moved over too rapidly, else the overload release will blow; neither must it be moved over too slowly, else the resistance wire in the rheostat is likely to be melted. The contact points on the face of the rheostat should be kept clean, as must also the sliding contact, or "shoe," on the rheostat arm.

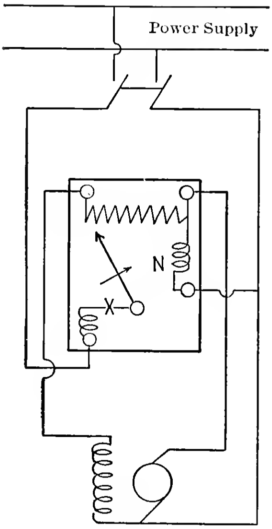


FIG. 290.—Inside connections of a starting box; the circuit-breaker, to take care of overloads, opens the circuit at X.

280. Automatic Starting of Motors.— As mentioned previously, if a motor-starting rheostat is cut out too slowly, it is likely to be overheated; and if it is cut out too rapidly, damage will very likely occur to the motor or to its connected machine. To obviate these possibilities, it is advisable to install automatic starters in cases where the starting and stopping operation is frequent enough to justify the extra cost. An automatic starter, properly adjusted, will give the motor just the right acceleration; the operator has merely to press a button to start or stop the machine.

One type of automatic starter is shown in Fig. 291. The starting resistance is cut out in suitable steps, at the right time, by magnetic relays with contactors C_1, C_2, and C_3. These contactors are closed by the magnetic action of solenoids A, B, and C, which close the contacts against the action of springs. When the power circuit is closed by a main contactor controlled by a push button (not shown), contacts C_1, C_2, and C_3 are all open; hence, all of the starting resistance is in series with the armature. When the starting current has decreased to a certain desired value, the voltage across A has risen sufficiently to enable it to lift the plunger carrying the contactor, C_1, thus cutting out of the armature circuit the resistance between points R_1 and R_2. With further acceleration of the motor, the voltage across B has risen sufficiently to operate its plunger, C_2 is closed, and the resistance between R_2 and R_3 is cut out. As many as five or six of the contactors may be used, but in the general motor installation three is a sufficient number to give reasonable acceleration and reasonable values of starting current.

281. Speed Control of Motors.—It is often necessary to vary the speed of a motor according to the changing requirements of the load. To make the electric motor suitable for railway service, operation of machine tools, etc., it must be possible to change its speed quickly and easily through a wide range; and the scheme for obtaining this variation in speed must be such that the motor operates with a good efficiency at any one of the speeds required.

282. Possibility of Varying Speed.—By inspection of Eq. (117), it is seen that for a given load on the motor, the speed may be varied by a change in the voltage, E, impressed on the armature, the resistance of the armature circuit, R_a, or the value of the field flux, ϕ.

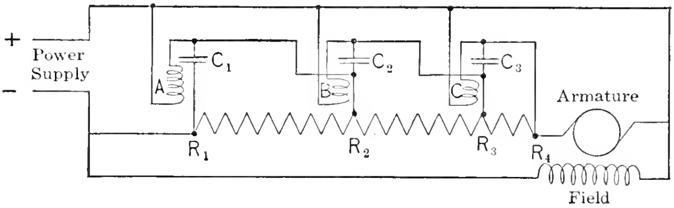


FIG. 291.—A simple automatic starter with three contactors. The solenoids A, B, and C, operate in sequence, cutting out the starting resistance at the proper time to keep the starting current equal to about 150 per cent full-load value during the starting period.

Increasing the voltage E impressed on the armature for a given value of armature current, requires an increase in N if the value of the field flux ϕ remains the same; the motor speed must increase. If the voltage impressed on the armature is decreased, the speed is lowered.

Increasing the resistance of the armature circuit, by the insertion of resistance in series with the armature, allows the c.e.m.f. to fall; with ϕ the same, a drop in speed results. The loss in heat in the armature circuit is equal to I^2R_a (where R_a is the resistance of the armature circuit), and if this quantity is large the motor must necessarily be inefficient; speed control by the addition of resistance in the armature circuit is therefore seldom used and will not be discussed here.

283. Multiple-voltage Control.—In the system using multiple-voltage control, the power line consists of several wires, and the voltage between various pairs is different. In one system, four wires are used to distribute power to the motors and the power supplied to these four wires is obtained from a set of three generators. The various voltages obtainable by using different pairs of supply wires are shown in Fig. 292. The voltage impressed on the armature in this system may be varied in steps from 60 volts to 250 volts, and the speeds obtainable would vary in about the same proportion, so that the highest speed obtainable would be about four times the lowest speed. This would be called a 1 : 4 speed control.

The field circuit is designed for a certain voltage, and is left connected to its proper line as the armature is shifted from one line to the other.

This scheme is used in the operation of motors for driving machine tools. If a high speed is desired for a certain operation on a lathe, the motor armature is connected to the 250-volt line, and, when lower speeds are desired, it is connected to one of the other pairs of wires, having a suitable voltage. Such a system of speed control involves complicated wiring and controllers for the motors, but its advantage lies in the fact that the motors operate at a comparatively high efficiency at any one of the speeds obtainable.

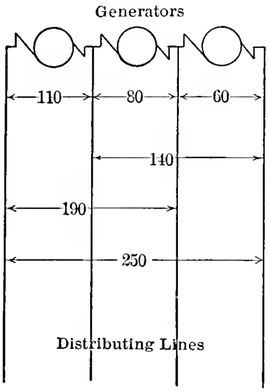


Fig. 292.—One method of getting various speeds with a shunt motor is to supply power at several different voltages; if the field circuit is left connected to one pair of lines the motor speed will be nearly proportional to the voltage of the lines to which the armature is connected.

284. Speed Control by Field Variation.—The most important method for obtaining various speeds consists in weakening the field of the motor; this is called *field control.* When first discussing the speed-load curves of motors it was shown that, under any condition of operation, the counter e.m.f. developed in the armature winding must be nearly equal to the impressed voltage. If, with the impressed voltage maintained constant, the field flux is varied, and after the field flux is varied, the load is readjusted so that the armature current is the same as at first, it is evident that the above condition can be fulfilled only if the speed of the motor follows the changes in field strength, a high speed corresponding to a weak field and vice versa.

Inspection of Eq. (117), together with the torque equation ($T=K'\phi I_a$), indicates that, with a given load torque on the motor, a decrease in the flux would require more armature current, in order that the value of generated torque be maintained. An increase in armature current, as we have already seen, generally requires some reduction in speed; whether the motor speed increases or decreases depends then upon the relative change in ϕ and I_a.

Consider again a 110-volt shunt motor, the full-load current of which is 40 amperes, and the armature resistance of which is 0.2 ohm; the full-load c.e.m.f. of this motor would be $110-40\times0.2=102$ volts. With no load this motor would require not less than 2 amperes, so that its no-load c.e.m.f. would be $110-2\times0.2=109.6$ volts.

The c.e.m.f. of the motor thus drops 7.6 volts when the armature current changes from 2 to 40 amperes. In other words, a small change in the c.e.m.f. produces a very large change in the armature current. Therefore, if a reduction of field flux does cause the motor to draw more armature current, there will be practically no change in the value of the c.e.m.f., and a reduction in flux is accompanied with an increase in speed.

285. Methods of Varying the Field Flux of a Motor.—The simplest method of changing the flux of a shunt motor is by varying the resistance of its field circuit.

The amount of speed variation which can thus be obtained from a shunt motor of ordinary design is not very great; the twisting of the main field by the armature reaction is not enough to be objectionable when the main field is operated somewhere near saturation, but when this field is weakened, the effect of the armature reaction is much exaggerated and the brushes will spark badly unless properly shifted with every change in load. A point is soon reached, however, where the field becomes so weak that with the exaggerated armature reaction there is not sufficient commutating flux. In the ordinary shunt motor, not designed for the purpose, a speed increase of about 30 per cent by field weakening is about the limit; beyond this limit excessive sparking will ensue.

Another method of changing the flux of a motor is to change the length of the air gap, thus changing the reluctance of the magnetic circuit. In one such type of " variable air gap " motors, somewhat used, the armature is slightly conical and is capable of being moved along its shaft relative to the pole pieces, which also have a conical bore. As the armature is moved along the shaft by a hand wheel, the length of the air gap is changed; with a constant field current, the flux varies practically inversely as the air-gap length.

The advantage of this method of varying the field flux is that armature reaction is not changed, since the relative strength of the field and armature m.m.fs. is not altered. The disadvantages are that the motor is mechanically complex, and that speed adjustments must be made at the motor, which is not always conveniently located.

286. Commutating Poles used with Field-weakening Control Scheme.—The type of motor best adapted for speed variation by the field-control method is the commutating-pole motor, and practically all motors intended for service where an adjustable speed is desired are equipped with commutating poles. In such motors the commutating poles provide the flux required for sparkless commutation, irrespective of the strength of the main field. Special starting rheostats are generally used for motors in which field weakening is employed for speed control; such a box is shown in Fig. 22, Volume III. Such *adjustable-speed* motors are designed for a speed range as great as 1 : 6; ordinarily, however, the speed variation required is not greater than 1 : 3 or 1 : 4.

The need of commutating poles, and the effect of armature reaction in twisting the field of an adjustable-speed motor, are shown by the oscillograms of field distribution, given in Figs. 293, 294 and 295. These films were taken by the use of an extra inductor threaded in one of the armature slots; the two ends of this inductor were connected to the oscillograph by two small slip rings put on the armature shaft for this purpose.

In each figure there is shown the field distribution for no load and full load, the motor being of modern design and intended for a speed range of 400 r.p.m. to 1600 r.p.m., by field control. For all cases, the impressed voltage was the same, and the scale for all oscillograms was the same. It may be seen that the field is much stiffer at the low speeds than at the higher, this, of course, being due to the greater field current required to bring the speed to the low value. As the full-load armature m.m.f. is of the same magnitude, whatever the speed, it follows that the twisting of the main field increases with speed; and this is seen to be the case by comparing Fig. 293 for a speed of 410 r.p.m. with Fig. 295 for a speed of 1650 r.p.m.; the higher speed requires about one-fifth as much field current as the lower, and the ratio of armature m.m.f. to field m.m.f. (which is the factor determining the amount of field distortion) is therefore five times as great for the high speed as for the low. The commutating poles furnish the requisite flux for sparkless commutation for all cases; but the motor is much more likely to have commutator trouble at the high speed than at

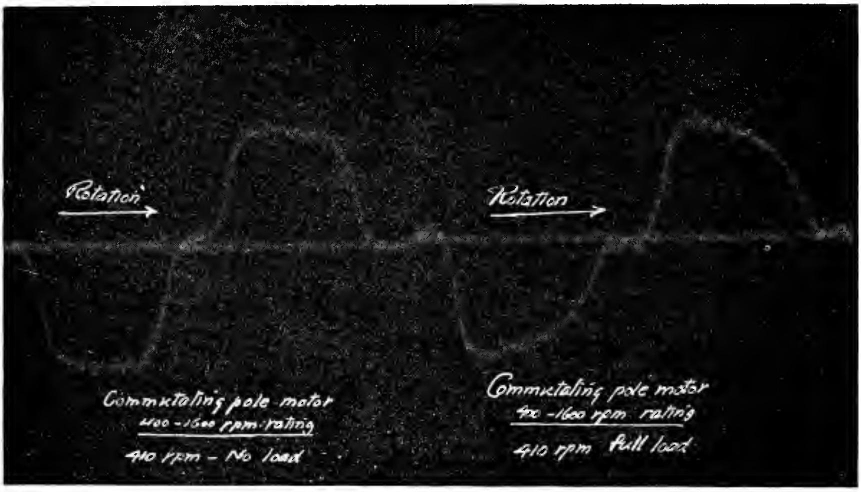


FIG. 293.—This film gives the field distribution of an adjustable-speed motor, for no-load and full-load conditions. The rated speed range is from 400 r.p.m. to 1600 r.p.m. and this oscillogram gives the conditions for the low speed. The main field being very strong there is not much distortion even at full load.

the low, because the maximum number of volts per bar is about twice as much at high as at low speed.

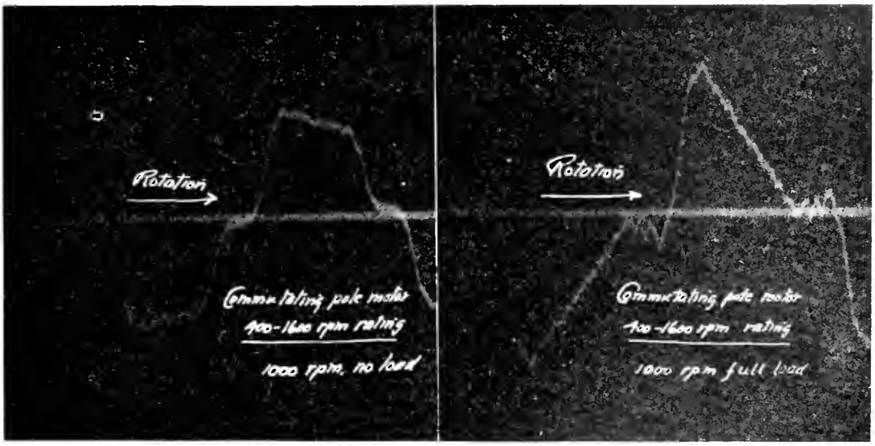


Fig. 294.—These films are for the same motor as used in getting Fig. 293; the speed is here 1000 r.p.m. and the main field is about one-third as strong as it was when getting Fig. 293. The field is much more distorted than it was with the strong field used for the speed of 400 r.p.m.

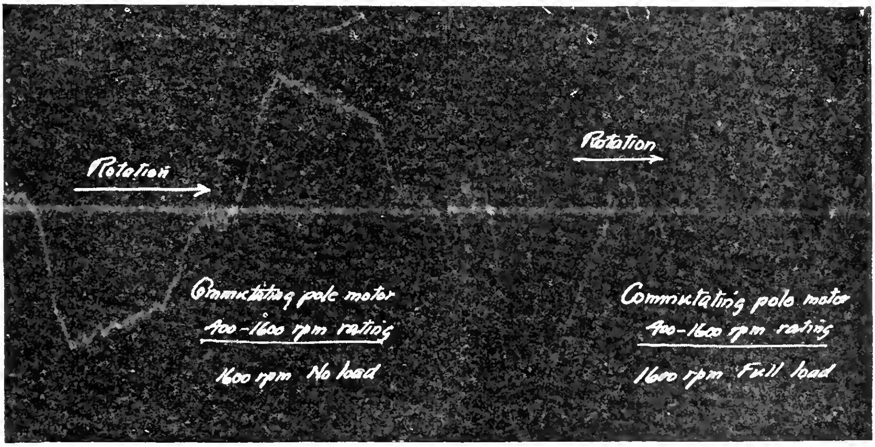


Fig. 295.—These oscillograms are for the same motor as used for the two previous figures but the speed is here 1600 r.p.m. The main field is only about one-fourth as strong as it was for Fig. 293 and the field distortion for this condition is excessive at full load, being very noticeable even at no load. Without commutating poles this motor could not operate at full load. For this, as for the two previous films, it is seen that the commutating poles give at the brushes the proper field for sparkless commutation in spite of the twisting of the main field.

In the conditions existing for the film of Fig. 295, at full load, the effect of armature reaction and commutating poles actually reverses the direction of the flux in the trailing tips of the main poles, so that as a conductor moves across a main pole it generates voltage in both directions. As the total e.m.f. generated in one path of the armature must be equal (practically) to the impressed voltage, it is necessary to have very high voltage generated in some of the conductors to offset the negative or opposing voltage generated under the trailing pole tips. This excessive voltage occurs under the leading pole tip where it is about twice the average voltage per bar.

287. Adjustable-speed Motor Compared to a Constant-speed Motor.—The torque obtainable from an adjustable-speed motor is greatest when the main field has its greatest strength, i.e., at minimum speeds. As the speed is increased by field weakening, the torque decreases in about the same ratio as the speed increases. This is due to the fact that the torque of a motor is proportional to the product of the armature current and the field density; the safe armature current is nearly as much at low speeds as at high speeds, so that the torque goes down as the speed goes up. The product of torque and speed is practically the same whatever the speed, and this means that the motor has the same capacity in horsepower over its whole speed range.

Now a motor, to give a certain output, must be of larger size the slower the speed; the size of all adjustable-speed motors is, therefore, larger than that of standard constant-speed motors of the same horsepower capacity. A field frame designed for a standard constant-speed 5-h.p. motor might be used in the construction of a 3-h.p. adjustable-speed motor; other sizes would be in about the same ratio.

288. Comparison of Field-control and Multiple-voltage Methods.—The field-control method of speed variation results in the efficient operation of the motor, and the wiring of the power supply to the motor is so much simpler than that of the multiple-voltage system that it is much preferred to the latter. The number of running speeds in the multiple-voltage scheme shown in Fig. 292 is limited to six, while the number obtainable when using field control depends only upon the number of contact points in the field rheostat of the motor; as many as 40 or 50 speeds are thus available with the rheostats ordinarily employed.

The multiple-voltage control may employ more or less field control, however, in which case the number of running speeds obtainable is greater than when field control alone is used. The chief advantage of the multiple-voltage control over the straight field control lies in the smaller size of motor required for a given horsepower capacity; if the field strength remains constant, the torque of the motor is independent of speed, hence the horsepower output of a motor using such a control goes up directly proportional to the speed.

289. Speed Control of Railway Motors. Series-parallel System.—

The speed control of railway motors is accomplished by the variation of the voltage impressed across the motor terminals by what is called the *series-parallel control.* There are always at least two motors on each electric car, both of the same rating. When the car is first started, all motors are connected in series, through a starting resistance, to the 600-volt line, the trolley or third rail being the positive side of the line and the track the negative side. The controller (which is simply a rotatable switch) has marked on its face various points, the number of these points depending upon the type of motor equipment.

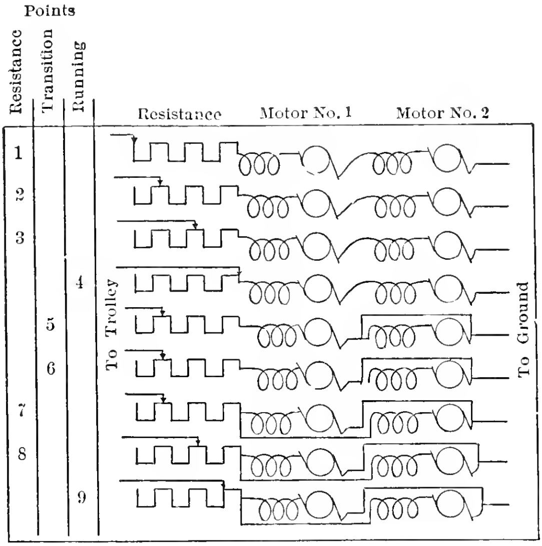


FIG. 296.—The scheme used generally in controlling the speed of a street car; the two motors are first connected in series and then thrown in parallel connection as the car speeds up. During the transition points only one motor is exerting torque.

For the two-motor control scheme the first three or four points are generally " resistance points," i.e., while the controller handle is pointing to any of them, more or less of the starting resistance is connected in series with the motors. On the next point, the two motors are connected in series with each other and directly to the line. This is called a " running point " because no power is being wasted as heat in the starting resistance, and so the equipment is operating at a fair efficiency. The motors are next thrown into parallel connection, and some of the resistance is again put in series with them. While the motors are being changed from series to parallel connection, the controller handle is moving through the " transition points," and the controller handle is so designed that it will not remain on one of these points unless held there. When the transition points have been passed through and the motors are connected in parallel, the resistance is again cut out in two or three "resistance points" and, finally, the two motors are each operating on the full-line voltage. This is the second "running point." The various connections used in the system of control are well shown in Fig. 296.

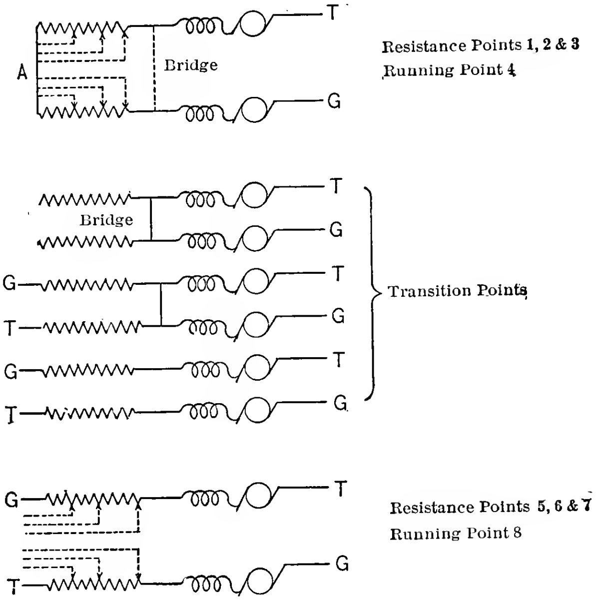


Fig. 297.—A better scheme of control than that shown in Fig. 296; during all of the acceleration period both motors are active, thus giving a smoother and more rapid acceleration to the car than the other method. The extra expense incurred in this scheme permits its use only on high-powered, expensive installations. It is called the "bridge" control.

The disadvantage in this scheme of control is that while going through the transition points, the tractive effort on the car, and hence its acceleration, is much decreased. On points 5 and 6, Fig. 296, the tractive effort will evidently be less than that on point 4, because on these points only one motor is active; the other has no current flowing through it. The acceleration of a car equipped with this system is somewhat uneven.

290. The Bridge System of Control.—In modern railway equipments, where the extra cost is warranted, a different set of connections is used in accelerating the car, which is known as the *bridge control.* A diagram of the connections used in this scheme is shown in Fig. 297. Here *T* stands for a trolley connection and *G* for a ground connection.

At first the two motors are connected in series with all the resistance, and there are three to five steps in cutting out this resistance as before. When the "bridge" is put in, the first running point is reached. Next come the transition points, on the first of which the connection *A* is removed; in the second, connection is made to the trolley and ground as indicated; and in the third, the bridge connection is removed.

During this transition period each motor continues to carry the same current as it did on running point 4. The starting resistances are so designed that when the bridge is removed no sudden change takes place in the current through the two motors. Then, on points 5, 6, and 7, the resistance is again cut out, and in point 8 the second running speed is reached. Much modern railway equipment is being furnished with this bridge system of control, as it gives a more uniform acceleration than the older method shown in Fig. 297.

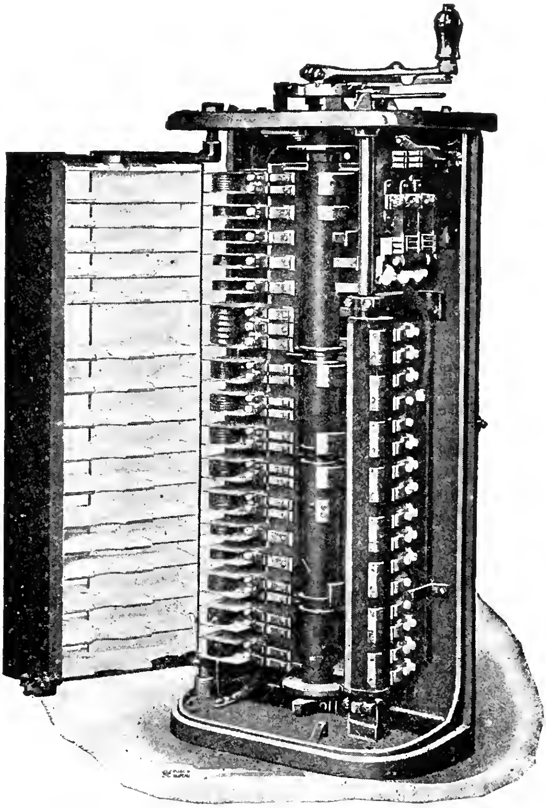


FIG. 298.—The interior appearance of a typical railway controller.

If the motors are equipped with commutating poles, two other running points may be obtained. After point 4 has been reached, a resistance is shunted across each motor field, thus weakening the field and increasing the motor speed and giving what we may call running point $4k$. In changing from point 4 to the first transition connection, these shunts are removed and not connected again until after point 8 has been reached. At present these shunts are not much used.

291. The Railway Controller.—A typical railway controller is shown in Fig. 298, and in Fig. 299 is given an enlarged view showing some of the contact fingers bearing on the copper segments which are carried on the main cylinder of the controller. As these fingers break contact with the segments, an arc is formed between a finger tip and a segment tip and, if not taken care of, this arcing would soon damage the controller. The controller is so designed that the arc *is formed in a magnetic field* which tends to lengthen the arc and so makes it rupture as soon as it is formed. This feature of the controller is called the *magnetic blow-out.* Also, the segment tips and fingers are so made as to be easily replaced by new ones when they have been burned so much as to be unserviceable.

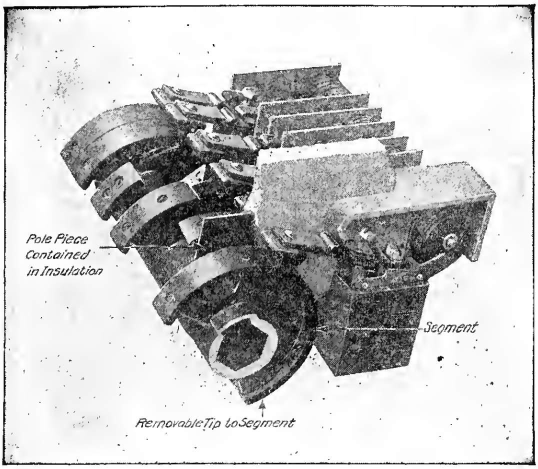


FIG. 299.—A few of the contact points of a railway controller, showing how the contact segments are easily replaceable. The arc which forms when the contactors open is formed in a magnetic field, which effectually "blows it out."

292. Use of a Flywheel with a Compound Motor.—The rate at which power is required by some kinds of machine tools is very irregular; in the punch press or forming press, for example, almost no power is required until the die comes in contact with the metal to be punched or formed. The rolling mill is another instance of a tool calling for an irregular power supply.

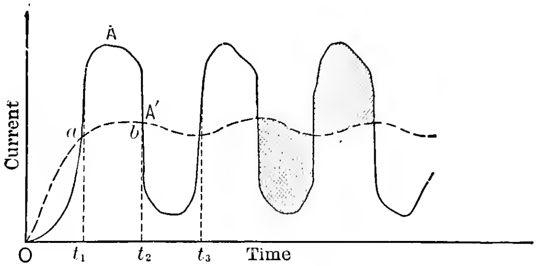


FIG. 300.—Typical curve of armature current required by a motor operating a punch press, shears, or similar duty. With no fly wheel the current is as shown by the full-line curve and when a sufficiently massive fly wheel is fitted to the motor shaft the current demand has about the appearance of the dashed curve.

If the mass of the moving parts is not very great in such machines, the power consumption of the driving motor will be very irregular. Figure 300 illustrates this point; the full-line curve shows the current consumption of a motor driving a shears for cutting large iron bars.

Now the motor in which such a current is flowing must be made sufficiently large to commutate successfully the *greatest value* of current, as at A. If the motor driving the shears were a compound-wound motor with a comparatively large number of turns in its series field (say 30–40 per cent compounding), and a heavy flywheel is put on the same shaft with the armature, the current to the motor, while doing the same work as before, will be given by the dashed curve of Fig. 300, in which the maximum value is much less than it was before. In fact, this motor equipped with the flywheel might be much smaller than the one without the flywheel. The motor through which current A', Fig. 300, flows need be only about two-thirds as large as if current A were the current input.

293. Effect of the Flywheel.—The action of the motor equipped with a flywheel is as follows: At time t_1, Fig. 300, the load on the motor suddenly increases and so causes the motor to begin to slow down. As it slows down, the rotating flywheel is slowed down also and so gives up some of its kinetic energy. During the time from t_1 to t_2, Fig. 300, the electrical input to the motor is not as great as the power demanded by the load, hence the motor slows down and the retarding flywheel assists the motor to carry the load. During this slowing down process, the current input to the motor must increase somewhat.

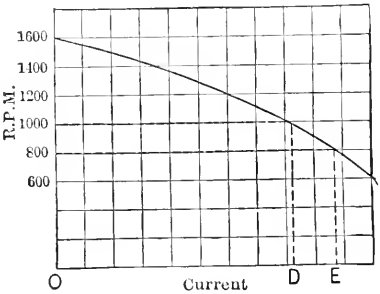


Fig. 301.—Speed-load curve of a heavily compounded motor.

If at the time t_1 the speed is 1000 r.p.m., and at the time t_2 the motor has slowed down to 800 r.p.m. and the speed-load curve of the motor is as given in Fig. 301, the current must increase during the same period from OD to OE. In Fig. 300 at_1 equals OD of Fig. 301 and bt_2 of Fig. 300 is equal to OE of Fig. 301. At the time t_2 the power demanded by the load is less than the input to the motor and so the motor begins to speed up and to increase the energy stored in the flywheel; the current begins to decrease at the same time. At the time t_3 the motor has regained its original speed of 1000 r.p.m. and the current has fallen to the value OD. The single-hatched area in Fig. 300 represents the amount of energy which the retarding flywheel gives up when the load is heavy, and the double-hatched area represents the energy which the motor returns to the flywheel when the load is light; these two areas are equal.

This use of a heavy flywheel to equalize the input to a motor supplying an intermittent load is becoming quite general; it decreases the size of motor required and makes the load of the generating station much more uniform. This last consideration is of importance only when the size of the motor is somewhere near that of the generator supplying its power.

294. Dynamic Braking.—When a motor is to be started and stopped frequently, the question of getting sufficient retarding force to bring it quickly to rest is to be considered. Ordinary frictional braking is possible, the pressure of the brake shoe being exerted by a powerful electromagnet, as shown in Fig. 58. Another method is to make the retarding motor act as a generator by disconnecting its armature from the power supply and connecting the brushes together through a suitable resistance. The amount of current supplied to the resistance depends upon the speed of the motor (acting as a generator), the field strength, and the value of the resistance; the braking effort of the motor is proportional to its field flux and the armature current. Sometimes the field circuit is disconnected from the line, leaving in the field poles only the residual magnetism. The armature is then short-circuited, and sufficient current flows to give the desired braking effort.

In electric railroads, regenerative braking has been tried and is generally successful. When continuous-current motors are used for the motive power, it is possible to make them pump power back into the trolley system when running downhill, thus not only acting as efficient brakes but also saving the mechanical braking equipment. As a series motor will not reverse the direction of its current, no matter what the speed may be, it is necessary to make the motors, which are to be used for braking, separately excited when braking. In the latest scheme, two of the motors of a train are used as a motor-generator set to supply field current to all the other motors of the train acting as brakes; each of these motors is connected to the trolley system and, if running at sufficient speed, and supplied with sufficient field current, it will supply power to the trolley. The amount supplied, and hence the braking effort, is controllable by the field current. In the C., M., and St. P. RR., this scheme decreases the total power required for operation of the road by about 15 per cent, thus cutting down the power bill by this amount. A more important advantage of this braking scheme, however, is to be found in the saving of brake shoes and wheel rims. Even if no power at all were saved by the scheme, it would still be worth while because of the saving in wear of shoes and wheels.

295. Balancers.—A balancer is a motor-generator set used for the purpose of maintaining equality, or approximate equality, between the voltages on the two sides of a three-wire system. In Fig. 302, a three-wire system is shown, the voltage across the outside of which is maintained constant at 220 volts by an ordinary 220-volt generator. The balancer set, consisting of two duplicate machines, mechanically direct-connected, have their armatures electrically in series across the two outside wires; the neutral wire is connected between the two armatures. The fields are also generally connected in series across the outside wires, as shown.

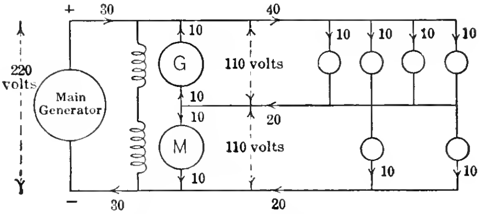


FIG. 302.—Showing the connection of a balancer set to a three-wire distribution system, the neutral wire carrying twenty amperes, due to the unbalanced load. This diagram assumes that the balancer consumes no power to run itself.

The action of the balancer set in maintaining the potential of the neutral midway between the two outside wires, may be seen from Fig. 302. When the load is unbalanced, the unit connected across the heavily loaded, lower voltage, side, will act as a generator while that across the lightly loaded side will operate as a motor. If the machines comprising the balancer set are assumed to have no losses, the neutral current will divide equally between them, and the power absorbed by the lower machine, or motor, will be given out by the upper one acting as a generator. In Fig. 302, the current distribution throughout the system is indicated by arrows and figures, the field currents of the machines being disregarded.

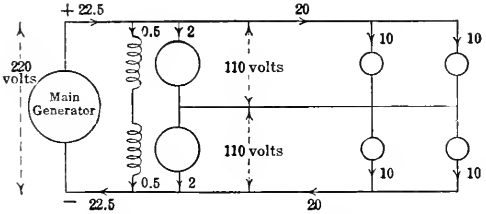


FIG. 303.—Conditions in a balancer set when the two loads are equal, the neutral carrying no current. It is assumed that the balancer units require currents as shown to supply the no-load losses.

Transferring the unbalance in the loads to the other side will simply reverse the function of the two units; the machine acting before as motor becomes a generator, and vice versa.

When there is no load on the system, or when the loads on the two sides are equal, the two machines of the balancer set act as motors in series, the voltage across each being in this case 110 volts, as shown in Fig. 303. In this figure the shunt fields of the two machines are also connected in series across the outside wires, giving both fields the same constant value of current. It is assumed that the field current is 0.5 ampere, and that with no load on the balancer an armature current of 2 amperes is required for each motor; the main generator therefore furnishes 20 amperes at 220 volts to the load and 2.5 amperes to the balancer set.

If, with the unbalanced load of Fig. 302, the losses of the balancer set are assumed the same as in Fig. 303, the current distribution will be as in Fig. 304; obviously the losses of the entire set must be supplied through that machine which is acting as a motor. Actually the losses will be somewhat greater when the balancer is loaded, causing a difference in the armature currents still greater than that shown.

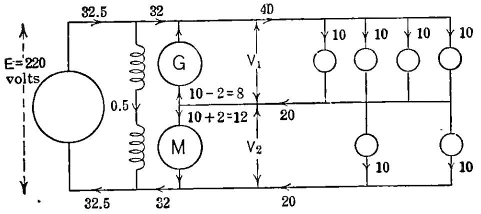


Fig. 304.—With the amount of unbalance indicated in Fig. 302, and with the losses assumed in Fig. 303, the current distribution in the system will be as shown here.

With an unbalanced load, the terminal voltage of the generator *G* will be

$$V_1 = E' - I'_a R'_a,$$

and that across the motor

$$V_2 = E'' + I''_a R''_a,$$

where E', I'_a and R'_a are respectively the generated e.m.f., armature current, and armature resistance of the generator *G*, and E'', I''_a and R''_a are corresponding quantities for the motor *M*. E'' is thus the c.e.m.f. of the motor *M*.

Since the machines are both operating at the same speed and have the same field current, E' must be equal to E''. It follows that the voltage V_1 across the heavily loaded side must be less than the voltage V_2 across the lightly loaded side by an amount equal to the sum of the two *IR* drops.

Thus, with the field connections so far shown, the automatic response of the balancer set depends for its action upon an unbalancing of the voltage. The amount of voltage unbalance may be somewhat reduced and the operation of a balancer set thereby improved by cross-connecting the shunt fields of the two machines as in Fig. 305. Now as the voltage across the loaded side falls, it will weaken the field of the machine operating as motor, holding up the speed of the set. An increase in the voltage across the lightly loaded side will at the same time strengthen the field of the generator, and thereby somewhat increase its generated e.m.f. There must, however, be still some voltage unbalance.

Perfect regulation may be obtained if compound-wound machines are used, their series fields being connected in series in the neutral in such an order that the neutral current flowing through the two series fields increases the field strength of the generator and weakens that of the motor. The connections for this arrangement are shown in Fig. 306, the upper machine being across the heavily loaded side and therefore acting as a generator. The connections are, however, such that if the load unbalance is shifted and the neutral current reverses, the lower machine, now acting as generator, has its field strengthened and the upper machine, now a motor, has its field weakened.

With this arrangement the actual amount of series-field current that each machine will receive must be carefully adjusted by means of German silver shunts. If the series fields are too strong, the voltage on the heavily loaded side will increase.

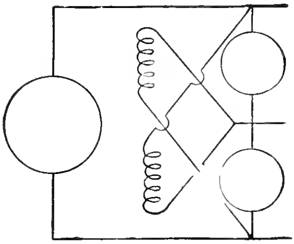


FIG. 305.

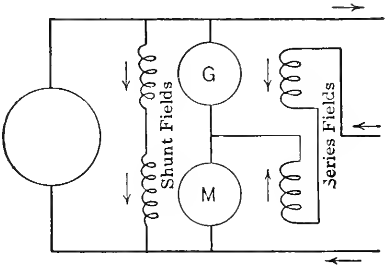


FIG. 306.

FIG. 305.—By cross-connecting the fields of the balancer set it is possible to reduce the amount of unbalance in the voltage for a given amount of unbalance in current.

FIG. 306.—By suitably compounding the units of the balancer set and connecting the series field in the neutral wire it is possible to keep the voltage division between the two sides of the line as nearly equal as may be desired.

In practice, balancer sets are not usually called upon to handle more than 15 or 20 per cent unbalance in load; i.e., the neutral current will not be more than 15 to 20 per cent of the full-load current of the main generator. The actual current through either unit of the set will generally be less than the neutral current, so that the balancer units are relatively small machines.

PROBLEMS

1. A four-pole lap-wound motor has 1.2×10^6 lines per pole, and 55 armature coils of 6 turns each. If the armature current taken from the line is 40 amperes, what will be the torque?

2. If the above motor is turning at 1200 r.p.m. and the armature current is 35 amperes, what horsepower is being developed? If the speed is cut down to 600 r.p.m., by resistance in series with the armature, and the armature current is 60 amperes, how many horsepower are being converted by the motor into mechanical power?

3. A bipolar motor has an armature 8 inches in diameter and 10 inches long; polar angle is 120°. There are 55 coils of 2 turns each, and the air-gap density is 5500 gausses. Neglecting pole fringe, how many horsepower are being developed, if the armature takes 65 amperes and turns 1350 r.p.m.? If the friction and core losses are 350 watts, what is the torque at the pulley?

4. A bipolar shunt-wound motor takes 52.2 amperes from 220-volt line; the field resistance is 125 ohms and the armature resistance is 0.12 ohm. The flux is 3.05×10^6 lines, and the number of conductors is 280. What is the c.e.m.f., and at what speed is the motor turning? Allow 2 volts for total brush drop.

5. The field winding of a shunt motor has a resistance of 105 ohms, and is connected to a 240-volt line. The total current taken by the motor is 55 amperes, and the armature resistance is 0.18 ohm. How much power is used as heat in the field, and how much in the armature? If the core loss and friction loss together are 280 watts, how much horsepower is being delivered? What is the speed, if the torque is 110 lb.-ft.?

6. A 110-volt shunt motor has an armature resistance of 0.42 ohm and brush contact drop of 3 volts. Field resistance is 130 ohms. When drawing 39.5 amperes from the line, the motor runs 1150 r.p.m. At what speed must the machine run as a generator to deliver 45 amperes at 110 volts. What is the difference in the generated voltage when taking 40 amperes as a motor and delivering 40 amperes as a generator, both on a 110-volt line?

7. Find the speed of a railway motor, in r.p.m., corresponding to a car speed of 25 miles per hour, the gear ratio being 15 : 64, and the car wheels being 33 inches diameter. Assuming 10 per cent of the motor torque used up in gear and axle friction, what is the motor torque in lb.-ft. if the car requires a tractive effort of 2400 pounds? What is the horsepower output of the motor?

8. A bipolar shunt motor has a full-load armature current of 95 amperes, an armature resistance of 0.092 ohm, and 280 conductors on the armature. The flux is 3.45×10^6 lines per pole. If the brush contact drop is 2 volts at no load, and 3 volts at full load, what is the no-load speed and the full-load speed? The friction and core losses are 450 watts. (Line voltage = 110.)

9. In the above problem, the field current is reduced 10 per cent by the field rheostat, and the reluctance of the magnetic circuit therewith is reduced by 2 per cent. Answer same questions as for Problem 8.

10. By how many ampere-turns would the effective field m.m.f. be reduced by armature reaction, if the brushes, were moved back, in the machine of Problem 8, by 20°, the armature carrying full-load current?

11. A series-wound armature for a four-pole, 220-volt shunt motor has full-load current of 90 amperes. The field resistance is 40 ohms, and the armature resistance is 0.15 ohm. The flux per pole is 3.8×10^6 lines, and there are 280 conductors on the armature. The no-load input to the armature is 900 watts. What is the no-load speed, full-load speed, and speed regulation? Assume brush drop as 2 volts at no-load and as 3 volts at full-load.

12. Solve the above problem for full-load speed, assuming that there is, in addition, a series-field winding of 0.022 ohm, shunted by a German silver strip of 0.018 ohm, the flux being the same as for the previous problem.

13. What would be the speed of the motor of Problem 11 with full-load current flowing, if there were inserted in the armature circuit an extra resistance of 1.2 ohms?

14. If the field-circuit resistance of the motor of Problem 11 was measured at 70° F., and after it has run for two hours the resistance of the field circuit is found to be 48.5 ohms, what is the temperature of the winding?

15. A certain shunt motor having an armature resistance of 0.042 ohm has a full-load current in the armature of 110 amperes. It runs at 450 r.p.m. with full load, when on the 110-volt line. What full-load speeds are available with the power supply given in Fig. 292, the field circuit remaining constantly on the 110-volt line? Assume 3 volts brush drop at full load.

16. Two 25-h.p., 1200-r.p.m., 110-volt shunt motors are directly connected mechanically, and their shunt fields are in series on a 220-volt line. At what speed will the motors run if the armatures, in series, are connected to a 220-volt line? If connected to a 110-volt line? If the fields are in parallel on a 110-volt line and armatures in series on a 220-volt line?

17. For the first case in the above problem, the field strength of one motor is reduced by shunting it with a resistance just equal to the field resistance. At what speed will the motors run, and how much power will each motor deliver when carrying full-load current in its armature? Assume permeability constant and disregard brush drop.

18. A four-pole, wave-wound, 600-volt railway motor has 140 turns per field coil. Armature resistance is 0.15 ohm, and field resistance 0.21 ohm. Full-load current is 90 amperes. Number of conductors on the armature is 912. With 30 amperes flowing, the flux per pole is 1.8×10^6 lines. Assume reluctance of the magnetic circuit constant. What is the speed for 50, 100, and 150 amperes?

19. At what speed will the motor in the above problem run, if 2.5 ohms resistance are inserted in series with it, and the current is 45 amperes?

20. A 110-volt motor is fitted with a steel flywheel weighing 700 pounds, the radius of gyration of which is 15 inches. If the current during the starting period is 20 amperes, and the average voltage on the armature during the starting period is 50 volts, and the armature losses are negligible, how long does it take to bring the flywheel to a speed of 500 r.p.m.? How much longer would it take to get it to 800 r.p.m.

21. The frictional and core losses of the above motor are 450 watts at 700 r.p.m. and may be assumed directly proportional to the speed. How long will it take for the motor to come to rest after the power is shut off, for the two speeds mentioned in Problem 20?

22. A 110-volt shunt motor having an armature resistance of 0.15 ohm has a normal speed of 950 r.p.m. when carrying full-load current of 50 amperes. How much resistance must be put in series with the armature if it is desired to give full-load torque at 450 r.p.m.?

23. A car weighing 10 tons is running down a 5 per cent grade at a speed of 20 miles per hour, being held at this speed by friction and regenerative braking. If the tractive effort to overcome friction at this speed is 500 pounds, how much current is the motor delivering to the trolley, this being at 600 volts above ground?

CHAPTER X

EFFICIENCY, HEATING AND RATING

296. Importance of a High Efficiency.—If the amount of electrical power put into a motor is measured, and the mechanical power output is measured at the same time, the *efficiency* of the motor may be obtained by finding *the ratio of the output to the input.* In just the same way, the efficiency of a generator is the ratio of the output to the input; in this case, however, the input is mechanical power and the output is in the form of electrical power. Both input and output must be expressed in the same unit before the efficiency may be calculated. Suppose, for example, that the input to a small electric motor is 1 kw. and the output is 1 h.p. The output, in terms of kilowatts, is 0.746, so that the efficiency is

$$\frac{0.746}{1} = 74.6 \text{ per cent.}$$

Or, we might say the input is equal to 1.34 h.p., so that the efficiency is equal to $\frac{1}{1.34} = 74.6$ per cent.

The efficiency of an electrical machine is one of its most important characteristics. To illustrate the importance of a high efficiency for a motor, let us consider the case of a factory requiring 100 h.p. to run the machinery installed in it. If the motor used to drive the shafting has an efficiency of 90 per cent, the necessary input when the motor is giving off 100 h.p. is equal to $100 \div 0.90 = 111$ h.p., which is equal to 83 kw. Suppose the price of power is $0.06 per kilowatt-hour (prices vary between $0.03 and $0.15 per kilowatt-hour, according to the size and location of the power station and the amount of power used by the customer). The cost of power per ten-hour day would be equal to $\$0.06 \times 83 \times 10 = \$49.80$.

If the motor has an efficiency of 80 per cent, we may calculate, in the same way, the cost for power to run the factory a ten-hour day and find it to be $56.00. The difference in cost in operating these two motors for a year would be $1860, and this is more than the probable cost of the motor. This one example serves to show how important the efficiency of a dynamo-electric machine is commercially.

297. Determination of Efficiency from Name-plate Data.—The full-load efficiency of a motor can always be approximately determined from the rating given on its name-plate. The A.I.E.E. rules specify that, among other things, the name-plate of a motor shall give the voltage and current for which the machine was designed at full load, and the output in horsepower (if the machine is a motor).

Suppose, for example, that the rating of a certain machine from the name-plate is 110 volts, 38.5 amperes, 5 h.p.

The watts input, full load $= 110 \times 38.5 = 4350$ watts.
The watts output, full load $= 5 \times 746 = 3730$ watts.
Hence the full-load efficiency $= 3730 \div 4350 = 85.7$ per cent.

298. Measured and Conventional Efficiency.—The efficiency of any machine being the ratio of its output to its input, expressed in terms of the same unit of power, we may write,

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{losses}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{losses}}. \quad . \quad (120)$$

The efficiency of any electrical machine can be determined by actually loading it and measuring its output and input; the efficiency determined from actual load measurements is called the *directly measured* efficiency.

Every piece of electrical apparatus carries a name-plate, on which is plainly marked its rating. In the case of generators, the rating is expressed as the kilowatts which the generator can furnish at its terminals; the rating of motors (except series railway motors) is usually given as the horsepower available at the shaft. It is necessary to specify also the terminal voltage of the machine and the speed at which it is to operate at full load; customarily the full-load current is also given. In the case of railway motors, only the full-load current, voltage, and speed are specified.

From Eq. (120), it is evident that if an input or an output is assumed for a machine from its name-plate, and the corresponding losses are determined, the efficiency at the input or output assumed may be obtained. The efficiency so determined is properly called the *conventional* efficiency, although it is also often termed the *predicted* efficiency.

It is not always possible to measure exactly the losses corresponding to any particular load; some are measured under different conditions of loading, some are calculated from resistance measurements, others may even be determined by empirical methods. It is because these losses are determined by conventional methods, that the value of efficiency resulting is called the conventional efficiency.

The ordinary electrical machine has a fairly high efficiency, generally over 80 per cent. In view of this fact, it is generally possible to get the efficiency of an electrical machine more accurately by a suitable conventional method than it is by making direct measure of input and output. Thus, with an efficiency of 90 per cent, if the input is read 1 per cent high in the direct measurement of efficiency and the output is read 1 per cent low, the efficiency, as found, will be 2 per cent lower than its actual value. On the other hand, in the conventional method, even if an error of 2 per cent is made in determining the losses, the efficiency will differ from its actual value by only 0.2 per cent.

This latter statement is based, of course, on the assumption that the conventional method does measure the actual losses as they would exist if the machine were loaded to the corresponding value. Actually, such is not the case, the core loss in a rotating armature, for example, is by no means the same in the conventional test as when actually running under load. But even if a considerable error of this kind is involved, the conventional efficiency is probably as near the actual value as is the directly measured value of efficiency. The more efficient the machine, the greater is the superiority of the conventional method.

299. Losses and their Variation with Load.—*Mechanical Losses.* There are several so-called *losses* in any electric machine. For example, in the operation of a motor, some of the power input is used in overcoming the friction of the shaft turning in the bearings, the friction of the brushes on the commutator, and the friction of the revolving armature on the air (called *windage*). These are usually designated as mechanical losses.

Electrical Losses. There are also other losses due to ohmic resistance, hysteresis, and eddy currents; these are called electrical losses. As losses due to ohmic resistance may be mentioned the I^2R losses in the field windings and field rheostat, in the armature windings, and in the brushes and brush contacts. Then there are the hysteresis and eddy-current losses in the armature core and the pole faces, where power is used and given out in the form of heat.

Variation of Losses with Load. Some losses are independent of the load the machine is carrying, and some vary with the load. If the speed of a machine (either motor or generator) remains constant as the load varies, we may assume that all of the mechanical losses are constant, i.e., independent of load. If they are measured at any one load they may be regarded as having the same value for any other load.

When the speed of a machine varies as the load changes, *the mechanical losses may be considered to vary directly with the speed.* If, for example, in a series motor, the mechanical losses are measured and found to be 800 watts when the armature is turning 1200 r.p.m., they may be assumed to equal 400 watts when the motor is turning 600 r.p.m.

The current flowing in a shunt field is nearly independent of the load. In the case of a shunt motor connected to a constant-potential line, the shunt-field current will be entirely independent of the load, and hence *the I^2R loss in the field coils will be constant.* This loss will be larger when the machine is first started than when it has run a sufficient length of time to get warmed up; in Chapter IV the effect of an increasing temperature upon the resistance of a conductor was discussed.

The current in the armature and series field of a motor (or generator) is proportional to the load the machine is carrying; the ohmic resistance loss is, therefore, *proportional to the square of the load,* and if the I^2R loss is plotted as one coordinate with the load as the other, the curve will be a parabola.

300. Loss at the Brush Contacts.—The resistance of the brush contacts (carbon brushes are assumed) is not a constant quantity, but depends upon the current density at the contact surface. If the current density is low, the resistance is high, and vice versa. This is entirely different from the armature and field circuits; except for the temperature change effects, the resistances of these circuits are constant and not affected by the current density. It has been found by experiment that the IR drop through *two brush contacts in series* may be calculated approximately by the formula,

$$\begin{aligned} IR\text{ drop} &= 1.0+(0.3\times\text{amperes per sq. cm.}) \\ &= 1.0+(0.047\times\text{amperes per sq. in.}) \quad . \quad . \quad . \quad (121) \end{aligned}$$

If we assume the normal current density at full load is 5 amperes per square centimeter, then the brush contact drop $=1.0+(0.3\times5)=2.5$ volts, *for both brushes together.* This drop is nearly the same on all sizes of machines, as it depends only upon the *current density* in the brush contacts and not upon the current itself; the current density at rated load is practically the same for all sizes of machines.

The loss due to brush-contact resistance is small compared to the other losses in a machine and it is often computed by taking *the IR,* or *contact resistance drop, as equal to 2 volts at all loads.* This makes the calculation of brush contact I^2R very simple, and even though it is a rough approximation, but very little error is produced in the final result. In a motor taking 40 amperes at full load, the contact resistance loss may be approximately obtained by putting I^2R loss $=40\times2=80$ watts, etc.

301. Hysteresis and Eddy-Current Losses.—In a shunt generator or motor, *the hysteresis and eddy-current losses may be considered as independent of the load, providing the speed does not change with the load.* They depend upon the flux density in the armature core and the speed with which the armature revolves. In the case of a motor the speed will generally change somewhat as the load changes; and in this case, however, but little error is made if these losses are considered to vary directly with the speed, as do the mechanical losses.

302. Stray Power.—All of the mechanical losses, and the hysteresis and eddy-current losses (commonly called *core loss*) are measured together, and the whole loss is called the stray power. The stray power therefore comprises all mechanical losses and core losses, and, if the field density is considered as constant, it may be assumed to vary directly with the speed.

303. Loss Curves.—We have, then, four losses to take into consideration when determining the efficiency of a motor or generator: the stray power, which varies slightly with the load; the shunt-field loss, which is generally independent of the load; the brush-contact resistance loss, which varies directly with the load; and the armature and series-field (if there is any) I^2R loss, which varies with the square of the load.

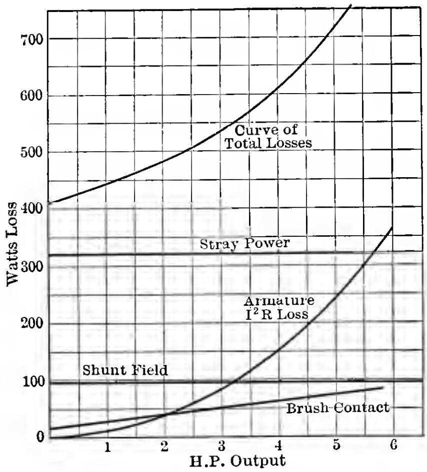


FIG. 307.—The loss curves for a five horse-power continuous-current motor.

These various losses are shown plotted as curves in Fig. 307. The results are taken from a test of a 5-h.p. shunt motor and are about right for any continuous-current machine of this capacity. The curve marked "total loss" is plotted by adding the ordinates of the other four curves, the significance of which is indicated in the figure. The shunt-field loss is 97 watts at all loads; and the stray power is 320 watts at no load and practically the same at full load; the brush-contact resistance loss is 3 watts at no load and 80 watts at full load; the no-load armature I^2R loss is less than one watt and at full load it is 242 watts.

304. Calculation of Efficiency from Loss Curves.—If such a set of curves is given for any machine, its efficiency curve may be at once obtained. The input to any electric machine must evidently be equal to the output plus all the losses occurring in the machine; hence the efficiency may be obtained by using the formula.

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{losses}}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (122)$$

At full load the output of the motor referred to in the previous paragraph is equal to $746 \times 5 = 3730$ watts.

The input must be

$$3730+(242+97+80+320)=4469 \text{ watts.}$$

Hence,

$$\text{The full load efficiency}=\frac{3730}{4469}=83.6 \text{ per cent.}$$

When the output = 4 h.p.,

$$\text{The efficiency}=\frac{2982}{2982+627}=82.7 \text{ per cent.}$$

When the output = 3 h.p.,

$$\text{The efficiency}=\frac{2238}{2238+550}=80.3 \text{ per cent.}$$

When the output = 2 h.p.,

$$\text{The efficiency}=\frac{1492}{1492+495}=75.1 \text{ per cent.}$$

When the output = 1 h.p.,

$$\text{The efficiency}=\frac{746}{746+455}=62.1 \text{ per cent.}$$

When the output = 0 h.p.,

$$\text{The efficiency}=\frac{0}{430}=0 \text{ per cent.}$$

Ordinarily, the loss curves are not obtained in terms of horsepower output, but in terms of the armature current, and hence the efficiency curve is obtained in terms of the armature current. It may, however, be readily converted into terms of horsepower if so desired.

305. Efficiency Determination without Actually Loading the Machine.—To determine the conventional efficiency of an electrical machine it is not necessary to actually load it; some method of determining the various losses is all that is required. *This is a great advantage in the testing of large machines;* for example, if a 1000-h.p. motor were to be tested, two difficulties would be encountered if it were attempted to actually load it up to its rated capacity. First, it would be difficult to find some way of putting a load of 1000 h.p. on the motor and then of measuring it accurately; secondly, it would require about 1000 kw. of power to run the test and, even though this power were available, *it would all be wasted in running the test* unless some special " pump-back " test was used.

The term " pump-back " test is used to designate a test in which two similar machines are being tested at the same time. In such a case, one machine is generally used as a motor to drive the other machine as a generator; the power from the generator is " pumped " back into the line from which the motor is drawing its power; hence the power actually used, even when both machines are operating at approximately full load, is only that amount necessary to supply the losses in the two machines.

306. Obtaining Data for the Determination of Efficiency.—To determine the losses, no facilities for loading the machine are required, and but little power is used in making the test. To get the stray power, the machine is run as a motor with no load and, after sufficient time has elapsed for the field coils to warm up, the input to the armature circuit is measured.

The resistance of the armature winding is determined, and the armature I^2R loss (with no-load current flowing) is calculated. This subtracted from the no-load input to the armature gives the no-load stray power, and this is assumed as the same for all loads. The resistance of the shunt-field circuit is measured, and the shunt-field I^2R loss is found. The brush-contact resistance is calculated after the area of the brush contact has been measured, and the brush-contact I^2R loss may be determined from the formula discussed in a previous paragraph. (Eq. 121.)

The readings obtained from such a test upon a 100-h.p. motor would give results about as follows:

Rated voltage = 230 volts;
Rated current = 350 amperes;
Shunt-field resistance (hot) = 49.8 ohms;
Armature resistance (hot) = 0.0112 ohm;
Armature current (running light) = 15.1 amperes;
Total area of all brush contacts = 140 sq. cm.;
Current density at full load = 4.93 amperes per sq. cm.;
Drop at brush contacts, full load $= 1.0 + (0.3 \times 4.93) = 2.48$ volts;
Drop at brush contacts, half load $= 1.0 + (0.3 \times 2.46) = 1.74$ volts;
Armature I^2R loss, no load $= 15.1^2 \times 0.0112 = 2.5$ watts;

Stray power at no load (assumed constant in this problem)

$$= (230 \times 15.1) - \left\{ 2.5 + 15.1\left(1.0 + \left(0.3 \times \frac{15.1}{70}\right)\right) \right\} = 3455 \text{ watts};$$

Shunt-field loss $= \dfrac{230^2}{49.8} = 1060$ watts.

From the data, the curves of Fig. 308 were plotted. Then the total-loss curve was constructed and from this the efficiency was readily computed. The shape of the efficiency curve is about the same for any electric motor or generator. The efficiency is low at light loads, reaches a fair value at half-load and, from this point up to one and one-quarter load, is practically constant. The full-load efficiency of large machines may be as high as 94 per cent, while for small machines of a few horsepower it is nearer 80 per cent.

307. Allowable Operating Temperatures.—It has been said before that the *capacity* of any electrical machine is limited by the ability of its insulation to withstand, continuously and without deterioration, the maximum temperature caused by its losses.

Experience shows that, for each type of insulation, there is a certain limiting temperature above which the insulation will deteriorate more or less rapidly. As far as the useful life of the insulation is concerned, however, there does not seem to be any advantage in operating at temperatures below this safe limit.

For the purpose of standardization, the American Institute of Electrical Engineers has, in its Standardization Rules, set certain maximum permissible limits for insulating materials, as follows:

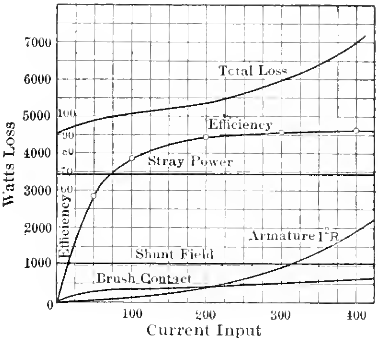


FIG. 308.—Loss curves and efficiency calculated from these curves; the results are for a 100-hp. 230-volt motor.

Class O.	Cotton, silk, paper and other fibrous materials, not so treated as to increase the thermal limit	90° C.
Class A.	Materials similar to Class O, but treated or impregnated, and including enameled wire	105° C.
Class B.	Mica, asbestos and other material capable of resisting high temperatures, in which any Class A material or binder is used for structural purposes only, and may be destroyed without impairing the insulating or mechanical properties	125° C.

The temperatures fixed are those which should not be exceeded in *any part* of the insulation. In every machine, certain spots will be hotter than the average for the part of the machine in which they occur; these are called the *hottest spots*. The temperatures of the table refer to these spots, and will generally be from 10° to 15° C. higher than the *observed* temperatures of a completed armature or field winding, depending on how the temperature is observed.

308. Capacity and Rating.—The rating of an electrical machine is determined by the service for which it is intended. For service in which the machine is never expected to operate above a definite limit, the rating may be the same as the capacity. Such machines are referred to as 50-degree; they are not intended to have any overload capacity.

When the service requires a machine to operate for the most part at or below a definite limit, but is likely to require it to carry certain overloads for periods of a few hours, the rating of the machine must be less than its capacity. Such machines are referred to as 40-degree machines; they are usually capable of carrying 25 per cent overload for two hours.

Machines intended for intermittent loads, as railway or hoisting motors, etc., are given an *intermittent*, or "short-time," rating. With such service, heat is generated intermittently, but is dissipated to the surrounding air at a more or less steady rate. Experience has shown that such service may be approximated, so far as heating is concerned, by allowing the machine to reach its limiting temperature after continuous operation at rated load in a definite time. Thus, a motor which, operated at full load, reaches the limiting temperatures in two hours, is given a two-hour rating. For some services, a twenty-minute rating approximates the intermittent duty for which a motor is intended; railway motors are generally given a one-hour rating.

Motors intended for intermittent duty (especially inclosed motors), are often built with more iron than is necessary for magnetic requirements, in order to increase their heat-storage capacity. There is obviously no gain in increasing the heat-storage capacity of a machine that is to be given a continuous rating.

309. Heating and Cooling.—The rise in temperature of a machine depends upon the rate of generation of heat, upon its heat-storage capacity, and upon the facility with which the heat is dissipated. When the rates of heat generation and dissipation become equal, the temperature will become stationary. The form of curve showing the temperature rise is a logarithmic curve of exactly the same shape as that given for the rise of current in an inductive circuit, in section 87. The heat dissipative quality of the machine corresponds to the resistance, and the heat storage quality of the machine corresponds to the inductance, of the electric circuit, the rate of heat supply corresponds to the voltage impressed in the inductive circuit multiplied by the final value of the current.

Cooling of a machine is facilitated by causing air currents to circulate in definite paths through the machine. In modern machines, a fan at the back end of the armature is quite common.

310. Open and Closed Ratings.—If a motor is installed in surroundings where the air is clean, it may be open, so that the air may circulate freely through its windings; but when there is much dust in the air, it becomes necessary to protect the motor from the dust as much as possible. In such cases, only one or two openings, covered with screens, are provided for the circulating air, resulting in a *semi-inclosed* motor. If the surrounding air is laden with dust, moisture, or fumes, or the motor is subject to splashing (as in railway or marine service) the motor must be totally inclosed.

Depriving a motor of free circulating air will naturally reduce its capacity; a motor with an open rating of 5 h.p. might develop only 3 h.p.

before reaching limiting temperatures when totally inclosed; its closed rating would therefore be 3 h.p.

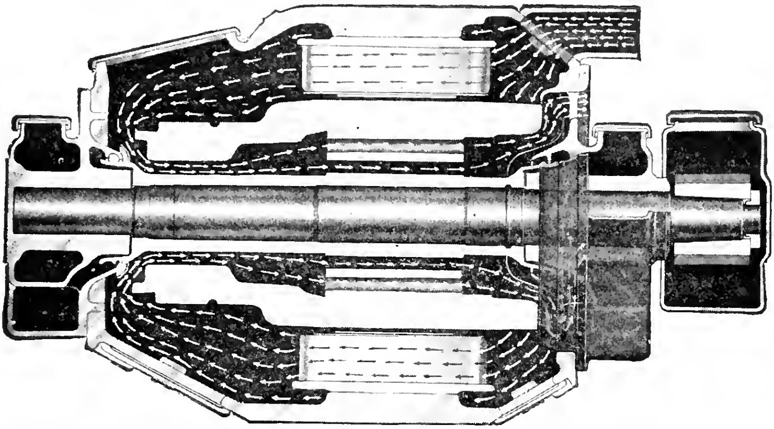


FIG. 309.—Showing the paths taken by the cooling air in a semi-inclosed motor.

The importance of properly directing the air currents within the frame of semi-inclosed and totally inclosed motors, to bring the heated air against cool surfaces, is thus apparent. In Fig. 309, the path of the air currents in a semi-inclosed motor are shown; Fig. 310 indicates them for the same motor totally inclosed.

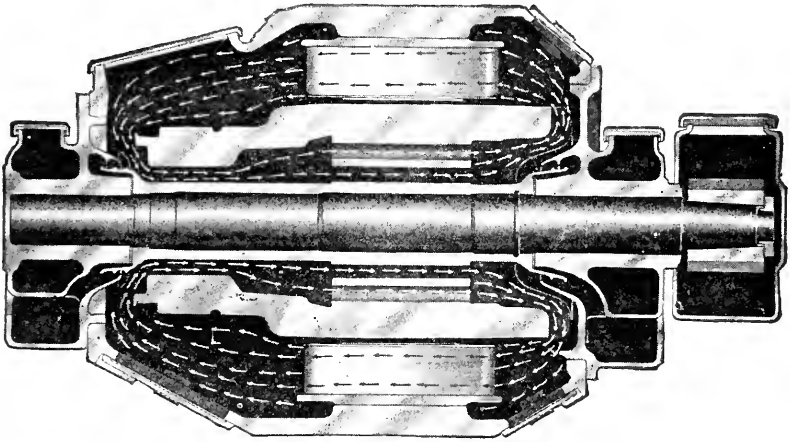


FIG. 310.—In a totally inclosed motor the air currents circulate about as shown here; the only way the heat can escape from the motor is by the comparatively poor convection through the steel frame. The air heats up as it passes through the armature and partly cools off as it flows by the frame while in the outer part of its path. The safe rating of such an inclosed motor is comparatively low.

311. Methods for Determining Temperature Rise.—There are two standard methods for determining the temperature rise of the various parts of c.c. machines: (1) by thermometers; and (2) by resistance measurements.

Thermometers are applied directly to the part of which the temperature is required. In c.c. machines, the available parts, while the machine is running, are the field coils, bearings, frame and pole tips. The final temperature of the armature winding, armature core, and commutator, are taken with the machine stopped, after it has been operated for a sufficient time for the armature temperature to become constant. Thermometers are generally held against a surface by felt pads or by putty; when so used for a field coil or armature winding, the hottest-spot temperature is estimated by adding a correction of 15° C. to the highest temperature observed.

The average temperature rise of a winding may be determined from resistance measurements, as was already shown in Chapter IV. When this method is used, a hottest-spot correction of 10° C. is added to the temperature calculated.

PROBLEMS

1. What is the full-load efficiency of a motor with a name-plate rating of 115 volts, 38 amperes, 1200 r.p.m., 5 h.p.?

2. If the only variable loss in a motor is that due to the armature I^2R loss, prove that the maximum efficiency occurs when this loss is equal to the sum of all the other losses.

3. A 110-volt motor has a shunt-field resistance of 68.3 ohms, and an armature resistance of 0.052 ohm. Rated full-load current is 110 amperes. When running light, it draws from the line 5.2 amperes. What is its efficiency and horsepower output, with an input current of 50 amperes? With 100 amperes?

4. The motor of a motor-generator set takes 110 amperes at 240 volts and delivers 91.2 per cent of its input to its generator. This machine has an efficiency of 92 per cent and is furnishing power to a 110-volt line. What is its output current?

5. The data for a 10-h.p., 220-volt motor are: shunt-field resistance 250 ohms; armature resistance 0.35 ohm; area of contact per set of brushes, 2 square inches; rated current, 40.3 amperes; stray power, 460 watts. Plot curves (points for every 10 amperes input, to 50 per cent overload) of armature loss, field loss, brush-contact loss, stray power, shunt-field loss, total loss and efficiency.

CHAPTER XI

AUXILIARY APPARATUS USED WITH CONTINUOUS-CURRENT MACHINERY

312. Switches.—A switch is a device for easily opening and closing a circuit. The simplest type consists of a copper blade, hinged at one end and fitting tightly between two copper plates, or jaws, at the other. A wooden handle is fastened to the end of the copper blade, by which the switch is operated. It is styled a knife, or lever, switch; the current is ruptured in the air.

These switches are made in various sizes, from 25 amperes to several thousand amperes. The larger ones use compound blades, as the contact surface available on one blade would not be great enough to carry the large currents without overheating. The switches used for generators supplying power to lighting circuits are generally double pole, one blade being in each side of the line. Railway generators, however, have the negative side continually connected to the negative bus, and so to ground, so that these machines require only a single-blade switch in the positive line.

The length of the blade, and the spacing between blades of a double-pole switch, depend upon the voltage for which the switch is designed, those to operate on a 600-volt circuit having considerably longer blades and greater spacing between blades than those for a 250-volt circuit. This is due to danger of a short switch not opening a 600-volt circuit even though the blade has been pulled back as far as it will go. When the switch is opened, an arc is formed, and the arc may run down the blade as the switch is opened and so hold over and burn from one post of the switch to the other. To prevent this disastrous arcing, *quick-break switches* are sometimes used. These are so designed that the switch is opened by a spring, snapping the blade back very quickly.

313. "Cutting" of a Switch.—Switches sometimes begin to "cut," especially in the hinge. This is caused by grit getting into the joint and wearing off little pieces of copper as the switch is opened and closed. Then the little bits of copper help this rubbing process, until the blade is worn very rough in the hinge and so makes poor contact and is caused to overheat. As soon as "cutting" is detected, a switch should be taken apart and the rubbing surfaces smoothed down with a file and emery. A little grease in the joint will generally keep the rubbing surface in good condition.

314. Remote-control Switches.—A switch which is opened directly by the operator is said to be a manually operated switch. Very large switches may be operated by means of small motors, the current supply of which is controlled by a small switch, placed conveniently for the operator; the main large switch may be far from the controlling switchboard. Instead of using a small motor for operating a remote-control switch, the action of an electromagnet may be used; in some types of switches (e.g., such as is used on many railway equipments) compressed air may be used to open and close the switch. It is the remote-control switch that makes the compact, *remote-control switchboard* of a large modern power plant possible.

315. Fuses.—When a machine or circuit is carrying more current than that for which it was designed, serious injury may result from overheating. The purpose of a fuse is to prevent such a possibility. A fuse consists of a piece of easily melted alloy, in the form of a wire or ribbon, connected in series with the machine or circuit to be protected. The size of a fuse is so selected that when a dangerous current is being carried by the circuit it is designed to protect, the heat generated by the I^2R loss in the fuse is sufficient to melt it and so the circuit is automatically opened.

316. Types of Fuses.—There are several types of fuses in common use. The earlier type was the *string fuse*, which consisted merely of a piece of fuse wire inserted in the circuit by suitable clamps and screws. The disadvantage of this kind is that there is some danger of starting a fire when the fuse blows and throws melted lead around.

Fig. 311.—A plug fuse. Insurance rules have done away with the open string fuse originally used. This fuse screws into a lamp socket; the cover is of mica, for inspection.

The *plug fuse* is designed to overcome this possibility; it consists of a short string fuse mounted in a porcelain plug fitted with a screw base like an incandescent lamp base. The cover to the plug is made of mica so that it may be seen whether or not the fuse has blown. The plug fuse is illustrated in Fig. 311.

The National Board of Fire Underwriters do not permit the use of plug fuses for currents greater than 30 amperes. For larger sizes the *cartridge fuse* must be used. This consists of a tube made of fiber, sometimes filled with borax, infusorial earth, or similar substances, through the center of which the fuse ribbon passes. The two ends of the paper tube are fitted with copper caps, to which the ends of the fuse are fastened. Short copper blades are fastened to these copper caps in the larger sizes and these fit into copper clips on the fuse block. When such a fuse blows, the arc is confined and smothered by the substance with which the fiber tube is filled.

In order to detect whether or not such a fuse is blown, a *tell-tale* is provided. This consists of a very small fuse, soldered to the copper terminals so that it is in parallel with the main fuse. This small fuse, however, for a short way passes on the outside of the paper tube, so that it can be seen. When the main fuse blows, the little one immediately melts and so gives evidence of the blowing of the main fuse. Such fuses are generally called N.E.C.S. fuses, meaning that they are designed in accordance with the National Electric Code Standard.

317. Replacing a Fuse.—When a fuse is blown, the circuit in which it is connected should be opened, by first opening the proper switch, and then a new fuse may be inserted. A new fuse should not be inserted until it has been ascertained that the circuit is dead; neglect of this point is likely to prove dangerous to the operator putting in the fuse, as the new fuse may blow (melt) while he is inserting it and so cause a dangerous burn.

Before leaving the subject of fuses, we must say a word regarding the practice of substituting copper wire, nails, etc., for fuses that have been blown, as a result of an overload on the line. A fuse is used *to protect against fire, overheating of machinery, etc.,* and when the fuse fails to work dangerous results may follow. If a fuse is replaced by a much larger one, or copper wire, etc., the circuit is no longer protected. An operator who replaces fuses by pieces of wire, etc., is just as foolish as a fireman who sits on the safety valve of his boiler and waits there for the boiler to blow up.

318. Circuit-Breakers.—In many kinds of service, overloads occur quite frequently; this is especially true in railway work. If several cars start up at the same time, or some cars start up just when two or three others are on an upgrade, the feeder supplying power for that section of the road is sure to be overloaded and, if it were fused, the fuses would be continually blowing, causing much work and annoyance for the operator. Also, during the time spent in replacing the fuses, the feeder would be dead and soon the cars would all be off their schedule time.

For this kind of service, where overloads occur frequently and an operator is present, fuses are not used for protection; a switch that opens automatically when an overload occurs, called a *circuit-breaker,* is used instead. The circuit-breakers used in continuous-current circuits always open the circuit in air.

The air-break circuit-breaker does not resemble a knife switch very closely, as may be seen by reference to Figs. 312 and 313. These illustrate a double-pole breaker for a low-voltage circuit of perhaps 100 amperes capacity.

319. Use of Multiple Contacts on an Air-break Circuit-Breaker.—The breaker shown in Fig. 312 is an air-break circuit-breaker, which has three sets of contact surfaces; when the breaker is closed practically all of the current is carried through a set of contacts made by a copper foil finger pressing against a copper block. When the breaker opens, these contact surfaces open and the current goes through another pair of copper contacts (not as carefully fitted as the first), and when these open, a final pair of contacts carry the current.

This last contact is made between two carbon blocks, the separation of which finally ruptures the current. The idea of thus breaking the circuit through a series of contacts is to preserve the main pair of contacts in good condition. If the current were ruptured by the main contacts, the arcing would soon spoil the contact surfaces. The arcing is really all done at the carbon blocks; when they are worn away a new pair may be put in.

The arm of the breaker is held in the "closed" position by a latch; it is tripped by the impact of a small plunger or similar device, which is operated by a solenoid located in the breaker. This solenoid carries the same current as the feeder is supplying, and when this current gets too large the solenoid lifts up the plunger, trips the latch, and a spring forces the breaker to quickly snap open. Figure 313 shows the breaker in the "open" position.

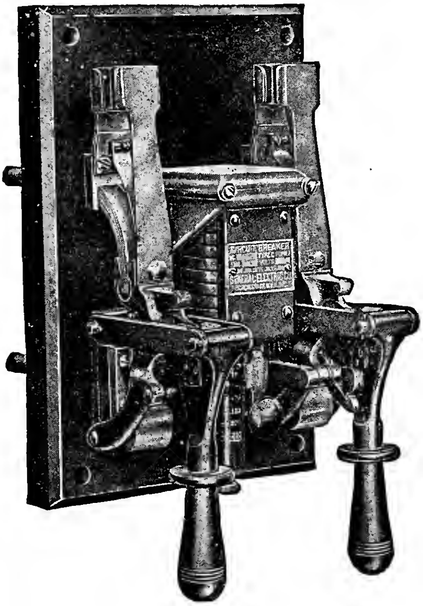


FIG. 312.—A double-pole air-break circuit-breaker. There are three sets of contacts which open successively as the breaker opens.

320. Procedure in Closing a Circuit-Breaker.—Each breaker in a station has a knife switch in series with it. When a breaker opens, *it must not be closed until the switch in the same feeder has been opened.* The circuit-breaker is for the protection of apparatus, etc., and cannot operate if the attendant holds the handle; hence, while *it is being closed it cannot protect the apparatus.* If, however, the switch in series with it is opened first, then the circuit-breaker closed, and then the switch closed, the circuit-breaker is free to operate, and protection against overloads is always obtained.

There is always an adjustment on a circuit-breaker which fixes the current at which it opens the circuit. A certain breaker rated, for example at 100 amperes may be set to trip at any current between 50 amperes and 150 amperes.

321. Overload, Time-limit Breaker.—Any electric machine will stand an overload for a short time without suffering injury. A manufacturing company will generally guarantee large generators to carry a 25 per cent overload for two hours and a 50 per cent overload for one minute.

In certain kinds of work, the load on an electric machine is intermittent, and for short periods of time there may be quite a heavy overload on the machine. If the duration of this overload is short, the machine will carry it safely and it is not desirable to have the circuit opened by a breaker or fuse. But if this overload should continue too long, the machine would be injured, and it is thus evident that a fuse or ordinary circuit-breaker could not properly take care of this kind of a load.

An *overload, time-limit breaker* is designed to fit such a service. It is essentially a circuit-breaker, to the plunger of which a dash-pot is attached. When an overload occurs, the plunger begins to move, but the damping is such that a considerable time is required for the plunger to move far enough to trip the breaker. The time elapsing from the moment the overload occurs to the tripping of the breaker is adjustable by a valve in the dash-pot, or similar device. It is evident from this description that such a piece of apparatus as the time-limit breaker just suits the needs of motors operating punch-presses, rolls, hoists, etc.

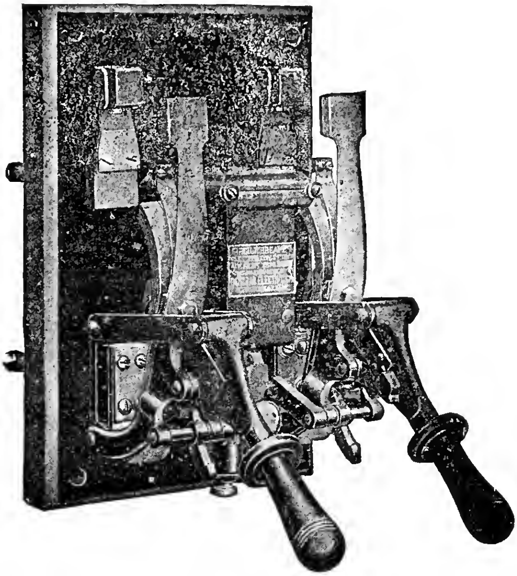


Fig. 313.—The breaker of Fig. 312 in the open position. The main contacts of copper leaves, the second contacts of block copper, as well as the third contacts of carbon block, can be seen.

322. Meters.—A complete description of the most important types of meters used in c.c. circuits has already been given in Chapter VI; something will now be said of their use.

An *indicating meter* is one in which the pointer deflects over a graduated scale, and so indicates at any instant the current or voltage in the circuit to which it is connected. It has no rotating parts and makes no record of the motion of its finger. These instruments show the operator how much current a feeder is carrying at any instant, or what voltage a machine is generating. From their indications the operator may properly adjust the voltage, re-distribute the load from one machine to another, etc.

A *recording meter* keeps a time record on a suitable chart, indicating by pen marks the magnitude of all fluctuations and the time at which they occurred; it must, therefore, include a time or clock mechanism within its parts.

Meters are subdivided into *switchboard instruments* and *portable instruments.* The first are fastened permanently on a switchboard and can generally be used only for indication on the machine or feeder to which they are attached. The portable meters are smaller and more compact than the switchboard instruments, and are made for laboratory work, or for carrying out to different parts of a distributing system to read the current or voltage.

323. Switchboard Meters.—A switchboard meter should be compact, have a large, well-marked scale of uniform graduations (except in some special cases), have a large, black pointer on the end of the indicating finger, and should be well damped. Of course, there are numerous other points to consider, such as permanency of calibration, freedom from temperature errors, etc., but only those mentioned above will be considered here.

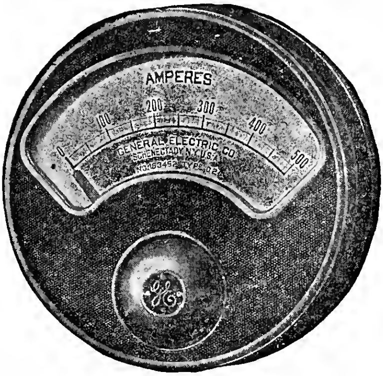


Fig. 314.

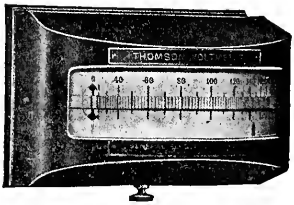


Fig. 315.

Fig. 314.—A switchboard meter of the round type; its pointer is large so as to be visible from a distance.

Fig. 315.—An edgewise type of switchboard meter; it is somewhat more compact than the round type and is used when many meters have to be assembled on a small board.

On a large switchboard, having many generators connected to it, and supplying many feeders, the size of a meter is of prime importance. There may be on one switchboard a hundred or more meters, and it is evident that if these meters are not comparatively small, the switchboard must be very large and so difficult for one operator to manage. Also, the expense for bus-bars, marble, copper feeders, etc., makes it advisable to keep the size of a switchboard as small as possible.

A round type of switchboard instrument is shown in Fig. 314 and in Fig. 315 is shown an edgewise type, having the scale horizontal. It will be noticed that the design of the meter has been carried out with the idea of getting compactness and still having a large, easily read, scale. The relative advantages of the round type and the edgewise types are much discussed; which is the better seems to be an open question.

324. Scale of a Meter.—The scale of an ammeter should be uniformly graduated, but sometimes it is advisable to have a voltmeter with a condensed scale on its lower ranges, for the purpose of getting a more open scale in the range where it is always used. For instance, a meter to be used on a 600-volt circuit would probably have a total range of 700 volts; from zero to 400 volts the scale might well be condensed, as the meter is practically never used on these ranges. From 400 volts to 700 volts, the scale could then be more open than if it were uniform throughout its range. The necessity for a clearly marked scale and large, easily-seen, indicating pointer is apparent when it is remembered that one operator may have to notice continually the indications of a hundred or more of these meters.

325. Portable Meters.—The portable type of meter differs from the switchboard type in that it is generally more accurately calibrated, has a more accurate and finely divided scale, and has a very thin indicating pointer. A switchboard instrument which indicates with an accuracy of 2 per cent is generally good enough; for laboratory tests, however, much higher accuracy is generally required. A common type of portable laboratory voltmeter is shown in Fig. 316.

326. Value and Importance of Recording Meter Records.—By inspection of the curve traced by a recording meter, the station superintendent can tell at a glance just what the load on his station has been, what its maximum and minimum values were, and when they occurred. Or, if the record is from a voltmeter, it serves to show how well the operator has maintained the voltage constant. These meters are excellent for keeping the operator at his task, as they infallibly indicate any variations in the station voltage.

327. Maximum Demand Meters.—In order to distribute the fixed charges of the power plant and distribution system of a power company more equitably among its customers, there is a growing tendency, in charging for electric energy, to take into account the power demands of the customers as well as their energy demands. A customer who requires a large amount of energy at a uniform rate evidently ties up less generator and feeder capacity than one who takes the same amount of energy at widely varying rates. Thus, the unit charge for energy should take into account not only the total kilowatt-hours used by a customer during a given period, but also the maximum power demanded at any time during the same period.

It would not, however, be fair to consider as the maximum of a customer, the greatest instantaneous peak that occurred; it is better to take the average of the power demanded over an appreciable time interval, so as not to include short-circuits or other abnormal consumptions of energy that last for so short a time as not to affect the generator or feeder capacity of the system. The time interval over which the maximum demand is taken depends upon the relation the maximum demand bears to the connected capacity of the station, so that the interval may be only a few minutes in the case of a large installation, and fifteen or thirty minutes in small ones.

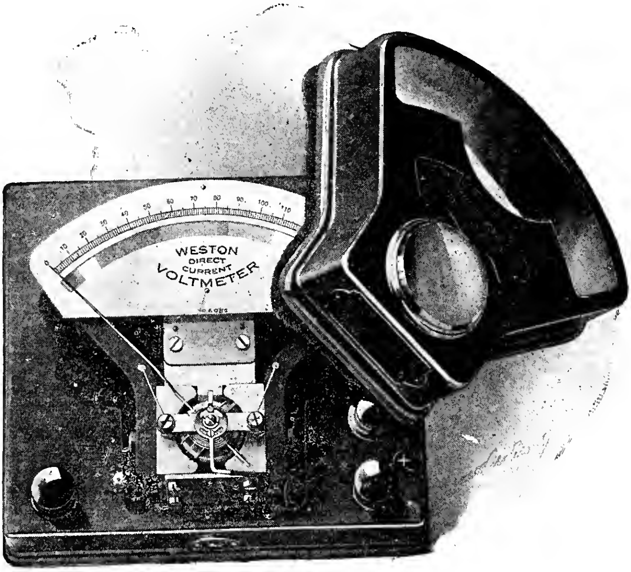


FIG. 316.—A portable type of continuous-current meter; the pointer is very fine and can be very accurately read with the assistance of the mirror mounted in the scale.

If no time interval were to be considered, the simplest kind of demand meter for a c.c. system, when the voltage at the customer's premises is practically constant, would be an ammeter, the needle of which pushes another friction pointer before it up the scale and leaves it at the maximum indication.

Where the average power demand over a given interval is desired, without any record of the exact time at which it occurred, a thermal type of demand meter is much used. One form operates by heat storage from an electrical heating element that includes a bi-metallic spring system; the two metals of the spring, having different rates of expansion, cause a pointer to move over a scale. This pointer pushes the maximum demand pointer before it, and leaves it at the highest reading.

Where both the maximum power demand and the time of occurrence are desired, the demand meter works in conjunction with a watt-hour meter; the demand meter may be in a separate case, or may be combined with the watt-hour meter in a single case. In one such form of demand meter, a set of cyclometer type wheels are electrically interlocked with the register of the watt-hour meter. The type wheels are moved forward at a rate which is exactly equal to the rate of energy flow through the watt-hour meter, and therefore give at any instant an indication which is equivalent to the reading of the watt-hour meter dial. At regular time intervals, a clock closes a circuit, energizing an electromagnet which pushes a rubber platen, a copying ribbon and a paper tape against the cyclometer type wheels, thus printing their reading on the paper tape. The electrical interlocking device between the demand meter and the watt-hour meter consists of a contact-making device attached to one of the spindles of the watt-hour meter register, the contact maker energizing an electromagnet which, for each impulse, pushes the cyclometer wheels ahead one unit.

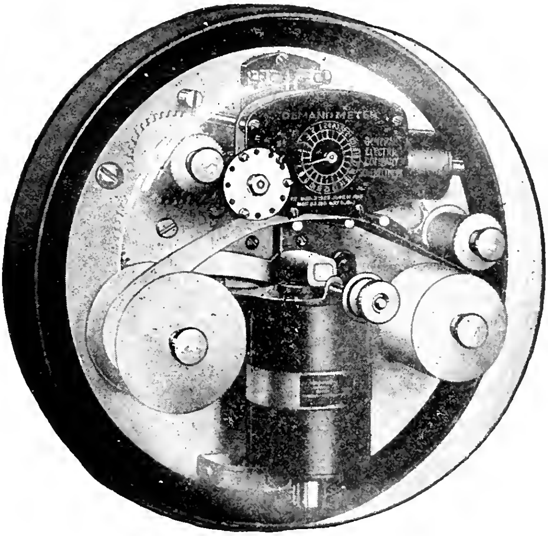


FIG. 317.—The printometer, one type of maximum demand meter.

A view of the printing type of demand meter is shown in Fig. 317, a plan of its electrical circuits in Fig. 318, and a twelve-hour record made by it, from 6 A.M. to 6 P.M, in Fig. 319. The left-hand figure indicates the time of printing, the second similar figure indicating the half-hour reading; thus the maximum kilowatt-hour consumption for a thirty-minute period is shown to be 46, which occurred between 2:00 and 2:30 o'clock, indicating an average power demand during this interval of 92 kw.

328. Switchboards.—Originally, the switchboard was a very crude affair, a wooden rack on the front of which were mounted the switches and fuses necessary for the operation of the plant. To-day, the switchboard is probably the most important part of a generating plant; if an accident happens at the switchboard, the whole plant may be crippled.

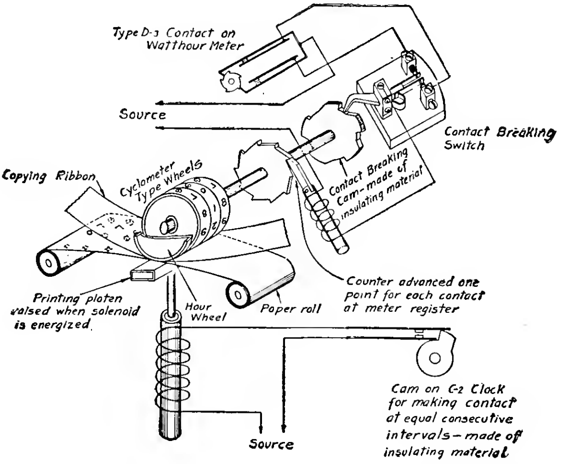


FIG. 318.—Schematic representation of the essential parts of the printometer, showing how the reading of the watt-meter is printed every half hour.

The switchboard is the place to which the electric power from the generators is supplied and metered and thence distributed to the outside system through a group of feeders. On it are located all the switches, meters, and protective devices of the plant, and from it the operation of the whole plant, both inside and outside the power house, is controlled.

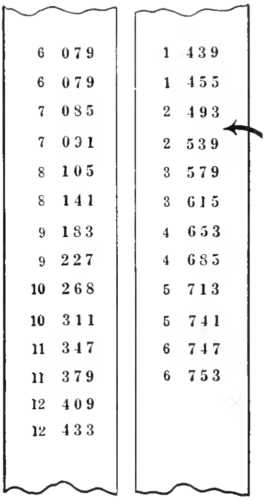
329. Panels.—A modern switchboard is divided into a number of *panels;* each panel serves for the control either of one generator or of a feeder or group of feeders. They are styled the *generator panels* and the *feeder panels;* in addition there may be panels for recording meters, etc.

330. Construction of a Switchboard.—The material of which the board is made must be a good insulator and of pleasing appearance; slate is sometimes used in cheap boards, but generally a high grade of marble is employed. The marble is a better insulator than slate (which is likely to have streaks of conducting mineral matter through it) and is much finer in appearance; of course it costs more than slate. The panels may be from 18 to 24 inches wide and perhaps 6 feet high. They are supported by a framework of structural steel anchored to the floor and wall of the station house.

331. Bus-bars.—Behind the whole length of the switchboard run a set (two or three) of heavy copper bars, called *bus-bars*, or sometimes merely *buses*. All the generator panels are on one side and the feeder panels on the other; the bus-bars then convey the total power of the station lengthwise along the board. For this reason they have a very large cross-section. They, as well as the rest of the connecting bars and wiring on the back of the board, are supported by porcelain channels and cleats fastened to the steel framework of the board.

332. Arrangement of Panels.—Each generator is connected through its respective circuit-breaker, ammeter, and switch to the bus-bars at a generator panel, and each feeder is connected to the bus-bars through its ammeter, circuit-breaker and switch. By having all generator panels on one end of the board and all feeders on the other end, the addition of more generators or feeders is easily accomplished without disturbing the arrangement of the board; the proper number of panels may be added at either end of the board.

At the center of the board, between the generator and feeder panels, is located the *station output panel* on which are the recording and watt-hour meters that show the total power output of the station. By daily records of these meters and of the records of the customers' meters, the station manager may obtain an idea of the efficiency of his system, i.e., the ratio of the amount of energy sold to customers, to the total energy sent out of the station. If this ratio is low he must improve it by better insulation of the outside lines, checking the accuracy of the customers' meters, etc.



6 079
6 079
7 085
7 091
8 105
8 141
9 183
9 227
10 268
10 311
11 347
11 379
12 409
12 433

1 439
1 455
2 493
2 539
3 579
3 615
4 653
4 685
5 713
5 741
6 747
6 753

Fig. 319.—Sample of record of printometer; the reading of the watt-hour element is printed every half hour and the difference of the successive readings shows the average demand for that interval of time. The maximum demand for the record shown here was between 2.00 and 2.30, and was at the rate of 92 kilowatts.

In Figs. 320 and 321 are shown the front and rear views of a railway switchboard; in the front view the meters, switches and circuit-breakers are seen and in the rear view the general construction of the board, angle-iron supports, cable connections, etc., is shown.

333. The Voltage Regulator.—When the load fluctuations on a plant are frequent and rapid, the automatic voltage regulation of a compound generator may not be sufficient to prevent momentary fluctuations in voltage. If a more nearly constant voltage is required than can be obtained from a compound generator by itself, an automatic voltage regulator is added.

For regulating the voltage of small generators of approximately 30 kw. and less, a regulator is used which intermittently short-circuits the field rheostat of the generator with which it is used. A view of a regulator of this type is shown in Fig. 322, and a diagram of its connections, in Fig. 323.

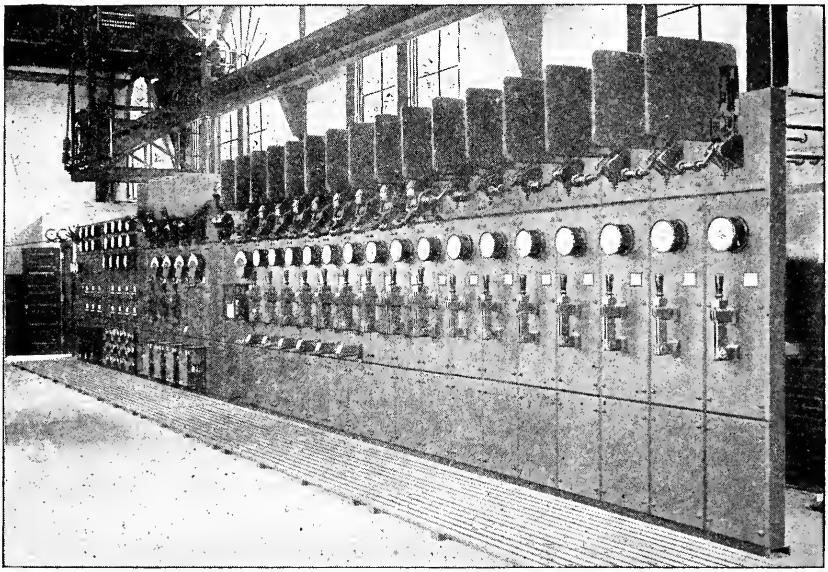


Fig. 320.—This front view of a switchboard for a railway power plant shows the feeder panels, with their switches and circuit-breakers in the foreground, the station metering panel in the center of the board and the generator panels in the background.

Two magnets, known as the main control magnet and the relay magnet, are provided, each magnet actuating a pair of contacts, known respectively as the main and relay contacts. Each set of contacts is opened by the action of its magnet and closed by a spring. Both magnets have laminated cores.

The winding of the relay magnet, in series with an external resistance, is connected directly across the terminals of the generator. It will be seen that when the relay contacts are closed, the field resistance of the generator is short-circuited. This field resistance is so adjusted that when the relay contacts are open and the resistance therefore in the field circuit, the voltage of the generator will be about 35 per cent lower than rated; when the contacts close and short-circuit the field resistance, the generator voltage rises above rated value.

The potential winding of the main control magnet, in series with an external resistance, is also connected across the terminals of the generator. When the main contacts close, a short circuit is applied to the relay winding.

Consider that the voltage of the generator is falling. The current through the main control magnet will therefore also be decreasing, and at some predetermined value of voltage, dependent upon the setting of the spring, the main control magnet will no longer be able to keep the main contacts open; the spring will then close them, short-circuiting the relay magnet. The relay magnet, being suddenly deprived of current, allows its spring to close the relay contacts very promptly. This action in turn

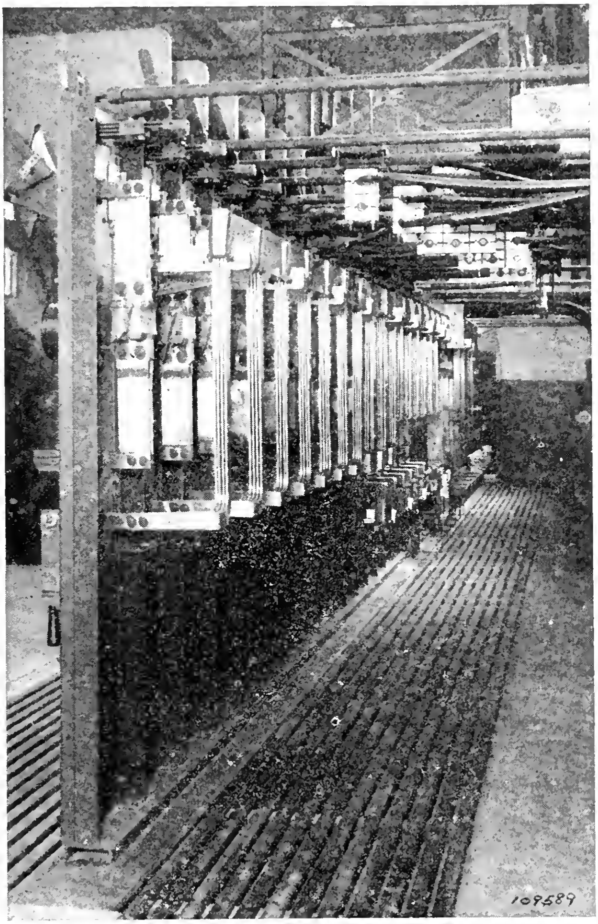


Fig. 321.—Rear view of the switch board illustrated in Fig. 320; this shows the general arrangement, method of supporting the bus-bars, ammeter shunts, etc. The panels themselves are supported by an iron pipe frame work.

short-circuits the field resistance, thereby causing the generator voltage to rise.

As the generator voltage rises, it soon reaches a value where the main control magnet current is sufficient to overcome the pull of the main control magnet spring. The main contacts open and remove the short-circuit across the relay winding, causing a sudden application of voltage across the latter, and opening the relay contacts very quickly. This action removes the short-circuit across the field resistance, and the generator voltage falls, so that the cycle repeats itself. The cycle of operations continues at a high rate, the main contacts vibrating at the rate of 500 to 800 times a minute, depending on the tendency of the voltage to vary from normal. Suitable condensers are used across the relay contacts to reduce sparking.

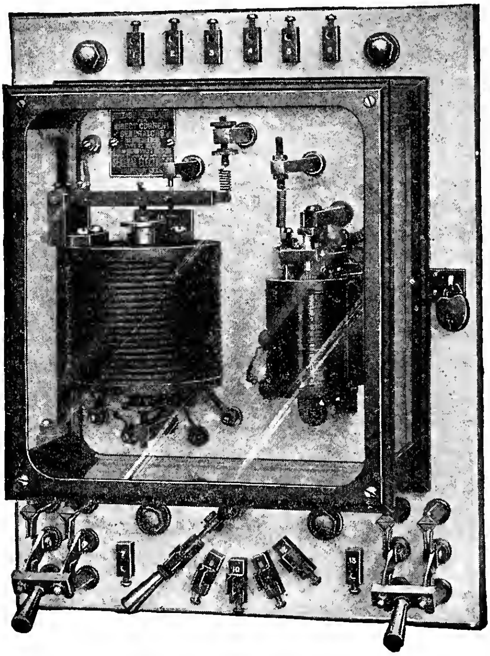


Fig. 322.—The form of voltage regulator originated by Tirrill, and frequently called the Tirrill regulator.

By proper construction and adjustment, the action of the control magnet may be made very delicate, and the generator voltage maintained quite constant. The closeness of the voltage regulation depends upon the range and rapidity of the load fluctuations; but even with the most severe load fluctuations, the voltage can be held constant within 1 or 2 per cent.

If it is desired to have the generator voltage rise with the load, a so-called compensating winding, connected across an adjustable compensating shunt (Fig. 324), is added to the control magnet; the compensating shunt is placed in series, either with the bus-bars or with one of the feeders. The compensating winding on the control magnet is so connected as to oppose the action of the potential winding, so that with load, the voltage across the potential winding must be greater in order to open the main contacts. About 15 per cent of compounding may be obtained by the use of a regulator using a compensating winding and shunt.

The regulator may be used to maintain the voltage constant at some distant distribution center, by connecting the main control magnet winding through pressure wires to the center. The voltage at the bus-bars therefore is raised to compensate for the IR drop of the feeders to the distribution center. (See Fig. 254.)

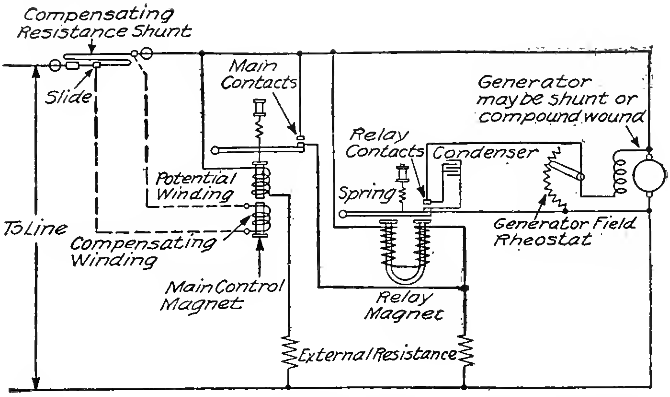


FIG. 323.—Essential circuits of the regulator shown in Fig. 322; its essential action is produced by the relay contacts continually short-circuiting the field rheostat, and then opening this short-circuit connection.

When the regulator described is used with several compound generators operating in parallel, it is used on only one machine at a time, the others being allowed to "trail" by means of their compound windings and equalizer connections; the regulated generator takes the instantaneous fluctuations in load, the average load being equalized over all the machines through their compound windings. The arrangement of regulating only one of several compound generators, is used only for maintaining constant bus-bar voltage; it cannot be used for compensating line drop.

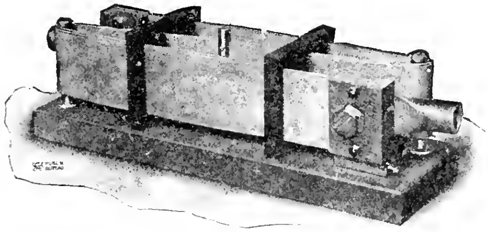


FIG. 324.—The kind of resistance put in series with a feeder, the drop across which serves to permit the compounding action of the regulator.

334. Counter E.M.F. Regulator.—With generators larger than 30 kw. the field currents to be handled become so large that arcing across the relay contacts takes place unless large condensers are used across these contacts. For regulating the voltage of c.c. generators above 30 kw., a regulator, in which the relay magnet is replaced by a "counter e.m.f." motor, the shaft of which carries a small eddy-current brake, is used. The motor field, in series with an external resistance, is connected across the generator terminals; and the brake coils, the motor armature, the generator field rheostat, and the generator field are all connected in series across the generator terminals. A front view of the regulator panel, carrying the main control magnet, is shown in Fig. 325; a rear view of the panel with the counter e.m.f. motor, is given in Fig. 326; a simplified diagram of connections is shown in Fig. 327.

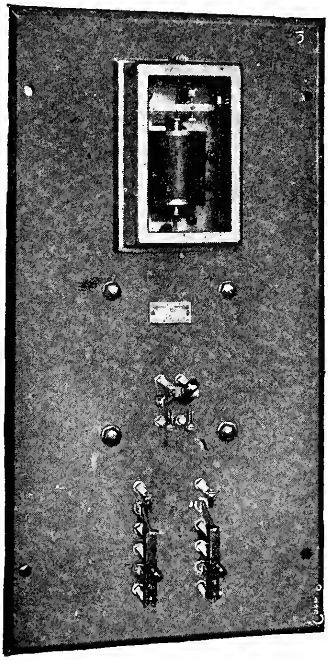


Fig. 325.

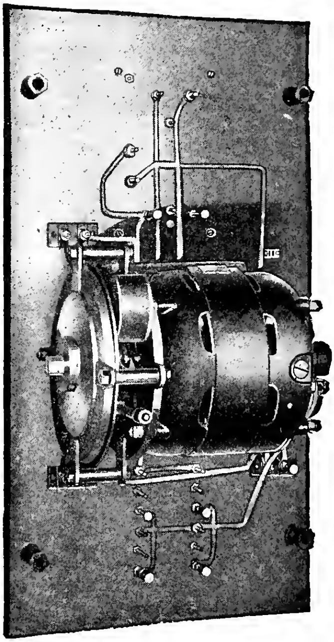


Fig. 326.

Fig. 325.—The front view of the control panel of the c.e.m.f. voltage regulator.
Fig. 326.—The c.e.m.f. motor used for field regulation, the near end of the motor is fitted with the eddy, current brake described in the text.

The action of the main control magnet is the same as before described; when the generator voltage is low the main contacts are closed. In this case, however, the main contacts short-circuit the field of the motor, instead of controlling the field resistance through the relay magnet. With no field current, the motor develops no counter e.m.f., and the field current of the generator is a maximum. The generator voltage, therefore, tends to rise, and at a predetermined voltage, slightly above the value to be maintained, the main control magnet opens the main contacts, removing the short-circuit from across the motor field. With current through its field, the motor develops a high value of counter e.m.f., which reduces the generator field current, and the generator voltage falls. The cycle repeats itself very rapidly, the main contacts vibrating from 500 to 800 times a minute.

If no brake were used in connection with the counter e.m.f. motor, the latter would have very little resisting torque, and would therefore run at too high a speed to function properly. Its generated torque would depend upon the average value of its field current and the current through its armature; the latter is, of course, the field current of the generator. The rapidly vibrating main contacts always give the motor a certain average value of field current, so that its generated torque will vary, for the most part, as the field current of the generator. The field current of the generator is to be varied as the load changes, so that the generated torque of the motor will vary with the load on the generator, resulting in a varying speed for the motor.

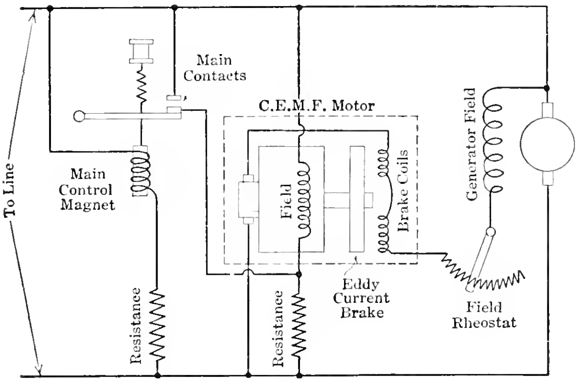


Fig. 327.—Essential circuits of the c.e.m.f. voltage regulator.

The eddy-current brake is added to maintain the speed of the motor fairly constant; it consists of a metallic disc rotating between electromagnets. Eddy currents are generated in the disc, forming a load for the motor, the resisting torque exerted varying directly as the speed of the motor and the strength of the magnets, which are themselves excited by the generator field current. Thus, any action that increases the generated torque of the motor also increases the resisting torque placed upon it, in about the same proportion.

This type of regulator may be developed for any size of c.c. generator. It varies the field excitation more rapidly, and therefore provides closer voltage regulation, than the first type described; it is obviously more expensive.

335. Automobile Equipment.—Electrical apparatus plays an important part in the operation of a modern automobile, the equipment serving to perform the functions of lighting the car, starting and igniting the engine, and keeping the battery charged. The various pieces of apparatus that carry out these functions will be discussed in a general way.

Lighting.—To light the car, a storage battery is provided. This is of the lead type, giving either 6 or 12 volts, the former being more generally used. The capacity of the battery is usually 60 to 100 ampere-hours.

A car may have either one of two types of wiring; these are the single-wire, or grounded-return, and the two-wire. In the single-wire, or grounded-return, system the various electrical circuits are completed by using the frame of the car as part of the circuit, each lamp or other piece of apparatus having one side grounded to the frame. Such ground connections must be perfect electrically and mechanically, so as not to be affected by corrosion and vibration. In the two-wire system, both sides of the electrical circuits are insulated from the frame, the circuits being completed with wires.

Starting.—To start the engine, a series motor is used, the motor being required to exert sufficient torque to start and turn the engine over at a speed ranging usually between 100 and 200 r.p.m., a speed sufficient to cause the engine to ignite. The series motor is connected to the engine flywheel, through some type of sliding pinion and overrunning clutch, which enables the engine to run away from the motor after it has been ignited. The operator usually pushes a pedal against spring action, putting the motor into mesh with the engine flywheel by means of the sliding pinion, and also closing the electrical circuit of the starting motor. As soon as the engine is ignited, it speeds up, overrunning the clutch. Release of the starting pedal allows the spring to take the sliding pinion out of mesh with the engine, and also opens the electrical circuit of the motor.

Ignition.—The primary object of ignition is to explode the compressed mixture of gasoline vapor and air in the engine cylinder, at the proper moment, by an electric spark. This is done by suddenly building up a voltage across a gap between the two terminals of the spark plug extending into the engine cylinder, the voltage being sufficiently high to cause a discharge across the gap.

Battery Ignition.—In Fig. 328 the principle of battery ignition is shown, the principal parts indicated being a battery, an induction coil, an interrupter, a distributor and a spark plug. The interrupter and the distributor are mounted on the same shaft, which is driven through gears from the engine. The interrupter is merely a contact which is momentarily made and broken by a cam, as many times per revolution as the engine requires sparks; the distributor consists of an arm or brush which rotates over a surface of insulating material, in which are imbedded a number of metallic strips, each connected to the spark plug of one of the engine cylinders.

The induction coil has a low-tension winding of a small number of turns and a high-tension winding of a large number of turns. Each time the interrupter makes a momentary contact, current flows through the primary or low-tension winding of the induction coil, which, when broken by the interrupter, induces a high voltage in the high-tension winding. At the same instant that the high voltage is induced in the high-tension winding, the distributor arm is over one of the metal contacts; the high voltage is thus applied to a spark plug, causing a spark to pass.

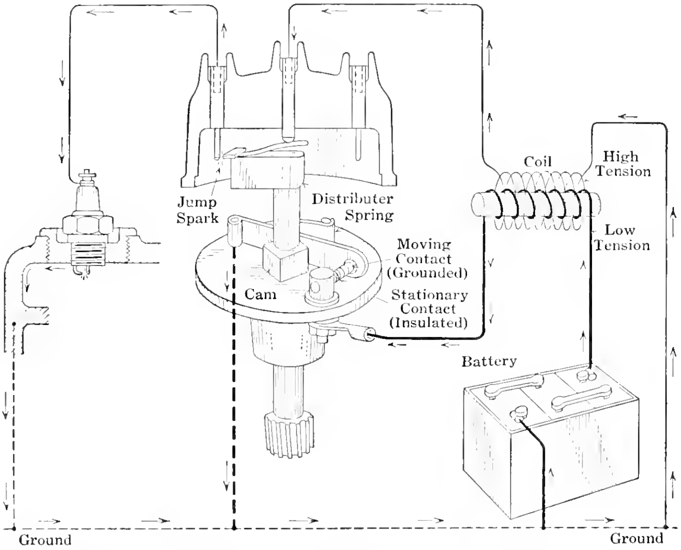


FIG. 328.—A schematic diagram of battery and coil scheme of ignition for an automobile engine.

In Fig. 328 there is a small gap between the distributor brush and the various contacts, and it is necessary that this gap be bridged by a spark in addition to the gap in the spark plug. Sometimes, no such gap is provided, the end of the distributor brush making actual wiping contact with the various contacts. Obviously, there must be as many contacts, evenly spaced, as there are engine cylinders.

The scheme shown in Fig. 328 gives only a single spark to each cylinder per explosion. In some systems, a vibrator is added to the induction coil, as in Fig. 329. When the circuit is closed by the interrupter, current passes through the low-tension winding and magnetizes the core; the keeper, K, of the vibrator carried on a steel spring, is thus attracted. As soon as the keeper moves toward the core a small amount, the circuit is opened at the contacts C, and the keeper is released, again closing the circuit. The vibrator thus rapidly opens and closes the circuit several times during the interval that the interrupter and distributor are in contact, and there will be a shower of sparks at the spark plug for each explosion.

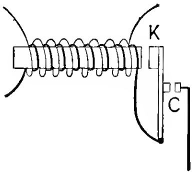
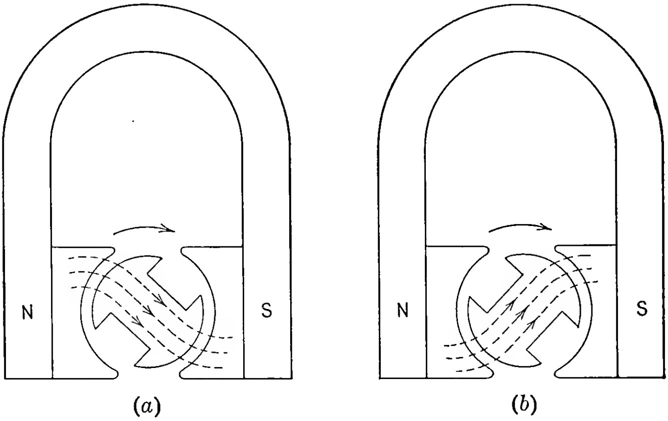


FIG. 329.—The scheme shown in Fig. 328 gives only one spark for igniting the mixture in the cylinder; if the primary circuit of the coil is fitted with a vibrator, as shown here, for each ignition a shower of sparks is furnished by the coil.

Magneto Ignition.—The ordinary magneto is a small generator having permanent magnets and a peculiarly shaped armature known as the Siemens H type. Two successive positions of the armature within the field are shown in Fig. 330; as the armature moves from position (a) to position (b), there is a very rapid reversal of the lines of force through it.

The armature generally carries two windings placed on the central portion; one is a low-tension winding of few turns and the other is a high-tension winding of many turns. The low-tension winding is closed on itself through an interrupter which breaks the circuit twice per revolution of the armature, while the flux through the armature is changing at its greatest rate. The collapse of the primary current is followed by the induction of a high voltage in the high-tension winding, this voltage being set up by the rapid reversal of the field. The high



(a) (b)

FIG. 330.—Showing how the rapid change in flux interlinkages of the Siemen's H armature occurs; from position a to position b, the flux through the armature has completely reversed.

tension is applied to each cylinder by a distributor, as before, one end of the high-tension winding being connected to the distributor arm through a slip ring, or similar connection, and the other end grounded.

A simple diagram of the magneto is given in Fig. 331. For a four-cylinder engine with two explosions per revolution, the magneto armature is rotated at engine speed, since it gives two sparks per revolution of its armature; the distributor rotates at one-half engine speed.

The magneto described is known as a high-tension magneto, since it carries both a high- and a low-tension winding. If a magneto carries only a low-tension winding on its armature, it is called a low-tension magneto; it is used in connection with an external induction coil.

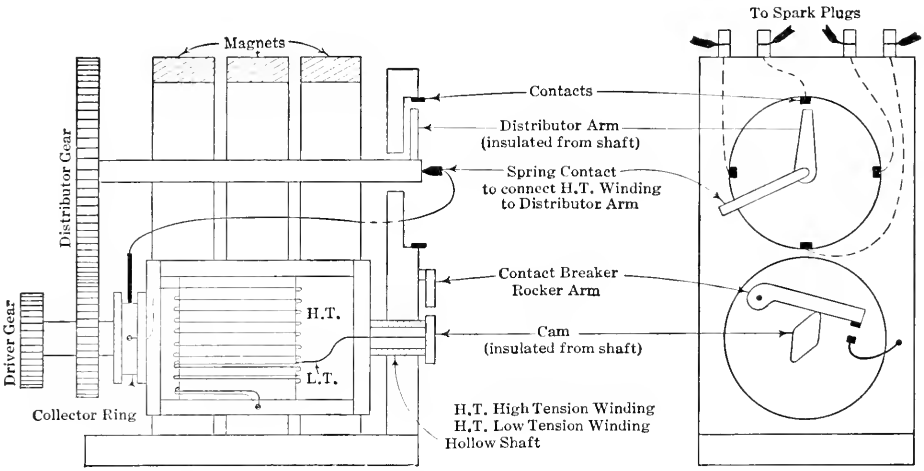


FIG. 331.—Showing how the double winding magneto is used for automobile ignition; the motion of the cam and rocker arm give one spark in the secondary when the primary circuit is opened and this spark is distributed in turn to the various cylinders of the engine. The distributor is generally mounted integral with the magneto, as shown here.

Charging the Battery.—In order that the storage battery may remain more or less charged at all times, a generator for the purpose is usually provided. As this generator is driven from the engine at varying speeds, it must be of the type described in section 229, furnishing more or less constant current at all speeds. The generator is connected in parallel with the battery through a cut-out switch; the battery thus maintains the voltage of the generator fairly constant.

The cut-out switch usually has two windings, as shown in Fig. 332. One winding is a potential winding, P, which closes the contacts, C, when the voltage of the generator has built up to a predetermined voltage. The second winding, A, is a current winding; its function is to open the circuit whenever the voltage of the generator drops below that of the battery, so that the latter begins to discharge into the generator. While the generator is furnishing current, both windings hold the cut-out switch closed.

It is important, with all types of generator used on automobiles, that the generator never become disconnected from the battery. Should this happen, the voltage of the generator may rise to as much as 50 volts, burning out the lamps if connected, its own field coils, and possibly the potential winding of the cut-out switch.

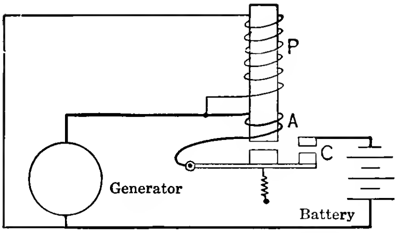


Fig. 332.—The "cut-out" relay of an automobile equipment; the battery is not connected to the generator until the generated voltage is greater than that of the battery. If the voltage of the generator falls below that of the battery, the reversed current flowing through coil *A*, demagnetizes the relay and so allows the spring to open the contact *C*.

336. Electric Welding.—The electric arc can be utilized very successfully for making welds and for the deposition of metal, the process being much used with iron and steel.

If a block of iron and a thin pencil of iron are connected to the two sides of a source of voltage, and the pencil of iron touched to the block and then slightly withdrawn, an arc will be formed. If this arc is maintained, a spot on the block and the end of the pencil are heated to fusion temperature, and the metal of the pencil vaporizes, is carried across and condenses on the block. The process is thus one of deposition of metal; a skillful operator is able by this method to deposit metal even overhead, filling up cracks and adding strengthening metal wherever desired.

For making welds or filling up cracks, the edges to be joined are cleaned and beveled, and metal is deposited to fill up the space. The pencil or rod of iron varies from about one-twelfth inch to one-eighth inch in diameter, depending upon the work; the rod is held in a suitable holder and is generally spoken of as the electrode. An experienced welder is able to maintain the arc with from 20 to 35 volts across the arc, and uses a current of from 50 to 175 amperes, both voltage and current values depending upon the nature of the work. The metal deposited by this process can readily be machined and has reasonable tensile strength.

The resistance of an electric arc varies inversely as the current it is taking, the greater the current the lower the resistance. This property results in an arc becoming fatter and drawing more and more current, if allowed to do so, as its resistance drops, resulting finally in a short-circuit.

It becomes necessary therefore to limit the current taken by an arc; this is done in one of two ways.

If welding is to be done, say, from a 110-volt constant-potential supply, a certain amount of dead resistance, called ballast resistance, is put in series with the arc. Now, if the arc tends to draw more current, the drop in voltage across the ballast increases and cuts down the voltage across the arc, thus limiting the current flow. The use of dead resistance in this fashion is, however, not economical.

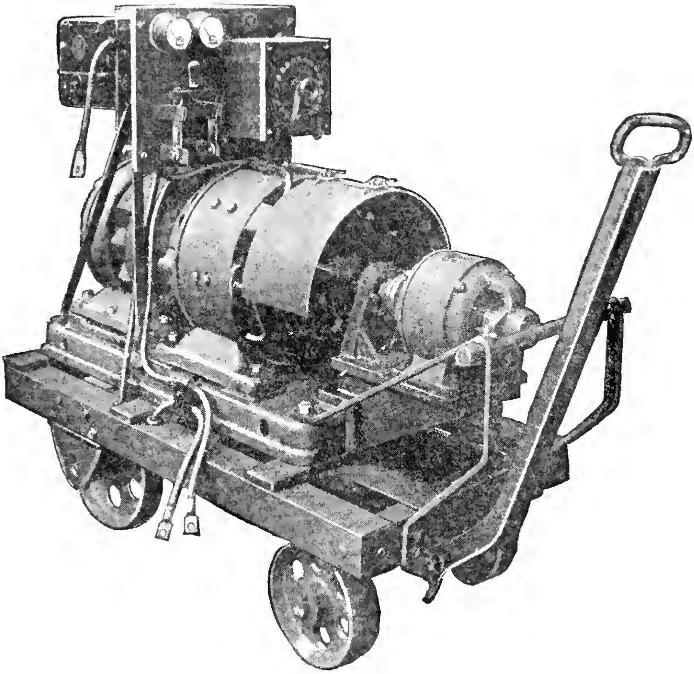


FIG. 333.—A portable welding set.

If the generator used as the source of power is to be used only for welding, it may be designed to have high armature reaction, which causes the voltage of the generator to fall rapidly as its load current is increased. There are available, on the market, portable welding sets consisting of a generator with high armature reaction, driven by a suitable prime mover. In Fig. 333 such a portable set is shown; the prime mover in this case is an alternating current motor. The generator used for such welding sets is generally separately excited; in the set shown in Fig. 333, a small exciter is furnished for the purpose.

In Fig. 334 a set of volt-ampere curves for the generator shown in Fig. 333 are given, the curves corresponding to different settings of the generator field rheostat. The generator is designed to supply 20 volts at the arc, but must, of course, also furnish the voltage drop in the cables leading from the generator to the work. Suppose that the two cables were each 400 feet long, and of size No. 2, and that 150 amperes were required at the arc. The cable drop would amount to approximately 20 volts, requiring the generator field rheostat to be so adjusted that the generator would give 150 amperes at 40 volts. This field setting corresponds to the right-hand curve of Fig. 334.

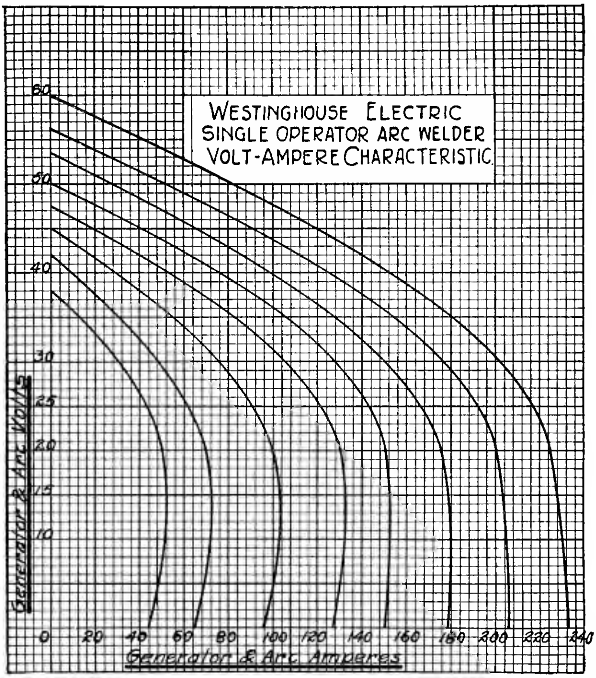


Fig. 334.—The external characteristics of the welding generator of Fig. 333, when using different values of field current.

For filling blow holes and cracks, in castings which are not to be finished, a carbon pencil is used and an arc struck between the casting and the carbon pencil. A thin pencil of iron is then thrust into the arc and melted, the molten metal dropping into the hole or crack. This process requires no skill and permits more rapid deposition of metal; it has the disadvantage that the metal deposited is hard and cannot be readily machined.

It is to be noted that an electric arc gives out a great deal of ultra-violet light, making it imperative that the eyes and every part of the body of the operator, and other persons in the vicinity, be protected. Exposure of the eyes results in severe pain and at least temporary blindness, and exposure of the skin results in severe burning, similar to sunburn. The skin is protected by ordinary clothing, gloves, and hoods or masks; sufficient protection for the eyes is provided by the use of suitable colored glass windows set into the hood or mask.

CHAPTER XII

BATTERIES

337. Classification of Batteries.—A battery is essentially nothing but two dissimilar metals or other conductors, dipping into a conducting solution of some kind. Two metals, such as copper and zinc, dipping into a solution of sulphuric acid, will show a difference of potential of about one volt; and if an external circuit is connected to the two metals, sufficient current will be delivered to ring a door-bell, operate a telegraph sounder, etc. As current flows, chemical action takes place, the zinc dissolving in the acid as zinc sulphate.

Batteries in which these chemical changes cannot be economically reversed, by forcing current to flow through the battery in the reverse direction, are called *primary batteries;* when they have supplied current for a certain length of time, their materials become exhausted. The old solution must be taken out and new supplied, and new electrodes must be furnished periodically for the ones which go into solution.

There are certain other combinations of metals or metallic salts which form, in the proper electrolyte, efficient batteries; these batteries, when exhausted, may be entirely recuperated by the passage of a current in the reverse direction for a sufficient time. Such combinations evidently constitute reservoirs for electric energy, and are hence called *storage batteries.*

338. Primary Batteries.—There are various combinations of metals and solutions which form efficient primary batteries, but at present only one or two types have any appreciable application. For telegraph lines the gravity battery is still used to a considerable extent; it consists of copper and zinc for the electrodes with a solution of zinc sulphate for the electrolyte. Its e.m.f. is just one volt, and the internal resistance varies from 0.1 ohm to a few ohms, depending upon the size and proximity of the electrodes. The Leclanché type of primary battery is important, because in a slightly modified form it constitutes the modern dry cell of which many millions are used each year. In this battery the electrodes are carbon and zinc, and the electrolyte is a solution of sal ammoniac (NH_4Cl). The Leclanché cell gives an e.m.f. of about 1.5 volts and generally has a resistance of from 1 to perhaps 5 ohms.

339. Polarization and Depolarizers.—As current flows from a cell, changes take place at the electrodes which reduce very materially the available e.m.f. of the cell. These changes may be merely an alteration in the density of the electrolyte near the electrodes, or may involve the production of new substances, such as hydrogen, at the surfaces of the electrodes. This effect is of sufficient importance to require the use of another element in the cell, in addition to the electrodes and electrolyte. This extra substance is generally an oxidizing agent, to reduce the hydrogen set free at the positive pole of the battery; it is called a *depolarizer*. In the gravity battery, the depolarizer is a solution of copper sulphate, and in the Leclanché cell it is a mass of manganese dioxide which surrounds the carbon electrode.

340. Dry Battery.—The dry battery, so much used for flashlights, radio sets, ignition, etc., is a modification of the Leclanché cell; the ingredients are the same, but they are put together in such a manner that the cell may be used in any position. The cell is not actually dry, but is filled with a moist paste or mass of some inert material containing the electrolyte, and is covered with a watertight seal of wax or pitch; it can thus operate perfectly well even if turned upside down.

In the form generally used in America, it consists of a tall cylindrical cup of zinc which serves as one electrode and also as a container for the other ingredients, about as indicated in Fig. 335, which shows a cross-section of the ordinary cell. Next to the zinc is a layer of pulp, blotting paper, or starch paste, which is saturated with sal ammoniac and zinc chloride. The carbon rod is centrally placed, as shown at *D*, and is surrounded by a mass of carbon granules and manganese peroxide. A seal of pitch, applied hot, serves to hold the carbon in place and to make the cell tight.

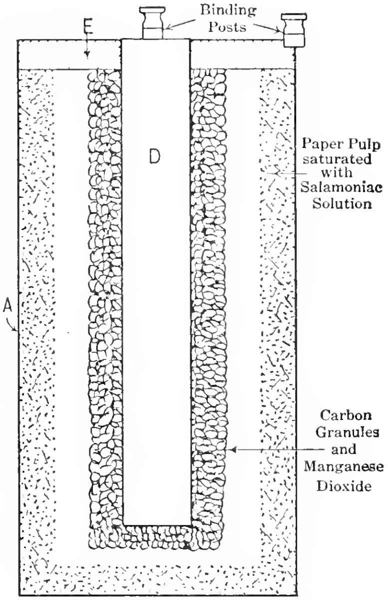


FIG. 335.—General construction of the ordinary dry cell; the outside zinc cup serves as one element of the cell as well as container for the electrolyte and carbon element.

341. E.M.F. and Short-circuit Current of a Dry Cell.—The e.m.f. of a new cell on open circuit is between 1.5 and 1.6 volts, but the terminal voltage is considerably lower, especially after being used a short time. In normal use, the average terminal voltage of the cell during its useful life is about 1.1 volts. A new cell may have an actual internal resistance of only 0.05 ohm, but this increases as the cell ages, and with an old cell may be several tenths of an ohm. The current which the cell delivers on short-circuit (say, external resistance less than 0.01 ohm) is from 20 to 30 amperes, with a new cell $2\frac{1}{2}$ inches by 6 inches, the ordinary size for a dry cell. As the cell ages, this short-circuit current rapidly decreases, even though the cell may not be in use. The shelf life of a 6-inch cell is only about one year; after the cell has stood idle for this length of time, the short-circuit current will generally be not more than 10 amperes. The smaller sizes of dry cells have a shelf life of much less than a year, unless especially well prepared.

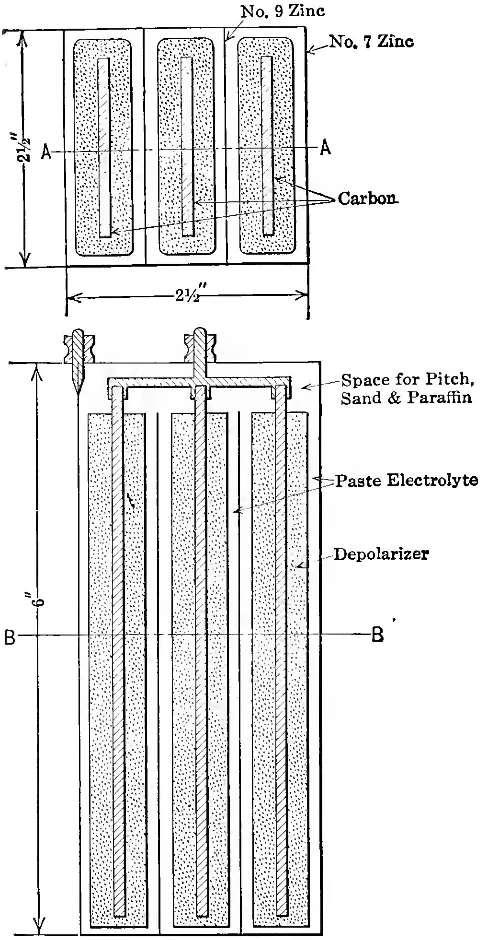


FIG. 336.—This construction of dry cell shows an attempt to use the material more economically; in the ordinary cell there seems to be a waste of depolarizer and carbon, which defect is here remedied by using thinner carbon and depolarizer. The square shape is however mechanically bad, the sides of the cell bulging out with age.

342. Ampere-hour Capacity of a Dry Cell.—The ordinary 6-in. cell has a capacity of from 20 to 40 ampere-hours, when discharged through a resistance of 16 ohms continuously until the terminal voltage falls to 0.5 volt. Actually, a cell becomes useless for ordinary purposes long before this quantity of electricity has been taken from it; when the terminal voltage has fallen to one volt, the cell is generally unserviceable. The more rapidly the cell is discharged the less is its capacity; discharged through 40 ohms, a certain cell had 1200 hours life (to terminal voltage of 1.0), and when discharged through 2 ohms it had only 9 hours life. The ampere-hours delivered are only about one-sixth as much with the lower resistance as with the higher. The ordinary 6-inch dry cell should not be used for a circuit requiring more than 0.1 ampere; under such a load, the cell will last about 300 hours. For intermittent service the cell will give more ampere-hour output than for continuous service; it seems to recover somewhat during the idle periods.

The cell, as ordinarily constructed, makes rather poor use of the materials; when the useful life of the cell is ended, much zinc and depolarizer are still unused. A cell $2\frac{1}{2}$ inches square and 6 inches high, having a more efficient assemblage of material than that customarily employed, will give as much as 700 hours life when discharged through 16 ohms resistance to a terminal e.m.f. of 1.0 volt. Figure 336 shows a cross-section through such a cell; it will be seen that the layer of depolarizer is thin, that three carbon electrodes are used in parallel, and extra sheets of zinc are placed inside the cell. When properly proportioned, all of the inside zinc, and all of the depolarizer, are used up at the end of the cell's life, and the outer containing case of zinc is eaten away to a uniformly thin shell.

A square cell like this does not retain its shape very well, however; the actions going on inside the cell bulge out the sides, tending to make it take a cylindrical form, and thus breaking the pitch seal at the top.

313. Storage Batteries or Accumulators.—Of the reversible, or storage, batteries, there are two types in use: the lead, or acid, battery, and the alkaline battery, of which the Edison battery is the only commercial type. In the lead battery, the electrodes are of lead and lead peroxide, both of which change to lead sulphate as the battery discharges. The electrolyte is a solution of sulphuric acid, generally contained in a glass or rubber jar. The Edison battery has one electrode of high nickel oxide and the other of powdered iron, the electrolyte being a solution of potassium hydroxide. By far the greater percentage of storage batteries used to-day are of the lead type.

314. The Lead Storage Battery.—There are two methods of preparing the electrodes for a lead battery. In one, the Planté type, a sheet of pure lead, properly corrugated to expose a very large surface to the electrolyte, serves as the original material for both electrodes. Two of these plates are used as the poles of an electrolytic bath, and by repeated charging and discharging cycles one plate becomes covered with lead peroxide (PbO_2) and the other with spongy lead. In the other type, this long forming process is practically done away with; the plates are formed by using a flat metal grid as container and filling it with a paste of low lead oxide, such as PbO or Pb_2O_3. Such plates are called pasted, or Faure, plates. These plates are made ready for service by one or two cycles of charge and discharge.

345. Action of the Lead Cell.—Although authorities differ as to the exact sequence of actions taking place in a lead cell, it is agreed that this action may be represented by the equation

$$\underset{\text{Positive plate}}{PbO_2} + 2H_2SO_4 + \underset{\text{Negative plate}}{Pb} = \underset{\text{Positive plate}}{PbSO_4} + 2H_2O + \underset{\text{Negative plate}}{PbSO_4}. \qquad . \quad (123)$$

This equation is to be read from left to right for discharge, and from right to left for charge. During discharge, the sulphuric acid is decomposed to form lead sulphate at both electrodes and the density of the electrolyte therefore decreases materially as the cell discharges; a corresponding increase in density occurs, of course, during the charging period.

346. Plates of the Lead Cell.—Most of the lead storage cells in use to-day are of the pasted variety; wherever it is necessary to get the greatest possible capacity from a cell of minimum weight and volume, the pasted is chosen in preference to the older type of plate, the Planté. Thus, for automobile ignition and for the propulsion of electric vehicles (if lead cells are used), the pasted plates are always used.

The grid which serves to hold the active material is generally of a stiff lead-antimony alloy; its exact form varies with the various manufacturers, but all of the forms are designed to hold the active material as firmly as possible, to have sufficient mechanical strength to prevent breaking and buckling, to have sufficient cross-section to carry the current without appreciable drop, and to bring the metal of the grid into as intimate contact with the active material as possible. This latter consideration is important, in view of the relatively poor conductivity of the active material.

These grids have their interstices filled with a paste of yellow lead oxide (PbO) for the negative plates, and red lead (Pb_3O_4) for the positives. These lead oxides are made up into a paste by the addition of some such substance as a mixture of glycerine and graphite, ammonium sulphate, potassium silicate, etc., any one of which is generally mixed with a dilute solution of sulphuric acid. This material mixed with the lead oxide should increase as much as possible the hardness, conductivity, and porosity of the oxide. Figure 337 shows one type of grid used, and the appearance of these pasted plates is shown in Fig. 338.

After drying, these lead oxide plates are assembled in a tank of dilute sulphuric acid for forming. As current is passed through the bath, the yellow oxide is changed to spongy lead, and the red lead is changed to lead peroxide; in case plates of only one kind are to be formed, pure lead plates are used for the other electrode of the bath, anode or cathode according as negative or positive plates are being formed. The positive plate of a storage battery (the one from which current flows during discharge) has a reddish-brown, or chocolate, color and is quite hard; the negative plate is of gray, spongy lead and is comparatively soft. The active material of this plate can be quite easily scraped off.

The Planté plates have a longer life than the pasted plates, and should be used in cases where the weight is not of so much importance and where discharges at excessive rates are likely to be demanded. They have a higher conductivity than is possible with the grid construction of the pasted plates.

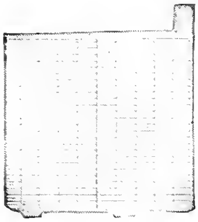


FIG. 337.

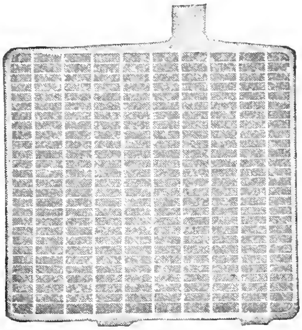


FIG. 338.

FIG. 337.—Form of grid used in making a pasted plate for a lead cell; a stiff lead-antimony alloy is cast into this form giving many interstices in which the active lead material may be held.

FIG. 338.—This shows the appearance of such a grid as that of Fig. 337, after it has been pasted full of the active material.

347. Separators.—If the positive and negative plates of a battery touch, it is short-circuited and so soon loses all of its charge. With charging and discharging cycles, the active material works and bulges, so that plates once straight soon become warped and crooked, especially if the charging and discharging take place at rapid rates. To prevent the plates from coming in contact with each other, which they would naturally do when sufficiently crooked, insulating separators of some kind are generally used; these are especially necessary when the battery must be made compact and hence with plates very close together. Thin sheets of wood are frequently used for separators; the wood must be treated chemically to remove all injurious substances, before being used in a battery. Thin, hard-rubber sheets, sometimes perforated with small holes, and sometimes made with imbedded threads, are also used. The separator must be very porous, so that acid may readily diffuse through it; otherwise the resistance of the battery is increased and its capacity cut down.

348. Electrolyte.—The electrolyte used in a lead battery is always a dilute solution of sulphuric acid; the proper strength of the solution varies somewhat with the use to which the battery is to be put, but is generally of specific gravity about 1.300 for automobile batteries, and 1.210 for batteries where more space for electrolyte is allowed, as station stand-by batteries. These figures for density are for the battery in fully charged condition.

The resistance of the electrolyte varies considerably with the density, about as shown in Fig. 339; it is seen that the densities specified above give the lower values for resistance. Pure water should be used in making up the electrolyte, as even small amounts of certain mineral salts are very injurious to a battery, materially shortening its life. In getting the right density, the acid should be added to the water slowly, as the heat of solution may crack the jar or even sputter the acid around; this is especially likely to occur if the water is poured into the acid, which should never be done.

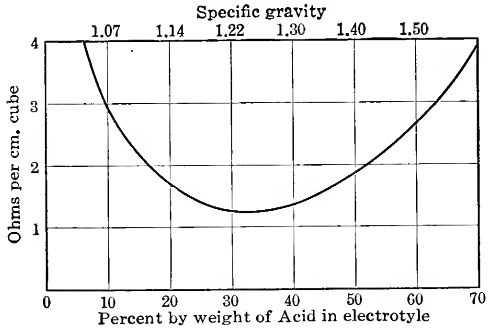


Fig. 339.—The resistance of the sulphuric acid solution used as the electrolyte in lead cells varies according to the amount of acid used as shown by this curve.

The resistance varies appreciably with temperature, increasing about 1.5 per cent for every degree Fahrenheit decrease in temperature; it is twice as much at zero as it is at 70° F. This effect very materially reduces the amount of current which can be drawn from a battery, as illustrated by the fact that a battery which operates the starting motor of an automobile easily in the summer, may not be able to turn the engine over in the winter, not only because of the increased viscosity of the lubricating oil, but also because the increase in battery resistance prevents the starting motor from getting its proper current.

349. Assembly of Battery.—For station batteries, the plates are generally assembled in large glass jars, these being placed on sand trays or similar supports. The plates are separated from each other by perhaps one-half inch, and no separator is required; a badly buckled plate, if such occurs, can be seen and replaced before giving trouble. The jars are sometimes covered with glass plates to prevent too rapid evaporation of the water from the electrolyte.

For automobile batteries, the plates are held perhaps one-eighth inch apart by suitable separators, are squeezed tightly together and forced into a rubber jar, which is itself held from breaking by being packed into a tightly fitting wooden box. The rubber jars are covered with sealed top pieces. The plates do not touch the bottom of the jars, but rest on several high ridges cast integral with the bottom of the jar; a deep sediment space is thus formed below the plates. As the battery charges and discharges, the active material gradually falls off; if the plates rested on the bottom of the jar, this material would soon short-circuit the plates. The sediment space is made deep enough to care for all the active material which is likely to fall off during the useful life of the battery.

For batteries of both types, the containers are so made that there is room for about one inch of electrolyte above the top of the plates; the plates should never be allowed to project above the electrolyte, as the projecting part of the plates would spoil if this occurred. In the case of the sealed type of automobile battery, provision must be made for the escape of the gas formed during charge; this is generally taken care of by small holes through the plug provided for filling the battery with electrolyte, and adding water.

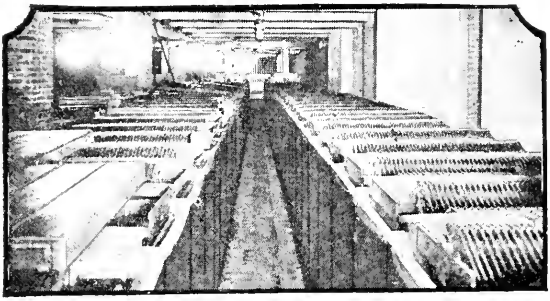


Fig. 340.—A large installation of lead cells to serve as a stand-by in case of the failure of the power supply to a large building.

A completely assembled battery of the station type is shown in Fig. 340, and in Fig. 341 is given a section view through an automobile battery, showing how compactly the plates are assembled in the jar, and also how the plates rest on the rubber ridges providing the sediment space.

350. Voltage of the Lead Cell.—The voltage of a lead cell may be taken roughly as 2 volts, but, of course, this varies with the condition of charge, whether the battery is being charged or discharged, and the amount of current passing through it. The exact form of the voltage curve of a battery, for discharge and charge, varies with the type and age of the plates, temperature, and similar factors, but is generally of the shape given in Fig. 342. The charge curve is naturally considerably higher than the discharge curve, because of the effect of the internal drop of the battery; this increases the terminal voltage of the battery on charge and decreases it on discharge.

During charge, the voltage rises rapidly at first, then holds nearly constant for several hours, to rise again rapidly as the active material is nearly all changed. At this time the battery begins to give off gas; when nearly all the active material has been transformed according to Eq. (123), the current begins to electrolyze the water. This gas which is given off is highly explosive, and because of this the battery room should be well ventilated; in certain installations on board ship, for example, blowers must be provided which continually renew the air in the battery chamber and carry off the old air in closed ducts to keep it away from commutator sparks, etc.

During the discharge, the battery voltage falls rapidly for a short time, then holds fairly constant for several hours, and then, at the end of the discharge period, falls rapidly again. The discharge should not be carried below about 1.8 volts per cell; if this is done repeatedly, the ampere-hour capacity of the battery is seriously diminished, and its internal resistance increased by the formation on the plates of excessive sulphate of a hard, insoluble nature.

As the battery is charged, the electrolyte increases uniformly in density; the continual release of the SO_4 ions at both plates during charge increases the acid density correspondingly. In station batteries, the density may increase form perhaps 1.160 to 1.210, and in the automobile battery from 1.200 to 1.300, the greater change being due to the correspondingly smaller amount of electrolyte present in the latter type.

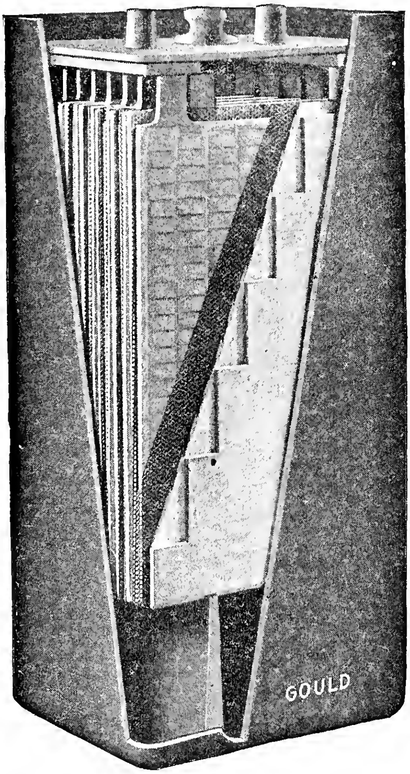


Fig. 341.—Cross-section of an automobile battery, showing how the plates are compactly fitted into the hard rubber container, and how the high ridges cast integral with the container serve to give a deep space below the plates for the accumulation of the active material which gradually falls off the plates with use.

351. Ampere-hour Capacity of a Cell.—The capacity of a storage cell is given in either ampere-hours or watt-hours; the latter takes into account the *IR* drop in the cell, and so is the quantity from which the efficiency of a cell must be reckoned. The ampere-hour rating has nothing directly to do with the resistance of the cell. In ordinary practice the ampere-hour rating is always used. The theoretical ampere-hour capacity can, of course, be figured from the known electrochemical equivalents of the constituents; thus, per ampere-hour capacity, there are required 4.42 grams of lead peroxide and 3.88 grams of spongy lead. It is customary with battery manufacturers to allow from two to three times this theoretical quantity to allow for lack of porosity (preventing all of the active material from being used) and also to allow for the sealing off of the active material with age. The negative plates, being soft, do not retain their active materials as well as the positives, so that in the assemblage of cells it is customary to have the number of negatives greater by one than the number of positives.

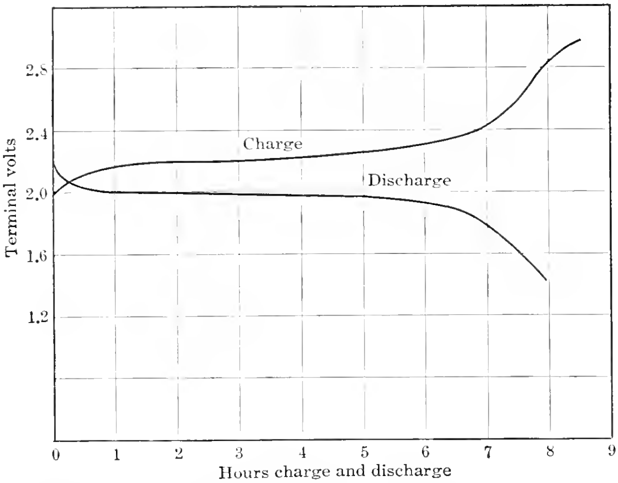


FIG. 342.—Typical charge and discharge curves of a lead cell, carrying normal current.

The ampere-hour capacity of an ordinary cell can be approximated by assuming that one ampere can be supplied for eight hours (8 ampere-hours) for each 25 square inches of surface of positive plate. In calculating the area, both sides of the plate are to be used, thus, a positive plate 4 inches by 6 inches could deliver 16 ampere-hours. These figures hold for plates about one-eighth inch thick; thicker plates have more capacity, the number of ampere-hours obtainable increasing about as the square root of the thickness.

352. Variation of Capacity with Rate of Discharge.—The ordinary lead cell is rated by the number of ampere-hours obtainable when the cell is discharged at the eight-hour rate; thus, an 80 ampere-hour battery is one that will deliver 10 amperes for eight hours. The capacity varies greatly with the time taken for discharge; if the above battery were discharged at one-half the normal rate, that is, with a discharge current of 5 amperes, about 95 ampere-hours capacity could be obtained. This change of capacity with rate is due principally to the non-porosity of the active material. The discharge at first changes the active material at the surface of the plates; this can be accomplished very effectively, but the material deep below the surface can only change as the electrolyte penetrates. As the rate of penetration is slow, it is evident that for rapid discharge rates the inner material will not be sufficiently supplied with fresh electrolyte, and so the ampere-hour capacity must be correspondingly less. A certain battery having an eight-hour rating of 100 ampere-hours had a capacity of 110 ampere-hours at the twelve-hour rate, 80 ampere-hours at the four-hour rate, and 55 ampere-hours at the one-hour rate.

353. Limiting Terminal Voltage.—A battery is regarded as discharged when its terminal voltage falls below a certain value, *this to be measured while the battery is delivering current.* As the terminal voltage will evidently depend upon the internal IR drop, as well as the condition of discharge, it is necessary to state the limiting voltage in terms of the current being delivered. It is ordinarily taken as

$$E_{\text{limiting}} = 1.66 + 0.0175T, \quad \ldots\ldots \quad (124)$$

in which T is the hours rate of discharge. Thus an 80-ampere-hour battery, discharging at the rate of 10 amperes, should not be discharged below a voltage of 1.80 volts (read with current flowing) or below 1.70 when discharging at the two-hour rate.

354. Indications of Charge.—In determining whether a battery is charged, it is of no avail to read the open-circuit voltage; neither is it always safe to depend upon the electrolyte density unless the density in the discharged condition is known. If the density of the electrolyte of a cell, in normal condition, is 0.100 above that of the discharged condition, it is safe to assume the cell fully charged. If, when discharging at normal rate, the terminal voltage is 2.05 volts or more, the battery is probably fully charged. The best criterion, however, is its behavior after it has been put on charge; if, when charging at normal rate, the battery gasses freely, it is completely charged.

355. Sulphation of Plates.—Whenever a battery discharges, the active material on both plates is changed to lead sulphate; this sulphate is fairly porous, and at once changes back to lead or lead peroxide on charge. If, however, a cell is left in the discharged condition for some time, or if too dense an electrolyte is used, or if the battery gets too hot, a deposition of hard dense sulphate forms, and this may seriously injure the battery. In a bad case of sulphation of this kind, light gray patches, can be seen on the plates, and the surface of these patches is very hard. Excessive internal resistance is thus caused by the isolation of the active material below the impervious sulphate layer; the difference between charge and discharge voltage is thus increased, and the electrolyte density is lowered, owing to the permanent loss of the sulphate ions.

Sulphation can be remedied, if it is not too severe, by giving the battery several overcharges; the gassing resulting from overcharge tends to crack off the hard sulphate layer. In case of bad sulphation, the plate must be either discarded or given some special treatment, such as the Glauber's salt treatment, for an explanation of which a book on storage batteries should be consulted.

356. Charging a Battery.—For charging a storage battery, it is advisable to use the eight-hour rate until gassing occurs, which will be after about nine hours, and the voltage per cell (measured with charging current flowing) is about 2.40. The rate should now be diminished to perhaps one-half its normal value, and this low value maintained until the voltage per cell is again 2.40 and remains at this value for about an hour. A battery should not be allowed to gas violently for any length of time, as gassing tends to crack off the active material, with consequent loss of capacity for the battery.

357. The Alkaline Storage Battery.—The Edison is the only commercial battery of this type now available. This battery depends for its action on the oxidation from a lower to a higher oxide of nickel in the positive plate, and the reduction of ferrous oxide to metallic iron in the negative plate. The electrolyte has a peculiar action, in that whatever change is caused by the electrolytic action at one plate is at once neutralized by the reverse action taking place at the other. The electrolyte (a mixture of solutions of potassium and lithium hydrates) is not used up at all during the charging operation, and hence but little of it is required. The plates can thus be put very close together.

The action of the Edison cell is probably represented by

$$\underset{\substack{\text{Positive plate}\\ \text{charged}}}{6\mathrm{NiO_2}} + 8\mathrm{KOH} + (\mathrm{H_2O}) + \underset{\substack{\text{Negative plate}\\ \text{charged}}}{3\mathrm{Fe}}$$

$$= \underset{\substack{\text{Positive plate}\\ \text{discharged}}}{2\mathrm{Ni_3O_4}} + 8\mathrm{KOH} + (\mathrm{H_2O}) + \underset{\substack{\text{Negative plate}\\ \text{discharged}}}{\mathrm{Fe_3O_4}}. \quad . \quad (125)$$

The equation read from left to right represents discharge, and from right to left gives the cycle of charge.

358. Plates of the Edison Cell.—The elements of the positive, or nickel oxide, plate, consist of perforated, nickel-plated, steel tubes, packed tight with nickel hydroxide, alternated with layers of flake nickel, the latter being mixed with the hydroxide to increase the effective conductivity of the active material. The nickel flakes come in contact with the steel container, and then, reaching through the body of the hydroxide, serve to conduct the current throughout the mass of the active material.

The negative plate is made up of perforated, nickel-plated, steel containers, filled with powdered iron oxide. The elements of both plates, of which several are required to make one plate of a battery, are fastened, by welding or pressing, into nickel-plated steel grids which serve as the plates of the cell. Fig. 343 shows the construction of the negative and positive plates of an Edison cell.

359. Assembly of an Edison Cell.—The steel grids containing the active elements are assembled and held together by insulated through-bolts, narrow strips of hard rubber being used to keep the plates a short distance apart; as in the lead cell, the number of negative plates is one greater than that of the positives. The containing jar is of thin nickel-plated steel, having corrugated sides to increase its rigidity. In the cover of this steel tank are three holes, two of which are fitted with insulating bushings for bringing out the battery terminals; the third hole, for filling with electrolyte and for adding water from time to time, is fitted with a cap and pop valve which lifts to let out gas but prevents entrance of extraneous matter. The cells are assembled in wooden trays, so built as to properly separate adjacent tanks; as these are conductors, they must not touch

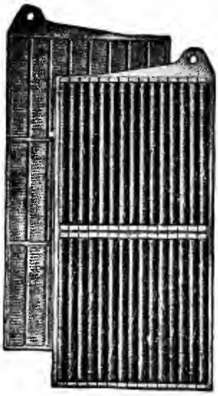


FIG. 343

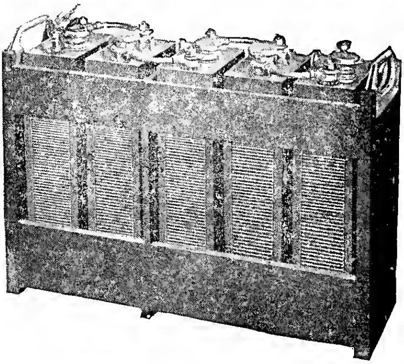


FIG. 344

FIG. 343.—In the Edison cell the active material is held in small steel containers, these being fastened into steel plates about as shown here. The size of the steel plates, and the number of the small elements used in building up the positive and negative plates depends upon the capacity the battery is intended to have. All the steel is heavily nickel plated.

FIG. 344.—As the Edison cell is held in a steel container an assembled battery must be such that the various cells are held from coming into contact with each other. This shows a typical assembly of five cells, in a wooden rack, in which the different cells are held apart by wooden spacers.

one another, as large leakage currents would otherwise flow. A completely assembled Edison battery is shown in Fig. 344.

360. Voltage of Edison Cell.—The e.m.f. of the Edison cell is much less than that of the lead cell; on discharge the terminal voltage starts at 1.50, falls rapidly to 1.30, has an average value during discharge of 1.20 and a useful limiting terminal voltage of 1.00. For a given battery installation using Edison cells, it is therefore necessary to put in about 70 per cent more cells than would be required if lead cells were used. Owing to the higher internal resistance of the Edison cell, there is a greater difference between the charge and discharge voltages than is the case with the lead cell. Typical charging and discharging curves for an Edison cell are given in Fig. 345.

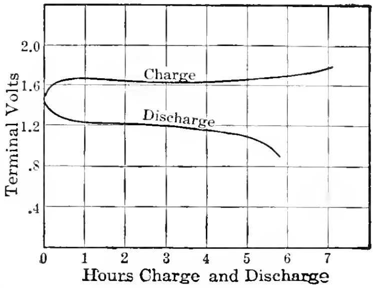


Fig. 345.—Typical charge and discharge curves of an Edison cell.

361. Charging Rate for the Edison Cell.—It is customary to rate an Edison cell at the five-hour rate; thus, a 75 ampere-hour cell will deliver 15 amperes for five hours. To completely charge such a cell requires a current of 15 amperes for about seven hours. The cell is regarded as charged when the charging voltage has remained constant at 1.8 volts for half an hour.

The alkaline cell will stand much more abuse than will the lead cell; it can be repeatedly discharged to complete short circuit, with no harm, whereas a lead cell would buckle and sulphate under such treatment. It will yield the same ampere-hour output, after being fully charged, at any discharge rate from one-quarter normal to four times the normal rate. It can be left in any condition of charge or discharge, with no deleterious effects, where a lead cell would be badly sulphated under the same treatment.

362. Effect of Temperature on Edison Cell.—A storage battery of either type shows a loss of capacity with temperatures lower than 70° F., but the effect is more marked with the Edison. The discharge capacity decreases somewhat more than 1 per cent per degree fall in temperature, until, at about 40° F., practically nothing can be obtained from the battery even though it may be fully charged. However, if the cell is warmed up, its normal output is available, the cooling not being permanently injurious. If Edison cells are used for vehicles in cold weather, they should be given a warming charge just before starting a run, and put in a covered box; the I^2R loss during discharge will be sufficient to keep them warm even in very cold weather.

During charge, an Edison cell should not be allowed to rise above

115° F., but during discharge a temperature of even 160° may obtain without doing the cells any injury. Lead cells should not be operated at such high temperatures, 110° F. being the upper limit if the life of the cell is not to be seriously diminished.

363. Resistance of Storage Batteries.—The internal resistance of a lead cell is extremely low, and varies approximately inversely with the ampere-hour capacity of the cell. At 70° F. the short-circuit current of a lead cell is about fifty times its rated current; this excessive current would be maintained for a short time only, because of the polarization of the cell. The Edison cell has comparatively more resistance than the lead cell; when short-circuited, it gives a current about ten times its rated value, showing the resistance of the alkaline battery to be about five times as much as that of a lead battery of the same rating. As mentioned before, the resistance of both types rises rapidly as the temperature falls.

364. Efficiency of Storage Batteries.—The efficiency of a cell may be measured by the ratio of ampere-hours output to ampere-hours input, or by the ratio of watt-hours output to watt-hours input. The ampere-hour output is measured by the ampere-hours delivered by a completely charged cell until its terminal voltage has fallen to the value given, for a lead cell, by Eq. (124), and the corresponding input by the ampere-hours required to charge a cell from this condition until the charging voltage remains constant, at the upper limit, for half an hour.

The ampere-hour efficiency of a lead cell depends somewhat upon the rate of discharge; for the eight-hour rate, it is from 80 per cent to 95 per cent. The corresponding figure for the Edison cell averages 80 per cent. The watt-hour efficiency shows up the lead cell much more favorably, because of its lower internal resistance; for the lead cell it is generally about 75 per cent, whereas the Edison cell on normal discharge rate shows from 55 to 65 per cent watt-hour efficiency. For both cells, this figure decreases for currents greater than the normal rated value.

365. Life of Cells.—With normal care, a lead cell using pasted plates should, according to certain authorities, last for at least 300 cycles of discharge and charge before showing appreciable loss of efficiency; for Planté plates 30 per cent more life can reasonably be expected. The life depends upon so many factors, such as purity of materials, regularity of charging and discharging, rates of discharge, amount of gassing permitted, variation of temperature allowed, etc., that much variation may be expected. As many as 1400 cycles for pasted plates have been reported by one experimenter. In automobile service, a good lead cell should last two years at least, unless it is continually overcharged, or allowed to freeze; in this service the battery seldom is completely discharged, but generally suffers from excessive gassing.

The Edison cell is so free from troubles that its life is much longer than that of the lead cell; the manufacturers generally guarantee their cells to show their rated capacity at the end of five years' use.

366. Comparison of Lead and Edison Batteries.—The weight and space required per kilowatt-hour capacity is nearly the same for both types. The Edison evaporates its water much faster than the lead cell, and so requires more attention for this reason. If the cells are in a confined space, so that artificial cooling is required to prevent overheating, the Edison requires much more blower capacity to keep the temperature within advisable limits. The watt-hour efficiency of the lead cell is much better than that of the Edison cell, especially at high rates of discharge; and at low temperatures the lead cell is incomparably superior to the Edison. The initial cost is greater for the Edison, but this is offset by the longer life guarantee.

367. Uses of Storage Batteries.—By far the greatest number of storage batteries in use to-day are in automobile lighting and ignition equipments; the total capacity of cells in such service, in the United States alone, is about three million kilowatt-hours. For farm-lighting and other small, isolated plants, about one million kilowatt-hours capacity is used, while for electric trucks, locomotives, and stand-by service in large stations, another million kilowatt-hours is installed.

For central station work, the cells have two distinct uses. In small stations they are used to carry the peak load, helping the generators to carry the load when the station is most heavily loaded, and then being charged when the generators would otherwise be underloaded. Their use in this service very materially reduces the amount of generating equipment required. In the large stations connected principally to lighting loads, they are used "floating" on the line, continually charged; their function here is to care for the entire load of the system for a short time, in case the generating equipment fails, thus ensuring continuity of service, a very important consideration in the case of a city like New York. The installed capacity in such stations is only sufficient to carry the load for perhaps twenty minutes, but in this time the generating apparatus would presumably be again ready to operate.

CHAPTER XIII

OPERATION AND CARE OF CONTINUOUS-CURRENT MACHINERY

368. Location.—In selecting the proper location for an electric machine, the two principal points to be kept in mind are, *first,* that the machine must be kept dry, and *second,* that it must be kept free from dust and dirt. If these two requirements are not satisfied, a motor or generator will soon deteriorate and develop faults.

Effect of Moisture.—If moisture is allowed to accumulate on or around a dynamo-electric machine, the machine will very soon be more or less *grounded.* A machine is said to be grounded when any part or parts of the electric circuits are connected to the framework (field casting, armature core, etc.) Such a grounding of the winding may be very dangerous for the operator; in the case of a high-voltage generator or motor it might cause a fatal shock if the operator should come in contact with the framework when he is standing upon a concrete floor or some other partially conducting surface. This point will be more fully discussed in the section on *faults.*

Besides the possibility of grounds, there is that of the windings becoming short-circuited because of the presence of moisture. This is especially likely to occur by the development of *two grounds,* on different parts of the windings. A machine which develops a short-circuit is sure to be seriously damaged by burning, unless the fault is at once discovered and removed.

Effect of Vibration.—Of course, in choosing the location for a machine, care must be exercised that a *firm foundation* can be provided. Even though an electric machine is very well balanced before leaving the factory, it is sure to vibrate more or less when running, unless it is fastened to a solid bed of some sort. If a machine is allowed to vibrate when in operation, there is likely to be sparking at the brush contacts, owing to the fact that the brushes make poor contact with the vibrating commutator; the vibration may also make the bearings heat excessively.

Effect of Dust and Dirt.—Any continuous-current machine is almost sure to develop commutator trouble if it is operated in such a location that dust or dirt can fall on the commutator. The dirt will work into the bearing surface of the brush, producing a rough surface and spoiling the commutator surface by scratching and undue wear. Also, it is likely to work into the insulation between the commutator bars and produce a partial short-circuit between adjacent bars. When a partial short-circuit occurs, it soon develops into a complete short-circuit, and the coil which is attached to the two segments is likely to burn out.

369. Faults Tend to Get Worse.—A point for the operator to keep constantly in mind is that nearly all of the faults developed by electrical machinery *tend to aggravate themselves if not immediately remedied.* Thus, a slight roughening of the commutator will produce but imperceptible sparking; unless, however, the rough spot is removed (and the cause also removed) the commutator will soon be unserviceable.

If it is absolutely necessary to run a machine in a room where dust accumulates, as, for example, in a cement mill or flour mill, an *inclosed type of machine* should be installed if possible. This type of machine is completely closed by end pieces fitted to the sides of the yoke. The armature, commutator, etc., are thus completely shut in, so that dirt and dust can collect only on the outside of the machine, where it can do but little harm. (All railway motors, e.g., are of this inclosed type, so that the working parts are kept free from the dust and moisture with which the car truck gets covered.) If it is not possible to use an enclosed type of machine, the attendant must exercise special care in keeping the commutator clean, and must blow the dust out of the windings by means of a bellows whenever the machine is shut down.

370. Precautions in Starting 'a Machine.—After a machine has been properly installed, a thorough inspection must be given *before the machine is started.* The operator must look over the machine very carefully, to see that there are no mechanical or electrical faults which would injure the machine as soon as started.

The machine should be thoroughly blown out by a bellows, so that any dust, dirt, chips, etc., which may have collected in the interstices of the windings and various parts, may be cleaned out.

Tightening of Parts, etc.—All parts of the machine should be thoroughly tightened; a pole may pull loose from the yoke very soon if its fastening bolts are not drawn up snug; the same precaution holds with respect to the pole shoes and any other parts held together by bolts. Care must be observed to see that there is nothing in the air gap; a small nut, nail, etc., in the air gap may cut the armature winding badly when the machine is started.

Bearings.—The machine must turn freely in its bearings, and the oil rings must be picking up the proper amount of oil. This latter point is extremely important; oil-ring bearings operate so reliably, in general, that they receive less attention from the operator than they really require. The oil used in the bearing should be the best grade of machine oil; if too thick, it does not flow freely through the oil ducts in the bearing, and if too thin it runs out too quickly and there is not a sufficient layer of oil between the shaft and bearings for proper lubrication. The bearings must be thoroughly cleaned from dirt, grit, etc.

371. Spacing the Brushes.—A machine may spark badly at the brushes, no matter how well the brushes may have been ground and how smooth the commutator may be. This may be due to the *improper spacing of the brushes.* Of course, many other causes may exist which produce sparking; some of them have been taken up already, and they will all be grouped together in the tabulated list of faults, which is given on a following page.

If a *two-pole* machine is considered, it is seen at once that the brushes should be spaced 180° apart on the commutator; on a *four-pole* machine they should be 90° apart, etc. On the earlier types of machines, where the interpolar space (space between the adjacent tips of consecutive poles) was large, the necessity for setting the brushes exactly the right distance apart did not exist. Around one side of the commutator of a two-pole machine, there might be 49 segments between brushes, and around the other side perhaps 51 segments, and still there might not be excessive sparking.

Proper Spacing Important on Commutating Pole Machines.—On the more modern machines, however, especially on machines having commutating poles, the brushes must be set exactly the right distance apart. On a high-voltage synchronous converter, if the spacing between one set of brushes is greater than that between another set by half the width of one commutator segment, bad sparking is likely to result. In addition to the sparking trouble, the armature of a machine on which the brushes are improperly spaced is very likely to overheat, even though the machine is not delivering full-load current to the outside circuit.

Uneven Spacing Produces Sparking.—The sparking, with unequally spaced brushes, may be due to either of two causes, or both. As we have shown before, the armature winding of any continuous current armature consists of two or more circuits in parallel, the lap-wound armature having as many paths as there are poles, and the wave-wound armature always having two paths. It may be that there is a greater e.m.f. induced in one path than in another if the brushes are unequally spaced. But if the several paths are in parallel and one path has a greater e.m.f. than the others, then this path will force current to flow through the others even though there is no load on the outside circuit.

372. Cause of the Sparking.—Consider the case shown in Fig. 346 (*a*) where the brushes A, B and C are correctly spaced, 90° apart, but D is misplaced so that the distance A–D is 80° and the distance D–C is 100°. The paths A–B and B–C generate the normal rated voltage of the machine, while the two paths A–D and D–C generate something less than this value as there are less effective inductors in the latter two quadrants than in the others. (In quadrant *D-C* various inductors are generating voltages in opposite directions.)

The electrical conditions in the armature are then the same as those shown in Fig. 346 (*b*) in which figure the cells *A* and *B* are supposed to develop a higher e.m.f. than the cells *C* and *D*. Even though there is no load on the battery, the load circuit being open, there will still be current flowing in the different cells, as indicated by the arrows. The amount of current which will thus flow in the different paths of the armature depends upon the inequality of the voltages in the different paths and upon the resistance of the armature. In large machines this resistance is very small and so, on large, high-voltage generators, it is very important that the brushes be spaced exactly right.

If the brushes on a commutating-pole machine are unequally spaced, it is impossible to eliminate sparking at all brushes, because when one brush or set of brushes is in the correct position with respect to the commutating poles, some of the other brushes cannot possibly be placed properly, because we have supposed the brushes unequally spaced while the commutating poles are always spaced equally. We have previously shown that a commutating-pole machine is very sensitive with regard to the proper placing of the brushes with respect to the commutating poles.

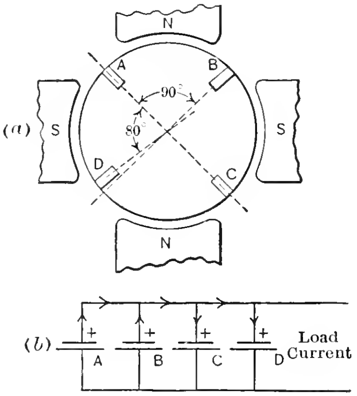


Fig. 346.—If the brushes of a four-pole machine are not equally spaced there will be less voltage between some brushes than between others; due to this effect a large circulating current may flow in the armature even when it is delivering no load.

Testing for Spacing.—The easiest way to obtain equality in spacing is to measure the total circumference of the commutator by laying a tape or strip of paper around the commutator surface under the brushes. This distance, divided by the number of brush studs, is the proper spacing to use in measuring *from the toe of one brush to the toe of the next brush.*

373. Fitting the Brushes.—The brushes must be thoroughly "sanded" to fit the commutator. This task requires a deal of time on the larger machines and must be done carefully. The proper method is shown in Fig. 347. *Sandpaper* (never emery cloth) must be used for this purpose. The sandpaper is torn into strips slightly wider than one brush and a strip is inserted between the brush to be fitted, and the commutator surface, the rough side of the paper being outside. (It is supposed that the brushes have been adjusted for the proper tension of about 1.4 pounds per square inch of contact surface.) Then, while the strip of sandpaper is held tightly against the surface of the commutator, it is worked back and forth, grinding the under surface of the brush to a shape that just fits the commutator.

Improper Fitting of Brushes.—While the brush is being ground it is *extremely important* that the sandpaper be so held that it lies tightly against the commutator for at least an inch or two both ahead of, and behind, the brush; otherwise the brush will be ground in such a fashion that only the center part of it fits on the commutator, the toe and heel having been cut away by the sandpaper. Figure 348 shows the sandpaper improperly held and the resultant shape of the brush. The surface of such a brush actually touching the commutator surface is very small and, when a load is put on the machine, excessive heating will develop at this point. Figure 347 shows how the sandpaper should be held and also how the brush, when so ground, fits the commutator over its whole cross-section. While the brush is being ground to a final fit, the sandpaper should be drawn under the brush only in the same direction as the machine is designed to run.

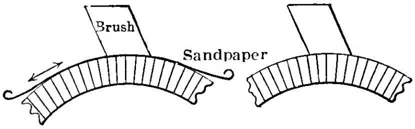


FIG. 347

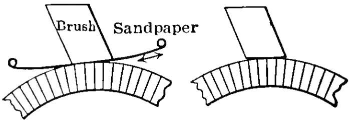


FIG. 348

FIG. 347.—How sandpaper should be used to fit the brush to the commutator.

FIG. 348.—If the sandpaper is improperly held, the brush will be ground round as shown here, making poor contact with the commutator, and causing excessive heating at the brush contact.

Use of Emery Cloth.—If emery cloth is used instead of sandpaper, two faults are likely to develop. The abrasive material used in making the rough surface of the emery cloth is a partial conductor, and small particles of it are likely to become imbedded in the insulation separating the copper bars, thus producing a partial short-circuit of the commutator. Or small particles of it may become imbedded in the contact surface of the brush and so, when the machine is in operation, may produce severe cutting (roughening) of the commutator surface, because the emery grains are extremely hard.

374. Effect of Unequal Air Gaps.—The inequality of voltage in the different paths will also occur even when the brushes are properly spaced, if the *air gap under some poles is less than under others.* As there is the same magnetizing force on all field poles (same number of turns and same current) the magnetic circuit having the smaller air gap will have the smaller reluctance and therefore the greater flux. Therefore, the inductors lying under those poles having the smaller air gaps will generate a greater e.m.f.

than the inductors lying under the poles with a larger air gap. Hence, those paths in the armature lying under the poles with the smallest air-gap length will generate a greater e.m.f. than the other paths, and so current may flow from one path to the other in such an armature, even though no load is being supplied by the machine.

The inequality in air gaps is also likely to produce heating in the bearings. That pole under which the magnetic field is most dense will exert a greater pull on the armature core than the others, resulting in excessive pressure on the bearing and thus tending to produce a hot bearing.

Development of Unequal Air Gap by Wear of Bearings.—This inequality in air gap may not exist when the machine is sent out from the factory but may develop if the bearings are allowed to wear down to an appreciable extent. This allows the armature to drop slightly, shortening the air gap for the magnetic circuits on the lower side of the armature and lengthening it for those above.

This same difficulty occurs if one field coil becomes short-circuited, from any cause. The inductors under the pole with the short-circuited coil will generate practically no voltage.

375. Faults Occurring in Electrical Machinery.—The difficulties which may be encountered in the operation of electrical machinery are tabulated below. The reasons for their occurrence and the means of remedying them will be explained later, with reference to the table on page 380.

1. A rough or dirty commutator always produces sparking at the brushes. The causes for the roughening of a commutator have been given before. When the roughening exists to a slight extent only, it may be remedied by polishing the surface of the commutator with sandpaper. If the machine is a motor, all brushes but one pair should be lifted out of the holders; the one pair left in contact with the commutator will carry enough current to run the motor with no load.

The motor is then run at a low speed and the surface first smoothed by coarse sandpaper held on the commutator surface by a block of wood, the face of which is perfectly flat. The sandpaper is moved slowly back and forth across the commutator as it revolves and this process is continued until all rough spots have been removed. Then finer sandpaper may be used in a similar fashion, to polish the surface; finally, a commutator stone should be used, or else fine sandpaper may be used with a small quantity of machine oil; this will produce a high polish. The commutator is then cleaned with a piece of rag and all copper dust is blown from the brushes and brush holders by a bellows. The windings and commutator connections must also be thoroughly cleaned out, as the copper dust is likely to produce short circuits.

Faults likely to occur in the operation of c.c machinery

- A. Sparking at the commutator
  - 1. Rough surface
  - 2. Brushes in the wrong position
  - 3. Insufficient brush tension
  - 4. Poorly fitted brushes
  - 5. Armature overloaded
  - 6. Vibration of the machine
  - 7. Short-circuited coil
  - 8. Open-circuited coil
  - 9. Unequal brush spacing
  - 10. Unequal air gaps
  - 11. Poor design
- B. Heating
  - 12. Bearings
  - 13. Commutator
  - 14. Armature
  - 15. Field coils
- C. Generator fails to build up
  - 16. Open field-circuit
  - 17. High resistance in the field rheostat
  - 18. Dirty commutator
  - 19. Wrong position of the brushes
  - 20. No residual magnetism
  - 21. Field connected incorrectly to the armature
  - 22. Speed low
- D. Motor fails to start
  - 23. Supply line dead
  - 24. Fuses blown
  - 25. Field or armature circuit open
  - 26. Too much resistance in the field rheostat
  - 27. Too much starting torque required by the load

If the commutator is too rough to be smoothed by sandpaper, a special turning tool is generally used. The tool holder fits on one of the brush-holder studs, and the commutator is turned down just as though it were in a lathe. (See Fig. 349.) Small armatures are generally removed from the field frame and swung in a lathe for turning the commutator.

2. If the brushes are in the wrong position they will all spark, but the difficulty is easily overcome by manipulation of the brush yoke rocker. On some machines, not equipped with commutating poles, the brush position must continually be changed as the load changes if sparkless commutation is desired.

3. The brush tension should be between 1 and $1\frac{1}{2}$ pounds per square inch of contact area. If the tension is not enough, the brush will not "follow" the commutator, and if the tension is too great, the commutator heats because of excessive loss due to friction. Loosen or tighten the springs used for holding the brush on the commutator.

4. These result from the unskillful fitting of the brushes, or by a faulty

brush having hard spots in it and so wearing unevenly. This trouble may be detected by a close examination of the wearing surface of the brush, after the brush has been taken from the holder. The surface should be smooth and polished all over; if the wearing surface is polished only in spots, it indicates a non-homogeneous brush, and a new one should be substituted.

5. A machine not equipped with commutating poles will always spark when overloaded, while a commutating-pole machine will not do so unless the overhead is very excessive. In the case of a station generator, inspection of the switchboard ammeter will locate this trouble, and if the meter indicates an overload, part of the load should be taken off.

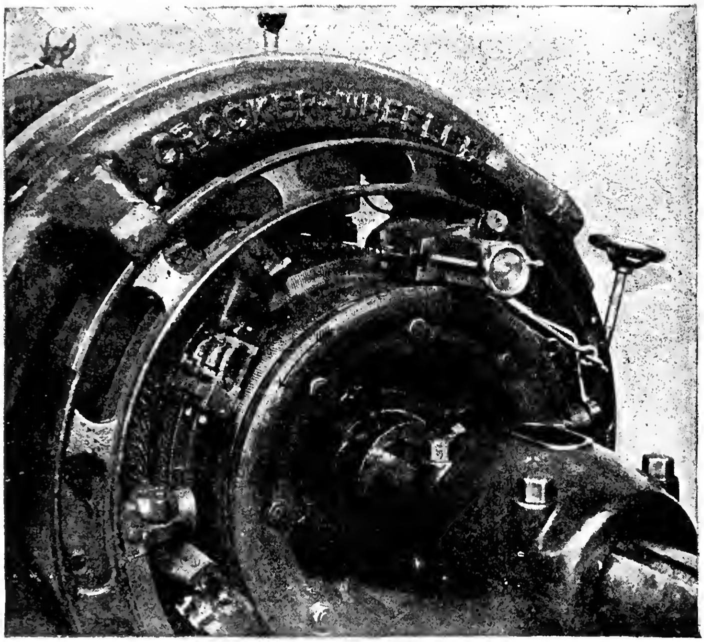


Fig. 349.—If a commutator becomes badly worn the armature may be taken out and put in a lathe for re-finishing the commutator, if the machine is a small one; for a larger one a commutator turning tool may be mounted right on the brush rigging, as shown here.

6. If a machine vibrates to any extent, it is likely to prevent the brushes from making firm contact with the commutator surface, with the result that sparking occurs. The machine must be put on a more solid foundation and balanced better, to get rid of this trouble.

7. A short-circuited coil will always produce bad sparking at the brush contact every time the short-circuited coil passes under a brush.

Unless the trouble is remedied very soon, the short-circuited coil will be burned out. Sometimes the cause of the short-circuit is a thin copper bridge spun over the mica insulation between the two copper segments to which the ends of the coil are attached. Or the insulation may have broken down on the armature where the end connections of the coils cross each other.

8. An open-circuited coil produces very vicious sparking every time the coil passes under a brush. The commutator bars to which the faulty coil is connected soon become very badly burned and, if the machine is allowed to run any considerable time with an open-circuited coil, the bars to which it is attached burn away so badly that it becomes necessary to turn down the whole commutator before sparking can be prevented, even though the faulty coil has been repaired. The method of locating a short-circuited or open-circuited coil will be described in the last section of this chapter.

9. This trouble has been discussed on a preceding page. The brushes may be tested for even spacing by measuring accurately the distance around the commutator surface, between brushes. If this proves to be uneven, the spacing may be changed by rocking the brush holder on the brush-holder stud and re-fitting the brush.

10. The air gap may be accurately measured by a tapered steel wedge. This wedge is chalked and then pushed into the air gap as far as it will go, in a direction parallel to the shaft. The pole rubs the chalk off and so the distance the wedge was inserted can be easily determined and its width at this point measured by a micrometer caliper. This distance must be practically the same for all poles; if it is not, a thin piece of sheet iron may be inserted between the poles and yoke, where the air gap is too long. This is generally called " shimming up the pole."

11. When discussing the subject of commutation, it was shown that when the coefficient of self-induction of the armature coil was too high or the width of the brush too small or the speed of the commutator too high, it would be impossible to obtain sparkless commutation. These points are well understood by designers, so that such a cause of sparking is not likely to exist. If it does, the operator can do nothing to remove it and the machine must be turned back to the factory as being improperly designed.

12. Heating of the bearings is generally due to lack of oil, to the poor grade of oil used, or to the dirt in the bearing. The attendant should keep constant watch on all bearings, and if one begins to heat excessively the cause should immediately be discovered and removed. If this is not done, the bearing is likely to get hot enough to melt the Babbitt metal lining, and then the machine is very likely to be injured seriously. If the Babbitt metal runs out, the armature may drop enough to rub on the pole

faces and tear the binding wires loose, etc., and the shaft is likely to be scored badly where it goes through the bearing.

If the bearing begins to get too hot, it is quite probable that its temperature will continue to increase until the lining melts. This is due to the fact that the excessive heating of the box thins the oil to such an extent that it no longer affords the requisite lubrication. If a bearing is discovered dangerously hot, oil (preferably heavy, such as cylinder oil) should be poured liberally into the holes on the top of the box; the shaft should be cooled by pouring water on it, close to the bearing when possible (care being used to keep the windings dry) and the machine should be kept turning slowly.

Water should never be poured over the box, and the machine should never be allowed to stop turning until the bearing has cooled sufficiently to be at a safe temperature. If these two precautions are not heeded, the shaft is likely to "freeze" in the bearing; i.e., the bearing metal solidifies tight to the shaft just as solder does in a soldered joint. When this happens, it takes a skillful mechanic to get it off; it should not be attempted by the machine operator, as the shaft is likely to be spoiled by scratching and scoring in removing the box.

13. If a commutator is allowed to become too hot, all of the coil connections are likely to be melted loose from the commutator bars. The over-heating of a commutator may be due to excessive brush pressure, sparking at brushes, too high a current density in the brush-contact surface, or improper lubrication.

If too high a current density is used in the brushes, either larger brushes must be employed or the rating of the machine must be decreased. A commutator often gets hot because of insufficient lubrication at the brush-contact surface; most brushes are made self-lubricating, but a little machine oil applied to the surface of the commutator with a piece of rag often helps to keep the commutator in good condition, and stops heating and sparking. All superfluous oil must be carefully removed from the commutator surface by a clean rag, or it will soon result in a dirty commutator and, therefore, in sparking.

14. The armature may become too hot either because of excessive core loss or excessive I^2R loss. If the first cause exists, the core has been improperly constructed, or else the machine is operating at a higher voltage than that for which it was designed. If the armature heats because of a high I^2R loss, there may be a short-circuited coil, or else the machine may be carrying too much load. In either case the remedy is evident.

15. Excessive heating of the field coils is likely to occur only if they are forced to carry more current than that for which they were designed. One field coil may become short-circuited and thus cut out of the circuit;

the rest of the coils will then carry more than the normal current. This fault very seldom occurs.

16. A self-exciting generator often fails to build up when started, but the cause generally can easily be located. If the machine fails to build up, the first thing to examine is the connection of the shunt field. It must, of course, be properly connected to the armature circuit. It may be that, even though the field circuit is properly connected to the armature, there is an open circuit somewhere. One of the connections between the various coils may be open. A bell-ringing magneto may be used to see if the field circuit is open, or a test may be made with any c.c. power line available; an incandescent lamp, connected to a 110-volt line through the field circuit to be tested, will light if there is no open circuit, but will not burn if the field is open somewhere. When this test is made, the field must be disconnected from the armature, otherwise the armature forms a short-circuit for the field.

Of course, the generator cannot build up if the field circuit is open, as its magnetic circuit cannot become excited.

17. There is always a rheostat in the shunt field of generator, and if this rheostat is turned to the " all in " position the generator will probably refuse to build up. This difficulty was discussed in section 221.

18. A dirty commutator produces the same effects as the field rheostat resistance. The high contact-resistance of the brushes on the commutator (due to the dirty surface) acts just the same as a high resistance in the field rheostat. The commutator may be cleaned by holding a piece of oily rag on the revolving commutator; the oil will generally loosen the sticky, black coating which forms on a commutator surface. The commutator should always be cleaned after oil is applied to its surface.

19. The brushes may have been disturbed and then set in the wrong position, with the result that, even if the field were excited, there would be no voltage on the armature terminals. The brush should rest on that commutator bar to which is connected the coil which lies in the interpolar space; whether or not it does so may generally be ascertained by tracing the end connections of the coils.

20. In the discussion of the " building up " of a self-exciting generator, it was shown that some residual magnetism is necessary to start the process. If, owing to some jarring, reversed current, or similar cause, the field frame has lost its residual magnetism, it is necessary to re-magnetize the field by supplying current to the field coils from some outside source. A few primary cells will generally be sufficient to give the requisite amount of residual magnetism. Before the cells are connected to the shunt-field circuit, the field must be disconnected from the armature or all the positive or negative brushes lifted from the commutator; otherwise the low-resistance armature forms a short-circuit for the field, and the cur-

rent from the cells, instead of flowing through the field, practically all goes through the armature.

21. This difficulty is only apt to occur in a laboratory, or after a generator has been reconnected for some reason or other. It was shown, in discussing the "building-up" of a self-excited generator that, with a given direction of rotation, there is a correct and a wrong order of connections of the field to the armature; when the field is incorrectly connected to the armature, what small current is sent through the field circuit, by the voltage due to residual magnetism, tends to decrease, instead of increase, the field magnetism. In this case, the connections of the field circuit to the armature must be reversed.

To test for this difficulty, the voltage of the machine should be noted with the field circuit open. If, on closing the field circuit, the voltage drops towards zero, it is probable that the field connections are reversed, although a dirty commutator will sometimes also give this same result.

In case the residual has reversed for some reason or other, with correct field connections, a generator will build up with reversed polarity, as shown in section 220. In practice, if it becomes necessary to reverse the residual magnetism, a few dry cells will generally suffice. The shunt field is disconnected from the armature, at one terminal (or all the positive or negative brushes raised), and the dry cells (perhaps half a dozen) are connected to the field circuit for a few moments. Then the cells may be disconnected, the field connected again to the armature (or brushes let back on commutator) and the generator may start to build up with the correct polarity. In case it does not, the operation should be repeated, but the cells should be connected to the field with polarity opposite to that first used.

If the residual of a generator, operated in parallel with others, should reverse, it may easily be restored by lifting all its positive or negative brushes and then closing all the switches connecting the machine to the bus-bars. If the bus-bars are excited in the proper polarity by another generator, field current with the proper direction will flow, and thus reverse the residual, in the first generator. The bus-bar switches should, of course, be opened before the brushes are dropped back on the commutator.

22. If a generator is running at a speed very much lower than rated speed, it is likely to refuse to "build up." The speed should be increased until the rated value is reached.

23. When a motor fails to start, the first thing to determine is whether or not the supply line is alive. This is readily tested by a voltmeter or by an incandescent lamp on a 110- or 220-volt circuit. On a 500-volt circuit, several incandescent lamps must be connected in series with each other and then connected across the line to test for voltage if lamps are to be used.

24. All motor circuits should be fused and it may, of course, be that the fuses are blown. This may be determined by an inspection of the indicator on an enclosed fuse or by testing the fuse in series with an incandescent lamp across the line. If the fuse is blown, of course, the lamp will not burn.

25. Either the field or the armature circuit may be open. The field circuit may be tested by holding the starting rheostat lever on the first contact button and, with the armature stationary, testing for magnetism on the pole shoes with a knife, keys, or similar article. If the field has no magnetism, it shows that the field circuit is open (provided the line is alive) and this open circuit must be located and removed.

The armature circuit may be open in the armature itself or in the starting box. The open circuit may be found with a bell-ringing magneto, by trying to ring through the different parts of the circuit.

26. If all of the resistance is "cut in" on the field rheostat, it may be that the torque the motor can develop with the weak field is not sufficient to start the load. The field rheostat should be turned to the "all out" position.

27. It may be, in some cases, that the starting torque demanded of a motor is greater than the motor can exert. On factory loads, where a great deal of belting is used, this is especially likely to be true. In such a case, some of the belting must be disconnected from the motor, by a releasing clutch or similar device, until the motor is up to its normal speed. If this is not possible, a larger motor must be installed.

376. Location of Armature Faults.—The three faults which are most likely to occur in an armature circuit are due to *short-circuits*, *open-circuits*, and *grounds*. An armature winding consists of a series of similar coils, all joined in series and insulated from the armature core. If the winding becomes electrically connected to the core at any place, by abrasion or the cutting of the insulation on the coils, the winding is said to be *grounded*. One ground in an armature winding does not generally interfere with the electrical operation of the machine, but the iron frame of the machine, being electrically connected to the armature winding, may give a fatal shock to an operator coming in contact with it.

The frame of any electric machine should be well insulated from the winding, and it is advisable to actually connect the framework of the machine to a ground connection, such as a waterpipe. Then the operator will not receive a shock upon coming in contact with the frame of the machine, even though the winding should become grounded on the iron frame of the machine.

The location of a short-circuited coil, open-circuited coil or ground is easily carried out with the help of very simple apparatus.

377. To Locate a Short-circuited or Open-circuited Coil.—Remove all the brushes from the commutator, with the exception of one pair. Connect a dry cell to the pair of brushes, as in Fig. 350. Then, with a low reading voltmeter (the full range of which is somewhat greater than the e.m.f. of the dry cell), read the IR drop between every adjacent pair of commutator segments. If there are no short-circuited coils or open-circuited coils the IR drops in the various coils will be the same. Of course, if there are more coils in one path of the armature winding than in the other, the drop per coil will be different. *But all the coils in series with each other in one path should have the same drop.* If a short-circuited coil is present it will be indicated by a zero IR drop, or, at least, by an IR drop much smaller than that of the other coils. An open-circuited coil is indicated by a zero IR drop across all of the coils in the same armature path, except the open-circuited coil, and the full voltage of the cell across the coil which is open-circuited.

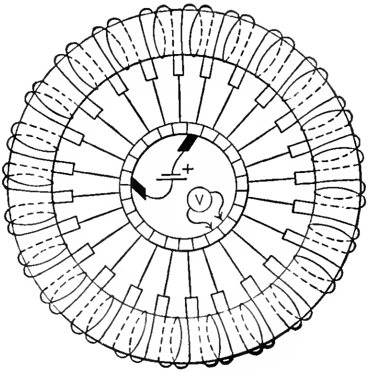


Fig. 350.—A battery and low-reading voltmeter may be used for locating open- or short-circuited coils in an armature.

After all coils have been tested as far as possible, the armature should be revolved slightly so that the coils connected to the commutator segments under the brushes may be tested.

As has been mentioned before, an open-circuited coil, on an armature which has been in service, can generally be located by the burned condition of those commutator bars to which it is connected.

378. Test for a Grounded Winding.—A test to see whether or not the winding is grounded may be made by connecting one side of a 110-volt line, through a voltmeter, to the shaft of the armature, and connecting the other side of the line to any bar of the commutator. If the meter gives an appreciable reading, it indicates a ground in the winding, because if the winding were perfectly insulated no current could flow through the voltmeter to make it indicate. Whether or not the winding is badly grounded is determined by the magnitude of the reading. If the meter gives a high reading, the winding is poorly insulated; a low reading signifies a well-insulated winding. A well-insulated armature should give, with the ordinary c.c. voltmeter, less than one volt when such a test is made. If the magnitude of the reading is the same as if the meter were connected directly to the 110-volt line, it indicates that there is a bad ground somewhere in the winding, a ground so bad that no insulation at all exists between the armature circuit and the core.

379. To Locate a Grounded Coil.—To locate the grounded coil of an armature, all brushes are removed but one pair, as for the previous test. The dry cell is connected to the brushes as before, and one terminal of the voltmeter is connected to the shaft of the armature and the other is connected in turn to each of the commutator segments, as indicated in Fig. 351. If there is no ground, the readings will all be practically zero, while if a ground exists, the voltmeter will give continually varying readings, it being practically zero on one of the segments. This segment is connected directly to the grounded coil.

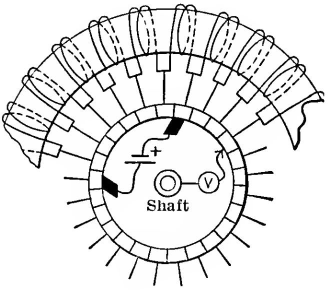


FIG. 351.—To locate grounds in an armature winding this connection scheme may be used. In general it is preferable to have diametrically placed brushes on the commutator, to increase the accuracy of the test.

Real Ground and Phantom Ground.—Now, it will be found that *each path indicates a grounded coil;* one of them is called a *phantom ground.* The two paths in the armature being in parallel, if there is a point in one path which has the same potential as the armature shaft (indicated by a zero reading of the voltmeter), there must be in the other path a corresponding point of zero potential which also will give a zero reading on the voltmeter.

The two segments giving zero reading are marked with chalk, and then the armature is rotated a fraction of a revolution and the test repeated. Two apparent grounds will again be detected, one in each path; but it will be noticed that one of the segments is the one on which a ground was detected before, while the other ground is now on a different segment from the one on which it was. That ground which persists on the same commutator bar is the only real ground; the one which shifts from one segment to another as the armature is revolved is a phantom ground only, as the winding is really not grounded in this place at all. As the armature is turned in several positions and tested each time, it will be noticed that the phantom ground moves on the structure in the opposite direction to that in which the armature has been moved, while the real ground turns just as the armature is turned.

380. Use of a Bell-buzzer for Making Tests.—Instead of using a voltmeter for these tests, a telephone receiver may be employed if the dry cell is connected to the brushes through a bell-buzzer. This is a very convenient way of testing, as a suitable low-reading voltmeter is not aways available.

381. Repairing an Armature.—Repairing a faulty armature generally requires considerable skill, and should not be attempted by the average operator. A repairman from the factory should be employed or, if the armature is a small one, it may be shipped back to the factory for repairs.

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